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A Type of Alxandroff space

BY
Gorgees shaheed Mohammad
University of Al-Qadisiya, Collage of Education, Dep. of Math.

Abstract :

In this paper we introduce the concept of A_2-space (which is a strong type of Alxandroff space) and investigate some of its properties. And also introduce concept of g- A_2-closed sets and investigate some of its consequences.

1-Introduction:

In 1937 (P. Alexandroff)[2] introduced Alxandroff space (a topological space such that every point has minimal neighborhood). Recently, Pratvlanapa [6] has introduced the concept of generalized closed sets of Alxandroff space. In this paper we obtain new concept of strong type of Alxandroff space (A_2-space). We investigate various properties of this concept and obtain g-close sets in this space.

2-Preliminaries

Definition 1: An Alxandroff space (A_1-space)[5] is a set X together with system τ of subsets of X satisfying the following:-
i) the union of any countable number of sets from τ is a set in τ
ii) The intersection of a finite number of a sets τ is also in τ
iii) X and Φ are in τ
the pair (X, τ) is called A-space, the members of τ are called A-open set

Definition 2 [1]: A topological space (X,τ) is said to be compact if every open cover of it, has a finite subset

Definition 3 [1]: A topological space (X,τ) is said to be T_0 space if for any distinct points x, y in X then there exists an open set U which contain one of them but not the other.

Definition 4 [1]: A topological space (X,τ) is said to be T_1 space if for any two distant point y in X ,there exists open sets U,V such that $x \in U$ and $y \in V, y \notin U$ and $x \notin V$.

Definition 5 [3]: A topological space (X,τ) is regular if for any $x \in X$ and any closed set F such that $x \notin F$ there exist open set $U,V \in \tau$ such that $x \in U, F \subseteq V$ and $U \cap V = \Phi$.

Definition 6 [5]: A subset M of x is called generalized closed if $cl(M) \subseteq U$ whenever $M \subseteq U$ and U is open .

3- A_2-space

Definition 7: An A_2-space is a set X together with system τ of subsets satisfying the following

i) the union of any number of sets from τ is a set in τ

ii) The intersection of any set from τ is also in τ

iii) X and Φ are in τ

a set $\mu \subseteq X$ is called A_2-open if $\mu \in \tau$ and its complementing is called A_2-closed or members of τ are called A_2-closed sets their complementary sets are called open

proposition 1: in general τ is a topology but the converse is not as can be easily see by the following example

Example 1: let $X = \left(\frac{-1}{n}, \frac{1}{n}\right) \subset R$, $n \in N$ with the ratio topology (natural topology) its clear that (X,τ) is topological space but not A_2-space since the $\cap\left(\frac{-1}{n}, \frac{1}{n}\right) = \{0\}$ which is not open.

Note 1: Every A_2-space is A_1-space but the converse is not true always.

Definition 8: with every $M \subseteq X$ we associate its A_2-closure ($A_2 - \overline{M}$)the intersection of all closed sets containing M .

Proposition 2: $\overline{M}$ is A_2-closed ; $M = \overline{M}$ if M is A_2-closed

Definition 9: A space (X,τ) is said to be A_2- compact if every A_2-open cover of it has finite subset

<u>Definition</u> 10: (X,τ) is said to be $T_0 - A_2$ space if for any distinct points x, y in X then there exists an A_2-open set U which contain one of the but not the other.

<u>Definition</u> 11: (X,τ) is said to be $T_1 - A_2$ - space if for any two distant point y in X ,there exists A_2-open sets U and V such that $x \in U$ and $y \in V, y \notin U$ and $x \notin V$.

<u>Definition</u> 12: (X,τ) is regular if for any $x \in X$ and any A_2-closed set F such that $x \notin F$ there exist A_2-open set $U, V \in \tau$ such that $x \in U, F \subseteq V$ and $U \cap V = \Phi$.

<u>Definition</u> 13: let M be a sub set of (X,τ). A point $p \in M$ is called A_2-interior point of M of $p \in G \subseteq M$ where G is A_2-open, the set of A_2-interior points of M denoted by A_2-int(M).

<u>Propositions</u> 3: i)
1) The A_2-interiors of a set M is the union of all A_2-open subset of M
2) A_2-int$(M)$is A_2-open
3) if G is an A_2-open subset of M then $G \subseteq A_2 - \text{int}(M) \subseteq M$
ii) M is A_2-open if $M = A_2 - \text{int}$

<u>Theorem</u> 1: (X,τ) is $T_0 - A_2$ space if $x \neq y$ in impels $\{\overline{x}\} \neq \{\overline{y}\}$.

<u>Definition</u> 14 : A subset M of X is called $g - A_2$ closed if $A_2 - cl(M) \subseteq U$ whenever $M \subseteq U$ and U is A_2-open .

<u>proposition</u> 4: Every A_2-closed is $g - A_2$ closed the converse is not true as shown by the following example.

<u>Example</u> 2: Let $X = R - Q$ and $\tau = \{X, \phi, \cup G_i\}$ where G_i runs over all countable subset of $R - Q$ then (X,τ) is an A_2-space
let B be the set of all irrational numbers in $(0,\infty)$ then B is not A_2-closed but B is $g - A_2$ closed since x is the only A_2-open and A_2-closed set contain B.

<u>Theorem</u> 2: A set M is $g - A_2$ closed if $A_2 - cl(M) - M$ contain no non empty A_2-closed set.

Proof : let M be a $g-A_2$ closed and let F be an A_2-closed set such that $F \subseteq A_2 - cl(M) - M$ and hence $F \subseteq A_2 - cl(M)$...(1)

since F is A_2-closed then F^c is A_2-open and $M \subseteq F^c$ (since $F \subseteq M^c$) since M is a $g-A_2$ closed (by Def of $g-A_2$ closed) $A_2 - cl(M) \subseteq F^c \Rightarrow$

and this show that $F \subset (A_2 - cl(M))^c$ ...(2)

then by (1) and (2) we get $F \subseteq A_2 - cl(M) \cap (A_2 - cl(M))^c = \phi$ and hence $F = \phi$,

Conversely :- supposed that $M \subseteq U$ when every U is A_2-open in X if $A_2 - cl(M) \not\subset U$ then $A_2 - cl(M) \cap U^c \neq \phi$

since $\overline{M}$ and U^c are both A_2-closed then $A_2 - cl(M) \cap U^c$ is A_2-closed (non empty) of $(A_2 - cl(M) - M)$ but $(A_2 - cl(M) - M)$ has no non-empty on A_2-closed and this contradiction and hence $A_2 - cl(M) \subseteq U$ and then M is $g-A_2$ closed .

Corollary 1: A $g-A_2$ closed set M is A_2-closed if $(A_2 - cl(M) - M)$ is A_2-closed.

proof : $\Rightarrow$ if M is A_2-closed then M is $g-A_2$ closed and by theorem (2) $A_2 - cl(M) - M = \phi$ and hence $(A_2 - cl(M) - M)$ is A_2-closed

conversely,

$\Leftarrow$suppose that $(A_2 - cl(M) - M)$ is A_2-closed, M is $g-A_2$ closed and $(A_2 - cl(M) - M)$ is A_2-closed sub set of itself by the theorem ( ) $A_2 - cl(M) - M = \phi$ and hence $A_2 - cl(M) = M$ and so M is A_2-closed.

Theorem 3: if M and N are $g-A_2$ closed then $M \cup N$ is $g-A_2$ closed

proof : suppose that $M \cup N \subset U$ when U is A_2 − open then $M \subset U$ and $N \subset U$ since both of M and N are $g-A_2$ closed then $A_2 - cl(M) \subset U$ $A_2 - cl(N) \subset U$ we have $A_2 - cl(M) \cup A_2 - cl(N) \subseteq U$ since $A_2 - cl(M) \cup A_2 - cl(N) = A_2 - cl(M \cup N)$ then $A_2 - (cl(M \cup N)) \subset U$ and hence $M \cup N$ is $g-A_2$ closed.

Theorem 4: suppose that $N \subseteq M \subseteq X, N$ is $g-A_2$ closed so relation to M and that M is $g-A_2$ closed subset as x then N is $g-A_2$ closed relation to x.

Proof : let $N \subset U$ and suppose that U is A_2-open in x then $N \subseteq M \cap U$

$N \cap M \subseteq M \cap U$ but $M \cap U$ is A_2-open and N is $g-A_2$ closed and by hypothesis $A_2-cl(N) \cap M \subseteq M \cap U \subseteq U$

$M \cap A_2-cl(N) \subseteq U \Rightarrow M \subseteq (A_2-cl(N))^c \cup U$ and since M is $g-A_2$ closed in x and $U \cup (A_2-cl(N))^c$ is A_2-open then

$A_2-cl(M) \subseteq (A_2-cl(N))^c \cup U$,

$A_2-cl(N) \subseteq (A_2-cl(M)) \subseteq (A_2-cl(N))^c \cup U$,

$A_2-cl(N) \subseteq (A_2-cl(M)) \cap (A_2-cl(N)) \subseteq U$ and hence $A_2-cl(N) \subseteq U$.

Corollary 2: let M be a $g-A_2$ closed set and suppose that F is a A_2-closed set then $M \cap F$ is $g-A_2$ closed set.

Proof : since M is $g-A_2$ closed in x and F is A_2-closed in x then $M \cap F$ is A_2-closed in M and hence $M \cap F$ is $g-A_2$ closed in M

since $M \cap F$ is $g-A_2$ closed in M and $M \cap F \subseteq M$ and M is $g-A_2$ closed in X by theorem (4) then $M \cap F$ is $g-A_2$ closed in X .

Theorem 5: Let (X, τ) be a A_2-space and $M \subseteq X$ if M is $g-A_2$ closed and $M \subseteq F \subseteq \overline{M}$ then F is $g-A_2$-closed

proof : since F is $g-A_2$-closed, $\overline{F}-F$ has no contain non empty $g-A_2$ closed set

since $M \subseteq F \subseteq \overline{M}$

$F \subseteq \overline{M}$ since $\overline{M}$ is A_2-closed $\Rightarrow \overline{\overline{M}} = \overline{M}$

$\overline{F} \subset \overline{M}, M \subset F, F^c \subset M^c$ then $\overline{F}-F \subseteq \overline{M}-M$ and hence $\overline{F}-F$ cannot has any A_2-closed (since M is $g-A_2$ closed).

Theorem 6 : Let (X, τ) be a A_2-space if $N \subseteq M \subseteq X$ and N is $g-A_2$ closed in X then N is $g-A_2$ closed in M

Proof : $N \subset U$ when U is A_2-open in M

we most prove that $\left(\overline{N}\right)_M \subseteq U$

since U is A_2-open in M then there is A_2-open G in X such that $U = G \bigcap M$ and $N \subset U = G \bigcap M \subseteq G \Rightarrow N \subseteq G$

since G is A_2-open in X and N is $g - A_2$ closed in X then $\overline{N} \subseteq G$ and hence $\overline{N} \subseteq M \subseteq G \bigcap M = U$ but

$\left(\overline{N}\right)_M = \overline{N} \bigcap M$ then $\left(\overline{N}\right)_M \subseteq U$

and hence N is $g - A_2$ closed in M .

Note 2: Every A_2-open and $g - A_2$ closed is A_2-closed .

Corollary 3: Let $N \subset M$ where M is A_2-open and $g - A_2$ closed then N is $g - A_2$ closed relative to M if N is $g - A_2$ closed in X .

Theorem 7: In $A_2 -$space (X, τ) then $\tau = F$ (where F the set of all A_2- closed) if every subset of X is a $g - A_2$ closed set

Proof : suppose that $\tau = F$ and that $M \subset F$ where U is $A_2 -$open and M subset of X

then $\overline{M} \subset \overline{U} = U$ since U is $A_2 -$closed $\overline{M} \subset U$ and hence M is $g - A_2$ closed

conversely suppose that every subset of x is $g - A_2$ closed let $U \in \tau$

then since $M \subset U$

and U is g-closed we have $\overline{U} \subset U \Rightarrow \overline{U} = U$ and hence $U \in F$ thus $\tau \subset F$ since every $A_2 -$closed is $g - A_2$ closed and hence $F \subseteq \tau$

Definition 15 :- Let (X, τ) be a $A_2 -$space a subset M of X is called compact if every $A_2 -$open cover of its has a finite sub cover ($A_2 -$open cover if there exist $A_2 -$open set such that [$M \subset \bigcup_{i=1} U_i$].

Theorem 8: Let (X, τ) be a $A_2 -$space such that X is $A_2 -$compact and M is $g - A_2$ closed sub set of X then M is $A_2 -$compact .

Proof : Let ζ be an $A_2 -$open cover of M then $M \subset \bigcup \zeta$ and hence $\overline{M} \subset \bigcup \zeta$ since M is $g - A_2$ closed and since $\overline{M}$ is $A_2 -$closed since any closed of

A_2 − compact is A_2 − compact then $\overline{M}$ is A_2 − compact and hence there is A_2 − open sates say $U_1, U_2, ..., U_n \in \zeta$ such that $M \subset \overline{M} \subset U_1 \cup U_2, ..., U_n$ so $M \subset U_1 \cup U_2, ..., U_n$ and hence M is A_2 − compact.

Definition 16 : (X,τ) is called A_2 − regular space if for any $x \in X$ and any closed set F such that $x \notin F$ there is $U, V \in \tau$ such that $x \in U, F \subseteq V$ and $U \cap V = \phi$.

Theorem 9: let (X,τ) is a and X is regular space then if M is compact set then M is $g - A_2$ closed

Proof : suppose that $M \subset U \in \tau$, since M compact then there is an finite cover $V \in \tau$ such that $M \subset V \subset \overline{V} \subset U$ and hence $\overline{M} \subset \overline{V} \subset U$ we have M is $g - A_2$ closed.

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الخلاصة

في هذا البحث قدمنا مفهوم فضاء- A_2 وناقشنا بعض خواصه كذلك قدمنا مفهوم المجموعات المغلقة العمومية - A_2 وناقشنا بعض صفاته.