Jordan Left Derivation and Jordan Left Centralizer of Skew Matrix Rings

A.H.Majeed* & **Rajaa .C.Shaheen****

*Department of Mathematics,College of Science,Baghdad University.

** Department of Mathematics ,College of Education ,Al-Qadisiyah University.

Abstract

In this paper, we determine the form of Jordan left derivation and Jordan Left Centralizers of a Skew matrix ring $M_2(R;\sigma,q)$ over a ring R .

Keywords: Skew matrix ring, Jordan Left Centralizer, Jordan left derivation

1-Introduction

Let R be a ring . An additive mapping $D: R \rightarrow R$ is said to be a derivation (resp.,Jordan derivation)if D(xy)= D(x)y + x D(y) for all x,y∈R (if $D(x^2) = D(x)x + xD(x)$ for all x∈R.An additive mapping $D: R \rightarrow R$ is said to be a left derivation (resp.,Jordan left derivation)if D(xy)=xD(y)+yD(x) for all x,y∈R (if $D(x^2) = 2xD(x)$ for all x∈R.the concept of left derivation and Jordan left derivation were introduced by Bresar and Vukman in [1] .For result concerning Jordan left derivations we refer the readers to [2,3 ,4,5].An additive mapping $T: R \rightarrow R$ is called left centralizer(resp., Jordan left centralizer)if T(xy)=T(x)y for all x,y∈R (resp., $T(x^2) = T(x)x$ .An additive mapping $T: R \rightarrow R$ is called a Jordan centralizer if T satisfies T(xy+yx)=T(x)y+yT(x)=T(y)x+xT(y) for all x,y∈R.For result concerning left centralizer we refer the reader to [6,7,8].In [9],Hamaguchi ,give a necessary and sufficient condition for a given mapping J of a skew matrix ring $M_2(R;\sigma,q)$ into itself to be a Jordan derivation also show that there are many Jordan derivations of $M_2(R;\sigma,q)$ which are not derivations and refer to the properties of Jordan derivations of $M_2(R)$,and derivations of $M_2(R;\sigma,q)$. Also the author consider invariant ideal with respect to these derivations .In this paper, we determine the form of Jordan left derivation and Jordan Left Centralizers of a Skew matrix ring $M_2(R;\sigma,q)$ over a ring R .Now, we shall recall the definitions of Skew matrix ring which is basic in this paper .

Definition 1.1 :-[10] Skew Matrix ring

Let R be a ring ,q an element in R and σ an endomorphism of R such that

$\sigma(q) = q$ and $\sigma(r)q = qr\ \forall r \in R$.Let $M_2(R;\sigma,q)$be the set of 2×2 matrices over R with usual addition and the following multiplication

$$\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} \begin{bmatrix} y_1 & y_2 \\ y_3 & y_4 \end{bmatrix} = \begin{bmatrix} x_1y_1 + x_2y_3q & x_1y_2 + x_2y_4 \\ x_3\sigma(y_1) + x_4y_3 & x_3\,\sigma(y_2)q + x_4y_4 \end{bmatrix}$$

$M_2(R;\sigma,q)$ is called a skew matrix ring over R .

We should mentioned the reader that a matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is denoted by $e_{11}\,a + e_{12}b + e_{21}c + e_{22}d$.

2-**Jordan Left Derivation of Skew Matrix Rings**

In this section ,we shall determined the form of Jordan left derivation of skew matrix ring .Let J be a Jordan left derivation of $M_2(R;\sigma,q)$ .First ,we set

$$J(e_{11}\,a)=\begin{bmatrix} f_1(a) & f_2(a) \\ f_3(a) & f_4(a) \end{bmatrix},J(e_{12}\,b)=\begin{bmatrix} h_1(b) & h_2(b) \\ h_3(b) & h_4(b) \end{bmatrix}$$

$$J(e_{21}c)=\begin{bmatrix} l_1(c) & l_2(c) \\ l_3(c) & l_4(c) \end{bmatrix}, J(e_{22}\,d)=\begin{bmatrix} g_1(d) & g_2(d) \\ g_3(d) & g_4(d) \end{bmatrix}$$

Where $f_i, h_i, g_i, l_i\colon R \to R$ are additive mapping .

Lemma2.1:- For any $a \in R$

1. f_1, f_2 are Jordan left derivations of R .
2. $f_3(a^2) = 0$
3. $f_4(a^2) = 0$

Proof:-Since

$$J(e_{11}a^2) = 2e_{11}aJ(e_{11}a)$$

$$\begin{bmatrix} f_1(a^{2)} & f_2(a^2) \\ f_3(a^2) & f_4(a^2) \end{bmatrix}=2\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} f_1(a) & f_2(a) \\ f_3(a) & f_4(a) \end{bmatrix}$$

$$\begin{bmatrix} f_1(a^{2)} & f_2(a^2) \\ f_3(a^2) & f_4(a^2) \end{bmatrix}=\begin{bmatrix} 2af_1(a) & 2af_2(a) \\ 0 & 0 \end{bmatrix}$$

Then $f_1(a^{2)} = 2af_1(a), f_2(a^2) = 2af_2(a), f_3(a^2) = 0$ and $f_4(a^2) = 0$.
So, we get the result .

Lemma2.2 :- For any d∈R.

1. g_3, g_4 are Jordan left derivations of R .
2. $g_1(d^2) = 0$
3. $g_2(d^2) = 0$

Proof:-Since

$$J(e_{22}d^2) = 2e_{22}d\, J(e_{22}d)$$

$$\begin{bmatrix} g_1(d^2) & g_2(d^2) \\ g_3(d^2) & g_4(d^2) \end{bmatrix} = 2\begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}\begin{bmatrix} g_1(d) & g_2(d) \\ g_3(d) & g_4(d) \end{bmatrix}$$

$$\begin{bmatrix} g_1(d^2) & g_2(d^2) \\ g_3(d^2) & g_4(d^2) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2dg_3(d) & 2dg_4(d) \end{bmatrix}$$

Then $g_3(d^2) = 2dg_3(d)$, $g_4(d^2) = 2dg_4(d)$, $g_1(d^2) = 0$ and $g_2(d^2) = 0$..

Lemma 2.3 :- For any a,b∈ R

1. $h_1(ab) = 2ah_1(b) + 2bf_3(a)q$
2. $h_2(ab) = 2ah_2(b) + 2bf_4(a)$
3. $h_3(ab) = 0$
4. $h_4(ab) = 0$

Proof:- Since

$$J(e_{12}ab) = J(e_{11}ae_{12}b + e_{12}be_{11}a)$$

$$\begin{bmatrix} h_1(ab) & h_2(ab) \\ h_3(ab) & h_4(ab) \end{bmatrix} = 2e_{11}aJ(e_{12}b) + 2\, e_{12}b\ \ J(e_{11}a)$$

$$= 2\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} h_1(b) & h_2(b) \\ h_3(b) & h_4(b) \end{bmatrix} + 2\begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}\begin{bmatrix} f_1(a) & f_2(a) \\ f_3(a) & f_4(a) \end{bmatrix}$$

$$= \begin{bmatrix} 2a\, h_1(b) & 2a\, h_2(b) \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2b\, f_3(a)q & 2b\, f_4(a) \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} h_1(ab) & h_2(ab) \\ h_3(ab) & h_4(ab) \end{bmatrix} = \begin{bmatrix} 2a\, h_1(b) + 2b\, f_3(a)q & 2a\, h_2(b) + 2b\, f_4(a) \\ 0 & 0 \end{bmatrix}.$$

Then ,we get the result .

Lemma 2.4 :-for any c,d∈ R

1. $l_1(dc) = 0$

2. $l_2(dc) = 0$

3. $l_3(dc) = 2dl_3(c) + 2c\sigma(g_1(d))$

4. $l_4(dc) = 2dl_4(c) + 2c\sigma(g_2(d))q$

Proof:-Since

$$J(e_{21}\,dc) = J(e_{22}\,de_{21}c + e_{21}ce_{22}\,d)$$

$$\begin{bmatrix} l_1(dc) & l_2(dc) \\ l_3(dc) & l_4(dc) \end{bmatrix} = 2e_{22}\,dJ(e_{21}c) + 2e_{21}cJ\,(e_{22}\,d)$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 2d \end{bmatrix}\begin{bmatrix} l_1(c) & l_2(c) \\ l_3(c) & l_4(c) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 2c & 0 \end{bmatrix}\begin{bmatrix} g_1(d) & g_2(d) \\ g_3(d) & g_4(d) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 2dl_3(c) & 2dl_4(c) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 2c\sigma(g_1(d)) & 2c\sigma(g_2(d))q \end{bmatrix}$$

$$\begin{bmatrix} l_1(dc) & l_2(dc) \\ l_3(dc) & l_4(dc) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2dl_3(c) + 2c\sigma(g_1(d)) & 2dl_4(c) + 2c\sigma(g_2(d))q \end{bmatrix}.$$

Then ,we get the result .

Theorem 2.5 :-Let R be a ring and J be a Jordan left derivation of $M_2(R;\sigma,q)$. Then

$$J\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} f_1(a) + h_1(b) + l_1(c) + g_1(d) & f_2(a) + h_2(b) + l_2(c) + g_2(d) \\ f_3(a) + h_3(b) + l_3(c) + g_3(d) & f_4(a) + h_4(b) + l_4(c) + g_4(d) \end{bmatrix},$$

such that

1. $f_3(a^2) = 0$,$f_4(a^2) = 0$,f_1, f_2 are Jordan left derivations of R.

2. $g_1(d^2) = 0, g_2(d^2) = 0$ g_3 and $g_4$are Jordan left derivations of R.

3. $h_1(ab) = 2a\,h_1(b) + 2b\,f_3(a)q$,$h_2(ab) = 2a\,h_2(b) + 2b\,f_4(a)$

$h_3(ab) = 0$and $h_4(ab) = 0$

4. $l_1(dc) = 0, l_2(dc) = 0$,$l_3(dc) = 2dl_3(c) + 2c\sigma(g_1(d))$and

$l_4(dc) = 2dl_4(c) + 2c\sigma\big(g_2(d)\big)q$.

Proof:-Since $J\begin{bmatrix} a & b \\ c & d \end{bmatrix} = J(e_{11}\,a) + J(e_{12}\,b) + J(e_{21}c) + J(e_{22}\,d)$

$$=\begin{bmatrix} f_1(a) & f_2(a) \\ f_3(a) & f_4(a) \end{bmatrix} + \begin{bmatrix} h_1(b) & h_2(b) \\ h_3(b) & h_4(b) \end{bmatrix} + \begin{bmatrix} l_1(c) & l_2(c) \\ l_3(c) & l_4(c) \end{bmatrix} + \begin{bmatrix} g_1(d) & g_2(d) \\ g_3(d) & g_4(d) \end{bmatrix}$$

$$=\begin{bmatrix} f_1(a)+h_1(b)+l_1(c)+g_1(d) & f_2(a)+h_2(b)+l_2(c)+g_2(d) \\ f_3(a)+h_3(b)+l_3(c)+g_3(d) & f_4(a)+h_4(b)+l_4(c)+g_4(d) \end{bmatrix}$$

By[Lemma 2.1], [Lemma 2.2] ,[Lemma 2.3]and [Lemma 2.4] we get the result .

3-Jordan Left Centralizer of Skew Matrix Rings

In this section ,we shall determined the form of Jordan left centralizer of skew matrix ring. Let J be a Jordan Left Centralizer of $M_2(R;\sigma,q)$ .First ,we set

$$J(e_{11}\, a)= \begin{bmatrix} f_1(a) & f_2(a) \\ f_3(a) & f_4(a) \end{bmatrix} ,J(e_{12}\, b)= \begin{bmatrix} h_1(b) & h_2(b) \\ h_3(b) & h_4(b) \end{bmatrix}$$

$$J(e_{21} c)= \begin{bmatrix} l_1(c) & l_2(c) \\ l_3(c) & l_4(c) \end{bmatrix} ,J(e_{22}\, d)= \begin{bmatrix} g_1(d) & g_2(d) \\ g_3(d) & g_4(d) \end{bmatrix}$$

Where $f_i, h_i, g_i, l_i: R \to R$ are additive mapping.

Lemma 3.1 :- For any a∈R

1. $f_1$is Jordan left centralizer of R .
2. $f_2(a^2) = 0$
3. $f_3(a^2) = f_3(a)\sigma(a)$
4. $f_4(a^2) = 0$.

Proof:- Since $J(e_{11}a^2) = J(e_{11}a)e_{11}a$

$$\begin{bmatrix} f_1(a^2) & f_2(a^2) \\ f_3(a^2) & f_4(a^2) \end{bmatrix} = \begin{bmatrix} f_1(a) & f_2(a) \\ f_3(a) & f_4(a) \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} f_1(a^2) & f_2(a^2) \\ f_3(a^2) & f_4(a^2) \end{bmatrix} = \begin{bmatrix} f_1(a)a & 0 \\ f_3(a)\sigma(a) & 0 \end{bmatrix}$$

Then $f_1(a^2) = f_1(a)a, f_2(a^2) = 0, f_3(a^2) = f_3(a)\sigma(a)$and $f_4(a^2) = 0$.
So ,we get the result .

Lemma3.2 :- For any d∈R

1. $g_2, g_4$are Jordan left centralizers of R .
2. $g_1(d^2) = 0$
3. $g_3(d^2) = 0$

Proof:-Since

$$J(e_{22}d^2) = J(e_{22}d)\, e_{22}d$$

$$\begin{bmatrix} g_1(d^2) & g_2(d^2) \\ g_3(d^2) & g_4(d^2) \end{bmatrix} = \begin{bmatrix} g_1(d) & g_2(d) \\ g_3(d) & g_4(d) \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}$$

$$\begin{bmatrix} g_1(d^2) & g_2(d^2) \\ g_3(d^2) & g_4(d^2) \end{bmatrix} = \begin{bmatrix} 0 & g_2(d)d \\ 0 & g_4(d)d \end{bmatrix}$$

Then $g_1(d^2) = 0, g_3(d^2) = 0$&$g_2, g_4$are Jordan left centralizers of R**.**

Lemma3.3 :- For any a,b∈ R

1. $h_1(ab) = h_1(b)a$
2. $h_2(ab) = f_1(a)b$
3. $h_3(ab) = h_3\,(b)\sigma(a)$
4. $h_4(ab) = f_3(a)\sigma(b)q$

Proof:-Since $J(e_{12}ab) = J(e_{11}ae_{12}b + e_{12}be_{11}a)$

$$\begin{bmatrix} h_1(ab) & h_2(ab) \\ h_3(ab) & h_4(ab) \end{bmatrix} = J(e_{11}a)e_{12}b + J(\,e_{12}b\,)\,e_{11}a$$

$$= \begin{bmatrix} f_1(a) & f_2(a) \\ f_3(a) & f_4(a) \end{bmatrix} \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} h_1(b) & h_2(b) \\ h_3(b) & h_4(b) \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & f_1(a)b \\ 0 & f_3(a)\sigma(b)q \end{bmatrix} + \begin{bmatrix} h_1(b)a & 0 \\ h_3\,(b)\sigma(a) & 0 \end{bmatrix}$$

$$\begin{bmatrix} h_1(ab) & h_2(ab) \\ h_3(ab) & h_4(ab) \end{bmatrix} = \begin{bmatrix} h_1(b)a & f_1(a)b \\ h_3\,(b)\sigma(a) & f_3(a)\sigma(b)q \end{bmatrix},$$

then $h_1(ab) = h_1(b)a, h_2(ab) = f_1(a)b, h_3(ab) = h_3\,(b)\sigma(a)$ and

$h_4(ab) = f_3(a)\sigma(b)q$, so, we get the result .

Lemma 3.4 :-For any c ,d∈ R

1. $l_1(dc) = g_2(d)\,cq$
2. $l_2(dc) = l_2(c)d$
3. $l_3(dc) = g_4(d)c$
4. $l_4(dc) = l_4(c)d$

Proof:-Since

$$J(e_{21}\,dc) = J(e_{22}\,de_{21}c + e_{21}ce_{22}\,d)$$

$$\begin{bmatrix} l_1(dc) & l_2(dc) \\ l_3(dc) & l_4(dc) \end{bmatrix} = J(e_{22}\, d)e_{21} c + J(e_{21} c)e_{22}\, d$$

$$= \begin{bmatrix} g_1(d) & g_2(d) \\ g_3(d) & g_4(d) \end{bmatrix} \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix} + \begin{bmatrix} l_1(c) & l_2(c) \\ l_3(c) & l_4(c) \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}$$

$$= \begin{bmatrix} g_2(d)\, cq & 0 \\ g_4(d)\, c & 0 \end{bmatrix} + \begin{bmatrix} 0 & l_2(c)d \\ 0 & l_4(c)d \end{bmatrix}$$

$$= \begin{bmatrix} g_2(d)\, cq & l_2(c)d \\ g_4(d)\, c & l_4(c)d \end{bmatrix}$$

$l_1(dc) = g_2(d)cq$, $l_2(dc) = l_2(c)d$

$l_3(dc) = g_4(d)\, c$, $l_4(dc) = l_4(c)d$

Theorem 3.5:- Let R be a ring and J be a Jordan left centralizer of $M_2(R; \sigma, q)$ Then

$$J \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} f_1(a) + h_1(b) + l_1(c) + g_1(d) & f_2(a) + h_2(b) + l_2(c) + g_2(d) \\ f_3(a) + h_3(b) + l_3(c) + g_3(d) & f_4(a) + h_4(b) + l_4(c) + g_4(d) \end{bmatrix},$$

Such that

1. f_1 is Jordan left centralizer of R , $f_2(a^2) = 0, f_3(a^2) = f_3(a)\sigma(a)$ and $f_4(a^2) = 0$.
2. g_2, g_4 are Jordan left centralizers of R , $g_1(d^2) = 0$ and $g_3(d^2) = 0$.
3. $h_1(ab) = h_1(b)a, h_2(ab) = f_1(a)b, h_3(ab) = h_3(b)\sigma(a)$ and $h_4(ab) = f_3(a)\sigma(b)q$

4. $l_1(dc) = g_2(d)\, cq, l_2(dc) = l_2(c)d, l_3(dc) = g_4(d)c$ *and* $l_4(dc) = l_4(c)d$.

Proof:- Since $J \begin{bmatrix} a & b \\ c & d \end{bmatrix} = J(e_{11}\, a) + J(e_{12}\, b) + J(e_{21} c) + J(e_{22}\, d)$

$$= \begin{bmatrix} f_1(a) & f_2(a) \\ f_3(a) & f_4(a) \end{bmatrix} + \begin{bmatrix} h_1(b) & h_2(b) \\ h_3(b) & h_4(b) \end{bmatrix} + \begin{bmatrix} l_1(c) & l_2(c) \\ l_3(c) & l_4(c) \end{bmatrix} + \begin{bmatrix} g_1(d) & g_2(d) \\ g_3(d) & g_4(d) \end{bmatrix}$$

$$= \begin{bmatrix} f_1(a) + h_1(b) + l_1(c) + g_1(d) & f_2(a) + h_2(b) + l_2(c) + g_2(d) \\ f_3(a) + h_3(b) + l_3(c) + g_3(d) & f_4(a) + h_4(b) + l_4(c) + g_4(d) \end{bmatrix}$$

Also, by [Lemma 3.1],[Lemma 3.2],[Lemma 3.3] and [Lemma 3.4],we get the result .

Theorem 3.6 :-Let R be a ring with identity and J a Jordan Left centralizer of $M_2(R;\sigma,q)$ .Then there exist $f_2, f_4, g_1, g_3 : R \to R$ and $\propto, \beta, \acute{\alpha}, \acute{\beta}, \lambda, \acute{\lambda}, \varepsilon, \acute{\varepsilon} \in R$.

Such that

$$J(e_{11}a) = \begin{bmatrix} \varepsilon a & f_2(a) \\ \beta\sigma(a) & f_4(a) \end{bmatrix}, J(e_{12}\, b) = \begin{bmatrix} \acute{\propto} b & \varepsilon b \\ \acute{\beta}\sigma(b) & \beta\sigma(b) \end{bmatrix}$$

$$J(e_{21}c) = \begin{bmatrix} \propto cq & \acute{\lambda} c \\ \lambda c & \acute{\varepsilon} c \end{bmatrix}, J(e_{22}\, d) = \begin{bmatrix} g_1(d) & \alpha d \\ g_3(d) & \lambda d \end{bmatrix}$$

and $J\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \varepsilon a + \acute{\alpha} b + \alpha cq + g_1(d) & f_2(a) + \varepsilon b + \acute{\lambda} c + \alpha d \\ \beta\sigma(a) + \acute{\beta}\sigma(b) + \lambda c + g_3(d) & f_4(a) + \beta\sigma(b)q + \acute{\varepsilon} c + \lambda d \end{bmatrix}$

Proof:-From [Lemma 3.1,3]

$$f_3(a^2) = f_3(a)\sigma(a)$$

Replace a by a+b

$$f_3\left(a^2 + ab + ba + b^2\right) = f_3(a + b)\sigma(a + b)$$
$$f_3(ab + ba) = f_3(a)\sigma(b) + f_3(b)\sigma(a)$$

Replace b by 1,since R has identity

$$f_3(2a) = f_3(a)\sigma(1) + f_3(1)\sigma(a)$$
$$f_3(2a) = f_3(a) + f_3(1)\sigma(a)$$

Then

$$f_3(a) = f_3(1)\sigma(a)$$

Now,Let $\beta = f_3(1)$ .Then

$$f_3(a) = \beta\sigma(a)$$

But $g_2(d^2) = g_2(d)d$

$$g_2((d + c)^2) = g_2(d + c)(d + c)$$
$$g_2(dc + cd) = g_2(d)c + g_2(c)d$$

Replace c by 1

$$g_2(2d) = g_2(d) + g_2(1)d$$
$$g_2(d) = g_2(1)d$$

Let $\propto = g_2(1)$,Then

$$g_2(d) = \propto d,$$

and since $g_4(d^2) = g_4(d)d$

By the same way ,we have

$$g_4(d) = \lambda d \text{ ,where } \lambda = g_4(1)$$

Also ,from [Lemma 3.1,1]

$$f_1(a^2) = f_1(a)a$$

Then

$$f_1(a) = \varepsilon a \text{ ,where } \varepsilon = f_1(1)$$

By[Lemma 3.3,1], $h_1(ab) = h_1(b)a$

Replace b by 1 ,to get

$$h_1(a) = h_1(1)a, \text{let } \acute{\alpha} = h_1(1)$$

Then

$$h_1(a) = \acute{\alpha} a,$$

since $h_2(ab) = f_1(a)$b

If b=1 then

$$h_2(a) = f_1(a) = \varepsilon a \text{ ,and } h_3(a) = h_3(1)\sigma(a), \text{ let } \beta' = h_3(1)$$

Then

$$h_3(a) = \acute{\beta}\, \sigma(a)$$

And since $h_4(ab) = f_3(a)\sigma(b)q$

Replace a by 1

$$h_4(b) = f_3(1)\sigma(b)q$$

Since $\beta = f_3(1)$ then

$$h_4(b) = \beta\ \sigma(b)q$$

Now ,by [Lemma 3.4,1]

$$l_1(dc) = g_2(d)cq$$

$l_1(dc) = \alpha dcq$,Replace c by 1

$$l_1(d) = \alpha dq$$

Since $l_2(dc) = l_2(c)d$,Replace c by 1

$$l_2(d) = l_2(1)d, \text{ let } \acute{\lambda} = l_2(1)$$

$l_2(d) = \acute{\lambda} d$,and since $l_3(dc) = g_4(d)\, c$,

Then

$$l_3(dc) = \lambda d\, c.$$

Replace c by 1 ,to get

$$l_3(d) = \lambda d$$

Now ,since $l_4(dc) = l_4(c)d$,Replace c by 1 ,to get

$$l_4(d) = l_4(1)d, \text{Let } \acute{\varepsilon} = l_4(1)$$

Then

$$l_4(d) = \acute{\varepsilon} d,$$

and since

$$J\begin{bmatrix} a & b \\ c & d \end{bmatrix} = J(e_{11}\, a) + J(e_{12}\, b) + J(e_{21} c) + J(e_{22}\, d)$$

$$=\begin{bmatrix} f_1(a) & f_2(a) \\ f_3(a) & f_4(a) \end{bmatrix} + \begin{bmatrix} h_1(b) & h_2(b) \\ h_3(b) & h_4(b) \end{bmatrix} + \begin{bmatrix} l_1(c) & l_2(c) \\ l_3(c) & l_4(c) \end{bmatrix} + \begin{bmatrix} g_1(d) & g_2(d) \\ g_3(d) & g_4(d) \end{bmatrix}$$

$$=\begin{bmatrix} f_1(a)+h_1(b)+l_1(c)+g_1(d) & f_2(a)+h_2(b)+l_2(c)+g_2(d) \\ f_3(a)+h_3(b)+l_3(c)+g_3(d) & f_4(a)+h_4(b)+l_4(c)+g_4(d) \end{bmatrix}$$

Then $J\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \varepsilon a + \acute{\alpha} b + \alpha c q + g_1(d) & f_2(a) + \varepsilon b + \acute{\lambda} c + \alpha d \\ \beta\sigma(a) + \acute{\beta}\sigma(b) + \lambda c + g_3(d) & f_4(a) + \beta\sigma(b) q + \acute{\varepsilon} c + \lambda d \end{bmatrix}$

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اشتقاق جوردان اليساري وتطبيق جوردان المركزي اليساري على حلقات المصفوفات التخالفية

عبد الرحمن حميد مجيد * و رجاء جفات شاهين**
***جامعة بغداد ،كلية العلوم ،قسم الرياضيات .**
**** جامعة القادسية ،كلية التربية ،قسم الرياضيات.**

الخلاصة : في هذا البحث حددنا شكل اشتقاق جوردان اليساري وتطبيق جوردان المركزي اليساري على حلقة المصفوفات $M_2(R;\sigma,q)$ على الحلقة R.