Some Generalized Sets and Mappings in IntuitionisticTopological Spaces

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ABSTRACT

In this paper we introduce new types of intuitionistic regular generalized closed set intuitionistic generalized pre regular -closed set,intuitionistic weakly generalized closed set , intuitionistic strongly generalized semi closed set,intuitionistic weakly closed set ,intuitionistic semi weakly generalized closed set,intuitionistic pre weakly generalized closed set, intuitionistic regular-weakly generalized closed set,intuitionistic regular w-closed set , intuitionistic regular generalized α-closed set and study the relations among them .Through the seconcepts we introduce a new class of mapping of intuitionistic generalized pre regular - closed map intuitionistic , weakly generalized closed map, intuitionistic strongly generalized semi closed map,intuitionistic weakly closed map ,intuitionistic semi weakly generalized closed map intuitionistic pre weakly generalized closed map, intuitionistic regular weakly generalized closed map,intuitionistic regular w-closed map , strongly $Irg\alpha$-continuous ,$Irg\alpha$-irresolute map, $Irg\alpha$-continuous map .We study and investigate some characterizations and relationships among them .

Keywords: *Intuitionistic regular generalized closed set, intuitionistic generalized pre regular -closed set ,intuitionistic weakly generalized closed set .*

INTRODUCTION

The concept of fuzzy set was introducedby Zadeh[15]in his classical paper 1965.After the discovery of the fuzzy sets much attention has been paid togeneralizethe basic concepts of classical topology in fuzzy setting and thus a subset naturally plays a very significant role in the study of fuzzy topology which was introduced by Chang 1968 [6] ,and later by Malghan and Benchalli in 1981 [10] . In 1983, Atanassov introduced the concept of " Intuitionistic fuzzy set " [1],[2],[3],[4] using a type ofgeneralized fuzzy set , Later, the concept is used to define intuitionistic fuzzy special sets byCoker [7], and intuitionistic fuzzy topological spaces are introduced by Coker [8]. In this direction,the concept of separation axioms in intuitionistic fuzzy topological spaces which was introduced byBayhan , S .and Coker,D [5] .Also concept of intuitionistic topological spaces which was introduced by Coker in 2000.[9]
In this paper and through this concepts of (intuitionistic regular generalized , intuitionistic generalized pre regular -closed setintuitionistic, weakly generalized closed set intuitionistic strongly generalizedsemi closed set intuitionistic weakly closed set , intuitionistic semi weakly generalized closed set,intuitionistic pre weakly generalized closed set, intuitionistic regular-weakly generalized closed set,intuitionistic regular w-closed set and intuitionistic regular generalized α-closed set)we introduce a new class of mappingof intuitionistic generalized pre regular -closed map intuitionistic , weakly generalized closed map, intuitionistic strongly generalizedsemi closed map,intuitionistic weakly closed map ,intuitionistic semi weakly generalized closed map,intuitionistic pre-weakly generalized closed map, intuitionistic regular-weakly generalized closed map,intuitionistic regular w-closed map , strongly $Irg\alpha$*-continuous ,*$Irg\alpha$*-irresolute map,* $Irg\alpha$*-continuous map .Also we study and investigate some characterizations and relationship among them .*

Preliminaries

Definition 1.1 [7]

Let X be a non empty set . An intuitionistic set A is an object having the form A =$\langle x, A_{1}, A_2\rangle$*,where* A_1 *and* A_2 *are subsets of X satisfying*$A_1 \cap A_2 = \emptyset$*. The set*A_1 *is called the set of members of A , while* A_2 *iscalled the set of nonmembers of A .*

Remark

Any subset A of X can be regarded as intuitionistic set having the form $\tilde{A} = \langle x, A, A^c \rangle$.

Definition 1.2[7]

Let X be a nonempty set, and let $A = \langle x, A_1, A_2 \rangle$ *and* $B = \langle x, B_1, B_2 \rangle$ *be intuitionistic sets respectively, furthermore, let* $\{ A_i ; i \in J \}$ *be an arbitrary family of intuitionistic sets in X, where* $A_i = \langle x, A_i^{(1)}, A_i^{(2)} \rangle$, *then*

1) $A \subseteq B$ *if and only if* $A_1 \subseteq B_1$ *and* $B_2 \subseteq A_2$,
2) $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$,
3) *The complement of A is denoted by* $\bar{A}$ *and defined by* $\bar{A} = \langle x, A_2, A_1 \rangle$,
4) $FA = \langle x, A_1, A_1^c \rangle$, $SA = \langle x, A_2^c, A_2 \rangle$,
5) $\cup A_i = \langle x, \cup A_i^{(1)}, \cap A_i^{(2)} \rangle$, $\cap A_i = \langle x, \cap A_i^{(1)}, \cup A_i^{(2)} \rangle$,
6) $\tilde{\Phi} = \langle x, \emptyset, X \rangle$, $\tilde{X} = \langle x, X, \emptyset \rangle$.

Definition 1.3 [7]

Let X be a nonempty set, $p \in X$ *a fixed element in X, and let* $A = \langle x, A_1, A_2 \rangle$ *be an intuitionistic set (IS, for short). The IS* $\dot{p}$ *defined by* $\dot{p} = \langle x, \{p\}, \{p\}^c \rangle$ *is called an intuitionistic point* $(IP\ for\ short)$ *in X. The IS* $\ddot{p} = \langle x, \emptyset, \{p\}^c \rangle$ *is called a vanishing intuitionistic point (VIP, for short) in X. The IS* $\dot{p}$ *is said to be contained in A* $(\dot{p} \in A, for\ short)$ *if and only if* $p \in A_1$, *and similarly IS* $\ddot{p}$ *contained in A.* $(\ddot{p} \in A, for\ short)$ *if and only if* $p \notin A_2$ *For a given IS A in X, we may write* $A = (\cup \{\dot{p} : \dot{p} \in A\}) \cup (\cup \{\ddot{p} : \ddot{p} \in A\})$, *an whenever A is not a proper IS (i.e., if A is not of the form* $A = \langle x, A_1, A_2 \rangle$ *where* $A_1 \cup A_2 \neq X$ *), then* $A = \cup \{\dot{p} : \dot{p} \in A\}$ *hold. In general, any IS A in X can be written in the form* $A = \dot{A} \cup \ddot{A}$ *where* $\dot{A} = \cup \{\dot{p} : \dot{p} \in A\}$ *and* $\ddot{A} = \cup \{\ddot{p} : \ddot{p} \in A\}$.

Definition 1.4 [7]

Let X and Y be two nonempty sets and $f : X \longrightarrow Y$ *be a function.*

a) If $B = \langle y, B_1, B_2 \rangle$ *is an IS in Y, then the pre image*$(inverse\ image)$*of B under f is denoted by* $f^{-1}(B)$ *is an IS in X and defined by*
$f^{-1}(B) = \langle x, f^{-1}(B_1), f^{-1}(B_2) \rangle$.

b) If $A = \langle x, A_1, A_2 \rangle$ *is an IS in X, then the image of A under f denoted by* $f(A)$ *is an IS in Y defined by* $f(A) = \langle y, f(A_1), \underline{f}(A_2) \rangle$, *where* $\underline{f}(A) = \left(f(A_2^c)\right)^c$.

Definition 1.5[7],[8]

An intuitionistic topology on a nonempty set X is a family T of an intuitionistic sets in X satisfying the following conditions.

$(1)\tilde{\Phi}, X \in T$.

$(2)T$ *is closed under finite intersections.*

$(3)T$ *is closed under arbitrary unions.*

The pair (X, T) *is called an intuitionistic topological space (ITS, for short). Any element in T is usually called intuitionistic open set*$(IOS, for\ short)$.

The complement of an IOS in a ITS (X,T) is called intuitionistic closed set $(ICS\ , for\ short)$.

Definition 1.6[13]

Let (X,T) be an ITS and let $A = \langle x, A_1, A_2 \rangle$ be an intuitionistic subset $(IS's, forshort)$ in a set X. The interior $(IntA\ , forshort)$ and closure $(ClA, forshort)$ of a set A of X are defined : $IntA = \cup\{ G: G \subseteq A, G \in T \}$, $ClA = \cap\{ F : A \subseteq F, \overline{F} \in T\}$. In other words: The IntA is the largest intuitionistic open set contained in A, and ClA is the smallest intuitionistic closed set contain A i.e., $IntA \subseteq A$ and $A \subseteq ClA$. In the following definition we give a product of an intuitionistic set and a product of an intuitionistic topological space.

Definition 1.7.*[13]*

Let (X,T) be an ITS and let $A = \langle x, A_1, A_2 \rangle$ be an intuitionistic set, A is said to be Ig-closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$and U is open set in X.

Definition 1.8.*[13]*

A map $f : (X, T) \rightarrow (Y, \gamma)$ is calledIg-continuousif the inverse image of every isIg-closed set of $(Y, \gamma)$for every intuitionistic closed set of (Y, γ) .

Definition: 1.9*.[5]*

Let (X,T)be an ITS and $A = \langle x, A_1, A_2 \rangle$ be an intuitionistic set .Then A is said to be

(i) intuitionistic regular open (intuitionistic regular closed) if A = Iint(Icl(A)) where (A = Icl(Iint(A))).

(ii) intuitionistic generalized closed (Ig-closed) if Icl(A)⊆ U whenever A⊆ U and U is intuitionistic open in X .

(intuitionistic regular α open (Ir α-open) if Icl(A)⊆U whenever A⊆U and U isiii) intuitionistic regular closed in X.

intuitionistic regular semi open (Irs-open) if IScl(A)⊆U whenever A⊆ U and U is(iv) intuitionistic semi regular closed in X .

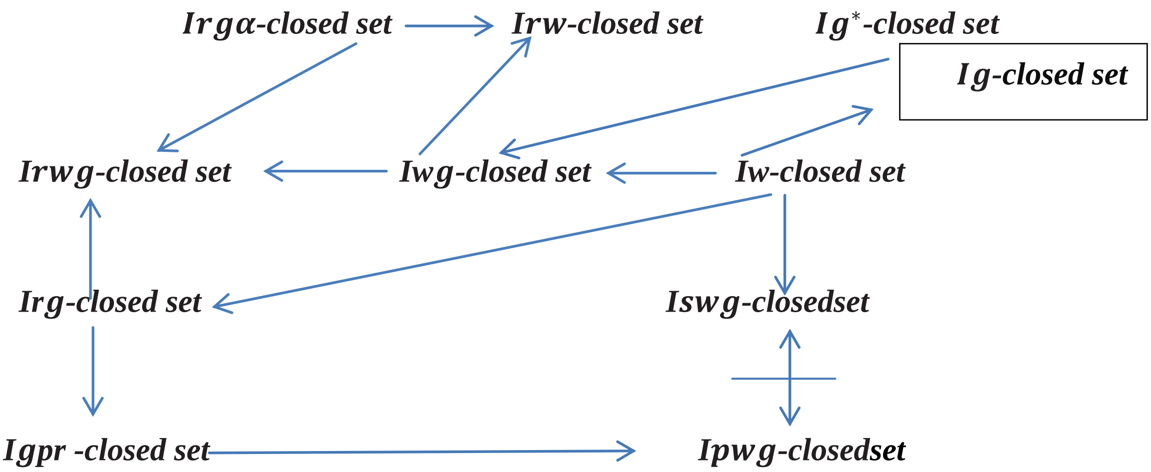
Section.2Some Generalized Sets in Intuitionistic Topological Spaces

In this section we introduce new types of intuitionistic regular generalized (Irg-closed set , for short) , intuitionistic generalized pre regular -closed set ($Igpr$-closed set , for short) intuitionistic weakly generalized closed set (Iwg-closed set , for short),intuitionistic strongly generalized semi closed set (Ig^*-closed set , for short) ,intuitionistic weakly closed set(Iw-closed, for short), intuitionistic semi weakly generalized closed set($Iswg$-closed, for short),intuitionistic pre weakly generalized closed set($Ipwg$-closed, for short), intuitionistic regular-weakly generalized closed set($Irwg$-closed, for short) ,intuitionistic regular w-closed set(Irw-closed, for short), intuitionistic regular generalized α-closed set($Irg\alpha$-closed, for short) .

Definition 2.1 *let (X ,T) be an ITS, and let $A = \langle x, A_1, A_2 \rangle$be an IS in X,then A is said to be:*

1) intuitionistic regular generalized closed set(Irg-closed,for short) if cl (A) $\subseteq$Uwhenever A $\subseteq$U and U is intuitionistic regular open in X.
2) intuitionistic generalized pre regular closed set($Igpr$-closed, for short) if pcl (A) $\subseteq$ Uwhenever A $\subseteq$U and U is intuitionistic regular open in X.
3) intuitionistic weakly generalized closed set(Iwg-closed, for short) if clint (A) $\subseteq$ Uwhenever A $\subseteq$U and U is intuitionistic open in X.
4) intuitionistic strongly generalized semi-closed set (Ig^*-closed, for short) if cl (A)$\subseteq$U whenever A $\subseteq$U and U isintuitionistic g-open in X.
5) intuitionistic weakly closed set(Iw-closed, for short)if cl (A) $\subseteq$U whenever A $\subseteq$U and U is intuitionistic semi open in X.
6) intuitionistic semi weakly generalized closed set(Iswg-closed, for short)ifclint (A) $\subseteq$ U whenever A $\subseteq$U and U isintuitionistic semi-open in X.
7) intuitionistic pre weakly generalized closed set(Ipwg-closed, for short), if clint (A) $\subseteq$ U whenever A $\subseteq$U and U isintuitionistic pre-open in X.
8) intuitionistic regular weakly generalized closed set($Irwg$-closed, for short)if clint (A) $\subseteq$U whenever A $\subseteq$U and U is intuitionistic regular open in X.
9) intuitionistic regular w-closed set(Irw-closed, for short)if cl (A) $\subset$U whenever A $\subset$U and U is intuitionistic regular semi open in X.
10) intuitionistic regular generalized α-closed set($Irg\alpha$-closed, for short)if αcl (A) $\subset$ U whenever A $\subset$U and U is intuitionistic regular α-open in X.
The complements of the above mentioned closed sets are their respective open sets.

Proposition 2.2.*let (X ,T) be an ITS, then the following implications are valid:*



Proof.$Irg\alpha$-closed set ⟶ **Irw-closed set**
Let $F = \langle x, F_1, F_2 \rangle$, such that $A = \langle x, A_1, A_2 \rangle \subseteq F$ then cl (A) $\subset$F**.** Since(X ,T)is$Irg\alpha$-closed then αcl (A) $\subset F$,where F is intuitionistic regular α-open in X. Sinceevery intuitionistic regular α-open in X is intuitionistic regular semi open in X. There fore (X, T) is Irw-closed set**.** The other proofs are same way. The converse of the above Proposition 2.2.need not be true, as seen from the following examples

Example 2.3. *Let* $X = \{a, b, c\}$ *with topology* $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, *where* $A = \langle x, \{a\}, \{b\}\rangle$ *,*$B = \langle x, \{a, c\}, \emptyset\rangle$ *,*$C = \langle x, \{a\}, \emptyset\rangle$ *Then (X, T) is* Irw*-closed setbut not*$Irg\alpha$*-closed set.*
Example 2.4.*Let*$X = \{a, b, c\}$ *with topology* $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, *where* $A = \langle x, \{a\}, \emptyset\rangle$ *,*$B = \langle x, \{a, c\}, \emptyset\rangle$ *.Then (X, T) is*$Irwg$*-closed setbut not Irw-closed set ,also* $Irwg$*-closedset but not* Iwg*-closed set* **.**
Example 2.5.Let*$X = \{a, b\}$ *with topology* $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, *where* $A = \langle x, \emptyset, \{b\}\rangle$ *,*$B = \langle x, \{a\}, \emptyset\rangle$ *,and SOX* $= \{\tilde{X}, \tilde{\emptyset}, A, B, C, D\}$, *where* $C = \langle x, \{b\}, \emptyset\rangle$ *,* $D = \langle x, \emptyset, \emptyset\rangle$*.Then (X, T) is* Ig^**-closed set but not* Iwg*-closed set* **.**

Example 2.6.Let*$X = \{a, b\}$ *with topology* $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, *where* $A = \langle x, \emptyset, \{b\}\rangle$ *,*$B = \langle x, \emptyset, \{a\}\rangle$ *,*$C = \langle x, \emptyset, \emptyset\rangle$*. Then (X, T)is* Iwg*-closed set but not Iw-closed set, also Then (X, T)is* Iwg*-closed setbut* Ig*-closed set*
Example 2.7.Let*$X = \{a, b, c\}$ *with topology* $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, *where* $A = \langle x, \{a\}, \{b\}\rangle$ *,*$B = \langle x, \{a\}, \emptyset\rangle$*. Then (X, T) is* Irg*-closed set but not Iw-closed set.*
Example 2.8.Let*$X = \{a, b\}$ *with topology* $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, *where* $A = \langle x, \{a\}, \{b\}\rangle$ *,* $B = \langle x, \{a\}, \emptyset\rangle$ *,*$C = \langle x, \emptyset, \emptyset\rangle$*,and SOX* $= \{\tilde{X}, \tilde{\emptyset}, A, B, C, D\}$, *where* $D = \langle x, \{b\}, \emptyset\rangle$*.Then (X, T) is* $Iswg$*-closed setbut not Iw-closed set.*
Example 2.9.Let*$X = \{a, b\}$ *with topology* $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, *where* $A = \langle x, \{a\}, \{b\}\rangle$ *,*$B = \langle x, \{a\}, \emptyset\rangle$ *,*$C = \langle x, \emptyset, \emptyset\rangle$ *,and POX* $= \{\tilde{X}, \tilde{\emptyset}, A, B, C, D, E\}$*,where* $D = \langle x, \{b\}, \emptyset\rangle$*,*$E = \langle x, \emptyset, \{b\}\rangle$*.Then (X, T) is* $Ipwg$*-closed set but not Igpr -closed set .*
Example 2.10.Let*$X = \{a, b\}$ *with topology* $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, *where* $A = \langle x, \{a\}, \{b\}\rangle$ *,*$B = \langle x, \{b\}, \{a, c\}\rangle$ *,*$C = \langle x, \{a, b\}, \emptyset\rangle$*.Then (X, T)is* Ig*-closed set but not Iw-closed set .*
Remark 2.11 ***.****by transitive:* $Irg\alpha$*-closedset* $\longrightarrow$ $Irwg$*-closed set* **,**
Iw-closed set $\longrightarrow$ $Irwg$*-closed set* **,***Iw-closed set* $\longrightarrow$ Irg*-closed set* **,**
Irg*-closed set* $\longrightarrow$ $Ipwg$*-closedset* **,***Iw-closed set* $\longrightarrow$ *Igpr -closed set and*
Igpr -closed set $\longrightarrow$ Ig^**-closed set* **.**

Remark 2.12 ***.****The following examples show that the* $Iswg$*-closed set and Igprclosed set are independent.*
Example 2.13.Recall Example 2.8.*we see that (X, T) is* $Iswg$*-closed setbut not Igpr -closed set.*

Example 2.14.Let*$X = \{a, b, c\}$ *with topology* $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, *where* $A = \langle x, \{a\}, \emptyset\rangle$*,* $B = \langle x, \emptyset, \emptyset\rangle$*. Then (X, T) is Igpr -closed set but not* $Iswg$*-closed set* **.**

Remark 2.15.*The following examples show that the* Iwg*-closed set and* $Iswg$*-closed set are independent.*

Example 2.16.Recall Example 2.8.*we see that (X, T) is* $Iswg$*-closed set but no tIwg closed set***.**

Example 2.17.Recall Example 2.6.*we see that (X, T) is* Iwg*-closed set but not* $Iswg$*closed set* **.**

Remark 2.18.*The following examples show that the* $Iswg$*-closed setand* $Ipwg$*-closed set are independent.*

Example 2.19.*Let* $X = \{a, b, c\}$ *with topology* $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$*, where* $A = \langle x, \{a\}, \{b\} \rangle$*,*$B = \langle x, \{a\}, \emptyset \rangle$ *,*$C = \langle x, \emptyset, \{b\} \rangle$*and* $SOX = \{\tilde{X}, \tilde{\emptyset}, A, B, C, D\}$*, where* $D = \langle x, \emptyset, \emptyset \rangle$ *and* $POX = T$ *. Then (X, T) is* $Iswg$*-closed setbut not* $Ipwg$*-closed set* **.**

Example 2.20.Recall Example 2.9.*we see that (X, T) is* $Ipwg$*-closed set but not* $Iswg$*-closed set.*

Remark 2.21.*The following examples show that the* Ig^{*}*-closed set and* Irw*-closed setare independent.*

Example 2.22.Recall Example 2.5.*we see that (X, T) is* Ig^{*}*-closed set but not* Irw*closed set.*

Example 2.23.Recall Example 2.3.*we see that (X, T) is* Irw*-closed setbut not*Ig^{*}*-closed set* **.**

Note :In general topology rw***-closed set*** $\longrightarrow$ rg***-closed set. [15]***

Section.3Some Generalized Mappings in Intuitionistic Topological Spaces

We introduce the following definitions :

Definition 3.1.*A map* $f : (X, T) \rightarrow (Y, \gamma)$ *is said to be intuitionistic regular generalized* α*-closed(briefly,* $Irg\alpha$*-closed) maps if the image of every intuitionistic closed set in* (X, T) *is* $Irg\alpha$*- closed in* (Y, γ) *.*

Definition 3.2.*A map* $f : (X, T) \rightarrow (Y, \gamma)$ *is called*

(i) $Irg\alpha$*-continuous : if the inverse image of every intuitionistic closed set in set V of*(Y, γ)*, is*$Irg\alpha$*-closed set in* (X, T)*,*

(ii)$Irg\alpha$*-irresolute map: if the inverse image of every* $Irg\alpha$*-closed set in*(Y, γ)*is* $Irg\alpha$*-closed in* (X, T)*,*

(iii)strongly $Irg\alpha$*-continuous : if the inverse image of every* $Irg\alpha$*-open setin* (Y, γ) *is open in* (X, T) *.*

Definition 3.3.*A map* $f : (X, T) \rightarrow (Y, \gamma)$ *is said to be*

(i) Irw*-closed: if every image is*Irw*-closed in* (Y, γ) *for each intuitionistic regular semi open set of* (X, T)*,*

(ii) Iw-closed: if every image is Iw-closed in (Y, γ) *for each intuitionistic closed set of* (X, T)*,*

(iii) Iwg*-closed: if every image is* Iwg*-closed in* (Y, γ) *for each intuitionistic closed set of* (X, T)*,*

(iv) Irwg-closed: if every image is Irwg-closed in (Y, γ) *for eachintuitionisticclosed set of* (X, T)*,*

(v) Irg-closed: if every image is Irg-closed in (Y, γ) *for eachintuitionistic closed set of* (X, T)*,*

(vi) Igpr-closed: if every image is Igpr-closed in (Y, γ) *for eachintuitionisticclosed set of*(X, T)*,*

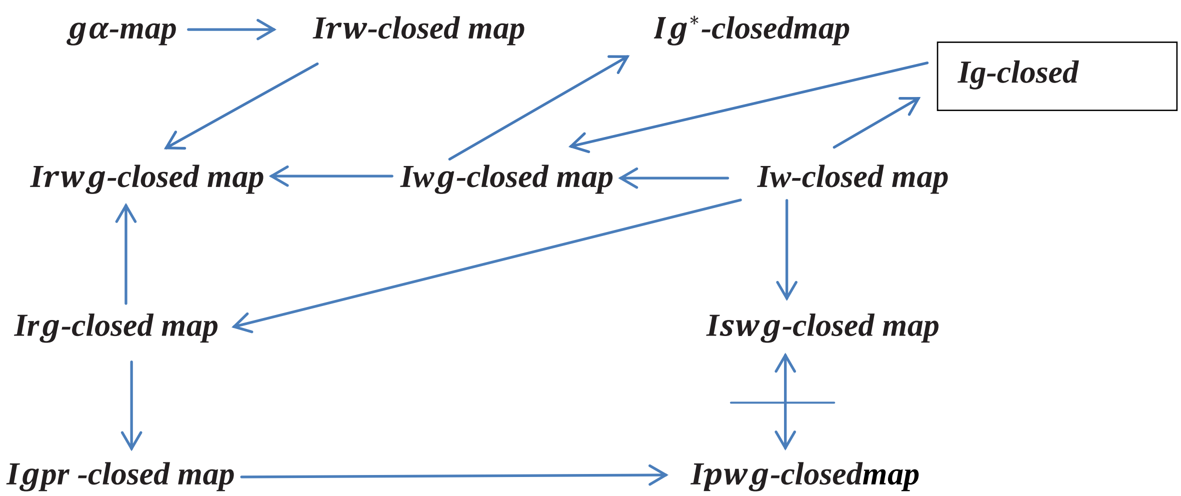
(vii) Ig^**-closedif every image is* Ig^**-closedin* (Y, γ) *for eachintuitionisticg-closedset of*(X, T)*,*

(viii)$Irg\alpha$*-closed if every image is* $Irg\alpha$*-closedin* (Y, γ) *for each intuitionistic regular* α*-closed in* (X, T)*,*

(viiii) Iswg-closed if every image is Iswg-closedin (Y, γ) *for each intuitionistic semi-closed in* (X, T)*,*

(viiiii)Ipwg- closed if every image is Ipwg-closedin (Y, γ) *for each intuitionistic pre-closed in* (X, T)*.*

Propostion 3.4. *Let* $f:(X, T) \rightarrow (Y, \gamma)$ *be a mapping, then the following implications are valid:*



Proof: ***Irgα-closed map*** $\longrightarrow$ ***Irw-closed map***

The prove follows from the definitions and fact that every $Irg\alpha$*- closedis Irw-closed .*

Irw-closed map $\longrightarrow$ ***Igpr -closed map***

The prove follows from the definitions and fact that every Irw-closed set isIgpr –closed.

Irw-closed map $\longrightarrow$ ***Irwg-closed map***

The proof follows from the definitions and fact that every Irw-closed setis Irwg-closed

Iwg-closed map $\longrightarrow$ ***Irwg-closed map***

The proof follows from the definitions and fact that every Iwg-closedset is Irwg-closed. **Iwg-closed map** $\longrightarrow$ $\boldsymbol{Ig^{*}}$**-closed map**

The proof follows from the definitions and fact that every Iwg-closedset is Ig^{*}-closed.The other proofs are same way.

The converse of the above Proposition 3.4.*need not be true, as seen from the following examples .*

Example 3.5.*Let*$X = \{a, b, c\}$ *with topology*$T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, *where* $A=\langle x, \{a\}, \{b\}\rangle$,$B =\langle x, \{a, c\}, \emptyset\rangle$,$C =\langle x, \{a\}, \emptyset\rangle$,$Y = \{1,2,3\}$*with topology* $\gamma = \{\tilde{Y}, \tilde{\emptyset}, D, E\}$, *where* $D =\langle x, \{1\}, \{2\}\rangle$, $E=\langle x, \{1, 2\}, \emptyset\rangle$. *Define a function f:* X→ Y *by* $f(a) = 1$, $f(b) = 3$, $f(c) = 2$.*Then f is Irw-closed map because every image is Irw-closedin* (Y, γ) *for every intuitionistic regular semi open set of* (X, T),*but not*$Irg\alpha$*-closed map because for every intuitionistic regular* α*-open in* (X, T)*there is no image satisfy*$Irg\alpha$*-closedin* (Y, γ) .

Example 3.6.*Let*$X = \{a, b, c\}$ *with topology*$T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, *where* $A =\langle x, \{a\}, \emptyset\rangle$,$B =\langle x, \{a, c\}, \emptyset\rangle$,$Y = \{1,2,3\}$*with topology* $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$, *where* $C =\langle x, \{1\}, \{2\}\rangle$, $D=\langle x, \{1, 3\}, \emptyset\rangle$. *Define a function by f:* X→ Y *by* $f(a) = 1, f(b) = 3, f(c) = 2$. *Then f is Irwg-closed map because every image is Irwg-closed in* (Y, γ) *for every intuitionistic closed set of* (X, T), *but not Iwg-closed map because for every intuitionistic closed set of* (X, T), *there is no image satisfy Iwg-closed in* (Y, γ) .

Example 3.7.*Let*$X = \{a, b, c\}$ *with topology*$T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, *where* $A =\langle x, \emptyset, \{b\}\rangle$, $B =\langle x, \{a\}, \emptyset\rangle$,$Y = \{1,2,3\}$*with topology* $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$, *where* $C=\langle x, \emptyset, \{2\}\rangle$,$D=\langle x, \{1, 2\}, \emptyset\rangle$,*SOX* $=\{\tilde{X}, \tilde{\emptyset}, A, B, E, F\}$, *where* $E =\langle x, \{b\}, \emptyset\rangle$, $F =\langle x, \emptyset, \emptyset\rangle$,*SOY* $=\{\tilde{X}, \tilde{\emptyset}, C, D, G, H\}$, *where* $G =\langle x, \{2\}, \emptyset\rangle$, $H =\langle x, \emptyset, \emptyset\rangle$,*Define a function f:* X→ Y *by* $f(a) = 1$, $f(b) = f(c) = 2$.*Then f is* Ig^{*}*-closed map because every image is* Ig^{*}*-closedin* (Y, γ) *for every intuitionistic g-closed set of*(X, T),*but no tIwg-closed map because for every intuitionistic closed set of* (X, T),*there is no image satisfy Iwg-closed in* (Y, γ) .

Example 3.8.*Let*$X = \{a, b\}$ *with topology* $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, *where* $A =\langle x, \emptyset, \{b\}\rangle$,$B =\langle x, \emptyset, \{a\}\rangle$,$C =\langle x, \emptyset, \emptyset\rangle$, $Y = \{1,2,3\}$*with topology* $\gamma = \{\tilde{Y}, \tilde{\emptyset}, D, E, F\}$, *where* $D =\langle x, \emptyset, \{1\}\rangle$,$E=\langle x, \emptyset, \{3\}\rangle$,$F =\langle x, \emptyset, \emptyset\rangle$.*Define a function f:* X→ Y *by* $f(a) = 1 = f(b)2$.*Then f is Iwg-closed set because every image is Iwg-closedin* (Y, γ) *for every intuitionistic closed set of* (X, T), *but not Iw-closed map because for every intuitionistic closed set of* (X, T), *there is no image satisfy Iw-closed in* (Y, γ) . *Also f is not Ig-closed map because for every intuitionistic closed set of* (X, T), *there is no image satisfy Ig-closed in* (Y, γ) .

Example 3.9.*Let*$X = \{a, b, c\}$ *with topology* $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, *where* $A =\langle x, \{a\}, \{b\}\rangle$,$B =\langle x, \{a\}, \emptyset\rangle$, $Y = \{1,2,3\}$*with topology* $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$, *where* $C =\langle x, \{3\}, \{1\}\rangle$,$D =\langle x, \{3\}, \emptyset\rangle$.*Define a function f:* X→ Y *by* $f(a) = 3$, $f(b) = 1, f(c) = 2$.*Then f is Irg-closed map because every image is Irg-closed in* (Y, γ) *for every intuitionistic closed set of* (X, T), *but not Iw-closed map because for every intuitionistic closed set of* (X, T), *there is no image satisfy Iw-closed in* (Y, γ) .

Example 3.10.*Let*$X = \{a, b\}$ *with topology* $T = \{\tilde{X}, \widetilde{\emptyset}, A, B, C\}$*, where* $A = \langle x, \{a\}, \{b\}\rangle$ *,B* $= \langle x, \{a\}, \emptyset\rangle$ *,C* $= \langle x, \emptyset, \emptyset\rangle$*,and SOX* $= \{\tilde{Y}, \widetilde{\emptyset}, A, B, C, D\}$*, where* $D = \langle x, \{b\}, \emptyset\rangle$*,*$Y = \{1,2,3\}$*with topology* $\gamma = \{\tilde{Y}, \widetilde{\emptyset}, E, F\}$*, where* $E = \langle x, \{1\}, \{2\}\rangle$ *,*$F = \langle x, \{1\}, \emptyset\rangle$*, SOY* $= \{\tilde{Y}, \widetilde{\emptyset}, C, E, F, G\}$*,where* $G = \langle x, \{2\}, \emptyset\rangle$*,Define a function f:* X→ Y *by* $f(a) = 1$, $f(b) = 2, f(c) = 3$ *.Then f is Iswg-closed map because every image is Iswg-closedin* (Y, γ) *for every intuitionistic semi-closed in* (X, T)*,but not Iw-closed map because for every intuitionistic closed set of* (X, T)*,there is no image satisfy Iw-closed in* (Y, γ) *.*

Example 3.11.*Let*$X = \{a, b\}$ *with topology* $T = \{\tilde{X}, \widetilde{\emptyset}, A, B, C\}$*, where* $A = \langle x, \{a\}, \{b\}\rangle$ *,B* $= \langle x, \{a\}, \emptyset\rangle$ *,C* $= \langle x, \emptyset, \emptyset\rangle$ *,POX* $= \{\tilde{X}, \widetilde{\emptyset}, A, B, C, D\}$*, where* $D = \langle x, \{b\}, \emptyset\rangle$ *,E* $= \langle x, \emptyset, \{b\}\rangle$ $Y = \{1,2,3\}$*with topology* $\gamma = \{\tilde{Y}, \widetilde{\emptyset}, C, E, F\}$*, where* $E = \langle x, \{1\}, \{2\}\rangle$ *,F* $= \langle x, \{1\}, \emptyset\rangle$*,POY* $= \{\tilde{Y}, \widetilde{\emptyset}, C, E, F, G, H\}$*, where* $G = \langle x, \{2\}, \emptyset\rangle$ *,H* $= \langle x, \emptyset, \{2\}\rangle$ *.Define a function f:* X→ Y *by* $f(a) = 1$, $f(b) = f(c) = 3$ *.Then f is Ipwg-closed map because every image isIpwg-closedin* (Y, γ) *for every intuitionistic pre-closed in* (X, T) *, but not Igpr -closed map because for every intuitionistic closed set of*(X, T)*, thereis noimage satisfy Igpr-closed in* (Y, γ) *.*

Example 3.12.*Let*$X = \{a, b\}$ *with topology* $T = \{\tilde{X}, \widetilde{\emptyset}, A, B\}$*,where* $A = \langle x, \{a\}, \{b, c\}\rangle$ *,B* $= \langle x, \{b, c\}, \{a\}\rangle$*,* $Y = \{1,2,3\}$*with topology* $\gamma = \{\tilde{Y}, \widetilde{\emptyset}, C, D\}$*, where C*$= \langle x, \{1\}, \{2,3\}\rangle$*,D* $= \langle x, \{1,3\}, \{2\}\rangle$*.Define a function f:* X→ Y *by* $f(a) = 1$, $f(b) = f(c) = 3$ *.Then fis Ig-closed map because every image isIg-closedin* (Y, γ) *for every intuitionistic g-closed in* (X, T) *, but not Iw -closed map because for every intuitionistic closed set of*(X, T)*, there is no image satisfy Iw-closed in* (Y, γ) *.*

Remark 3.13 .*by transitive: Irgα-closedmap* ⟶ *Irwg-closed map****,***

Iw-closed map ⟶ *Irwg-closed map* ***,****Iw-closed map* ⟶ *Irg-closed map* ***,***

Irg-closed map ⟶ *Ipwg-closedmap* ***,****Iw-closed map* ⟶ *Igpr –closed map*

and Igpr -closed set ⟶ *Ig***-closed set*

.Remark 3.14 .*The following examples show that the Iswg-closed map and Igpr-closed map are independent.*

Example 3.15.Recall Example3.10.*we see that fis Iswg-closed map because every image is Iswg-closedin* (Y, γ) *for every intuitionistic semi-open in* (X, T)*,but not Igpr-closed map because for every intuitionistic closed set of* (X, T) *there is no image satisfy Igpr-closed in* (Y, γ) *.*

Example 3.16.*Let*$X = \{a, b, c\}$ *with topology* $T = \{\tilde{X}, \widetilde{\emptyset}, A, B\}$*, where* $A = \langle x, \{a\}, \emptyset\rangle$*,B* $= \langle x, \emptyset, \emptyset\rangle$ *,* $Y = \{1,2,3\}$*with topology* $\gamma = \{\tilde{Y}, \widetilde{\emptyset}, C, D\}$*, where* $C = \langle x, \{3\}, \{1\}\rangle$*,D* $= \langle x, \{3\}, \emptyset\rangle$*. Define a function f:* X→ Y *by* $f(a) = 3$, $f(b) = f(c) = 1$*.Then fis Igpr -closed map because every image is Igpr-closed in* (Y, γ) *for every intuitionistic closed set of* (X, T)*,but not Iswg-closed map because for every intuitionistic closed set of*(X, T)*, there is no image satisfy Igpr-closed in* (Y, γ) *.*

Remark 3.17.*The following examples show that the Iwg-closed map and Iswg-closed map are independent.*

Example 3.18.Recall Example3.10.*we see that f is $Iswg$-closed map because every image is $Iswg$-closedin (Y,γ) for every intuitionistic semi-open in (X, T),but not Iwg-closed map because for every intuitionistic closed set of (X, T),there is no image satisfy Iwg-closed in (Y,γ)* **.**

Example 3.19.Recall Example3.8. *we see that f is Iwg-closed map because every image is Iwg-closed in (Y,γ) for every intuitionistic closed set of (X, T),but not $Iswg$-closed map because for every intuitionistic semi-closed in(X, T),there is no image satisfy $Iswg$-closedin (Y,γ) .*

Remark 3.20.*The following examples show that the $Iswg$-closed map and $Ipwg$-closedmap are independent.*

Example 3.21.*LetX = $\{a,b,c\}$ with topology $T = \{\tilde{X},\tilde{\emptyset},A\ ,B,C\}$, where $A = \langle x,\{a\},\{b\}\rangle$,B $= \langle x,\{a\},\emptyset\rangle$,C $= \langle x,\emptyset,\{b\}\rangle$, SOX $= \{\tilde{X},\tilde{\emptyset},A\ ,B,C,D\}$, where $D = \langle\ x,\emptyset\ ,\emptyset\rangle$,POX = $TY\ =\ \{1,2,3\}$with topology $\gamma = \{\tilde{Y},\tilde{\emptyset},E,F,G\}$, where $E = \langle x,\{1\},\{2\}\rangle$,$F = \langle x,\{1\},\emptyset\rangle$,C $= \langle x,\emptyset,\{b\}\rangle$, SOY $= \{\tilde{Y},\tilde{\emptyset},A\ ,B,C,D\}$, POY = T Then f is $Iswg$-closed map because every image is $Iswg$-closed in (Y,γ) for every intuitionistic semi-closed in (X, T),but not$Ipwg$-closedmap because for every intuitionistic pre-open in(X, T),there is no image satisfy $Ipwg$-closedin (Y,γ) .*

Example 3.22.Recall Example3.11.*we see that f is $Ipwg$-closedmap because every image is $Ipwg$-closedin (Y,γ) for every intuitionistic pre-open in (X, T),but not $Iswg$-closed map because for every intuitionistic semi-closed in(X, T),there is no image satisfy $Iswg$-closedin (Y,γ) .*

Remark 3.23.*The following examples show that the Ig^*-closed map and Irw-closed map are independent.*

Example 3.24.Recall Example3.7. *we see that f is Ig^*-closed map because every image isIg^*-closedin (Y,γ) for every intuitionistic g-closed set of(X, T),but not Irw-closed map because for every intuitionistic regular semi closed set of (X, T),there is no image satisfy Irw-closed in (Y,γ) .*

Example 3.25.Recall Example 2.3. *we see that f is Irw-closed map because every image is Irw-closedin (Y,γ) for every intuitionistic regular semi closed set of (X, T),but not but notIg^*-closed because for every intuitionistic g-closed set of(X, T),there is no image satisfy Ig^*-closedin (Y,γ)* **.**

Proposition 3.26.*If a mapping $f:(X,T)\rightarrow(Y,\gamma)$ is $Irg\alpha$-closed, then$Irg\alpha - cl\big(f(A)\big) \subset f(cl(A))$ for every subset A of (X, T) .*

Proof.*Suppose that f is$Irg\alpha$-closed and $A \subset X$. Then $cl(A)$ is intuitionistic closed in X and so $f(cl(A))$ is $Irg\alpha$ -closed in (Y,γ). We have $f(A) \subset f(cl(A))$,so that $Irg\alpha$-cl(f(A)) $\subset Irg\alpha\text{-}cl(f(cl(A)))\rightarrow$ (i) . Since $f(cl(A))$ is $Irg\alpha$-closed in (Y,γ), so that $Irg\alpha\text{-}cl(f(cl(A))) = f(cl(A))\rightarrow$ (ii), From (i) and (ii), we have $Irg\alpha\text{-}cl(f(A)) \subset f(cl(A)$ for every subset A of (X,T) .*

Remark 3.7*The converse of the above Proposition 3.25.is not true in general as seen from the following example.*

Example 3.28.*Let*$X = \{a, b, c\}$ *with topology* $T = \{\tilde{X}, \tilde{\emptyset}, A , B\}$, *where* $A = \langle x, \{a\}, \{b\} \rangle$,*B* $= \langle x, \{a\}, \emptyset \rangle$, $Y = \{1,2,3\}$*with topology* $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$, *where* $C = \langle x, \{3\}, \{1\} \rangle$,*D* $= \langle x, \{3\}, \emptyset \rangle$.*Define a function f:* X→ Y *by* $f(a) = 3$, $f(b) = 1, f(c) = 2$. *Then* $Ig\alpha\text{-}cl(f(A)) \subset f(cl(A))$ *for every subset A of* (X, T) .*But f is not* $Irg\alpha$*- closed, because there Is not* $Irg\alpha$*-closed in* (Y, γ) .

Theorem 3.29.*Let* (X, T) *and* (Y, γ)*be two intuitionistic topological spaces where* $Ig\alpha\text{-}cl(A) = Iw\text{-}cl(A)$ *for every subset A of Y andf :* $(X, T) \rightarrow (Y, \gamma)$*be a map, then the following are equivalent.*

(i) f is $Irg\alpha$*-closed map.*

(ii) $Irg\alpha\text{-}cl(f(A)) \subset f(cl(A))$ *for every subset A of* (X, T) .

Proof.*(i) ⇒(ii) Follows from the Proposition 3.25.*

(ii) ⇒(i) Let A be any intuitionistic closed set of (X, T) .*Then* $A = cl(A)$ *and so* $f(A) = Irg\alpha\text{-}cl(f(A)) \subset f(cl(A))$ *by hypothesis. We have* $f(A) \subset Irg\alpha\text{-}cl(f(A))$.*Therefore*$f(A) = Irg\alpha\text{ -}cl\big(f(A)\big)$. *Also* $f(A) = Irg\alpha\text{- } cl(f(A)) = Iw\text{-}cl(f(A))$,*by hypothesis. That is*$f(A) = Iw\text{-}cl(f(A))$ *and so* $f(A)$ *is Iw-closed in*(Y, γ) . *Thus* $f(A)$ *is* $Irg\alpha$ *-closed set in* (Y, γ)*and hence f is* $Irg\alpha$ *-closed map .*

Theorem 3.30.*A map f :* $(X, T) \rightarrow (Y, \gamma)$*is* $Irg\alpha$*-closed if and only if for each subset S of* (Y, γ)*and each an intuitionistic open set U containing*$f^{-1}(S) \subset U$, *there is an* $Irg\alpha$*-open set Vof*(Y, γ)*such that* $S \subset V$ *and*$f^{-1}(V) \subset U$.

Proof.*Suppose f is* $Irg\alpha$ *-closed. Let* $S \subset Y$ *and U be an intuitionistic open set of* (X, τ_1, τ_2)*such that* $f^{-1}(S) \subset U$. *Now* $X - U = \langle x, U_2, U_1 \rangle$*is an intuitionistic closed set in* (X, T) .*Since f is* $Irg\alpha$ *- closed,* $f(X - U)$ *is* $Irg\alpha$*-closed set in* (Y, γ).*Then* $V = Y - f(X - U)$*is an*$Irg\alpha$*-open set in* (Y, γ). *Note that* $f^{-1}(S) \subset U$ *implies* $S \subset V$ *and*$f^{-1}(V) = X - f^{-1}(f(X - U)) \subset X - (X - U) = U$. *That is* $f^{-1}(V) \subset U$.

For the converse, let F be an intuitionistic closed set of (X, T) .*Then* $f^{-1}(f(F)^c) \subset F^c$ *and*F^c *is an intuitionistic open in* (X, T) . *By hypothesis, there exists an* $Irg\alpha$*-open set V in* (Y, γ)*such that* $f(F)^c \subset V$ *and* $f^{-1}(V) \subset F^c$ *and so*$F \subset (f^{-1}(V))^c$.*Hence* $V^c \subset f(F) \subset f(((f^{-1}(V))^c) \subset V^c$ *which implies*$f(V) \subset V^c$. *Since* V^c *is* $Irg\alpha$*-closed,* $f(F)$ *is* $Irg\alpha$*-closed.That is* $f(F)$*is*$Ig\alpha$*-closed in* (Y, γ)*and therefore f is* $Irg\alpha$*-closed .*

Remark 3.31.*The composition of two*$Irg\alpha$*-closed maps need not be an intuitionistic closed map in general and this is shown by the following example.*

Example 3.32.*Let* $X = \{a, b, c\}$, *with topology* $T = \{\tilde{X}, \tilde{\emptyset}, A , B\}$, *where A* $= \langle x, \{a\}, \{b, c\} \rangle$,*B* $= \langle x, \{a\}, \emptyset \rangle$, $Y = \{1,2,3\}$*with topology* $\gamma = \left\{\tilde{Y}, \tilde{\emptyset}, C, D\right\}$, *where C* $= \langle x, \{3,2\}, \{1\} \rangle$, *D* $= \langle x, \{3\}, \emptyset \rangle$, $Z = \{e, f, g\}$*with topology* $\beta = \left\{\tilde{Z}, \tilde{\emptyset}, E, F, G, H\right\}$,*where* $E = \langle x, \{e\}, \{g\} \rangle$, $F = \langle x, \{f\}, \emptyset \rangle$, $G = \langle x, \emptyset, \{g\} \rangle$, $H = \langle x, \{e, f\}, \emptyset \rangle$.*Define f :* $(X, T) \rightarrow (Y, \gamma)$*by f(a) = 1, f(b) = 3and f(c) = 2and g :* $(Y, \gamma) \rightarrow (Z, \beta)$ *by f(1) = f(2) = f(3)= e . Then f and g are* $Irg\alpha$*-closed maps,but their composition*$g \circ f$ *:* $(X, T) \rightarrow (Z, \beta)$ *is not* $Irg\alpha$*-closed map, because* $K = \langle x, \{b, c\}, \{a\} \rangle$*is an intuitionistic closed in* (X, T), *but* $g \circ f(F) = g \circ f(K) = g(f(K)) = g(K) = \langle x, \{f\}, \{g\} \rangle$*which is not* $Irg\alpha$ *-closed in* (Z, β) .

Theorem 3.33.*If f : $(X, T) \to (Y, \gamma)$is an intuitionistic closed map and g : $(Y, \gamma)\to (Z, \beta)$ is $Irg\alpha$-closed map, then the composition $g\circ f$: $(X, t) \to (Z, \beta)$ is $Irg\alpha$-closed map .*

Proof. *Let F be any an intuitionistic closed set in (X, T) . Since f is an intuitionistic closed map, $f(F)$ is bi closed set in (Y, γ) .Since g is $Irg\alpha$-closed map, $g(f(F))$ is $Irg\alpha$-closed set in (Z, β) . That is $g \circ f(F) = g(f(F))$ is $Irg\alpha$-closedand hence $g \circ f$ is $Irg\alpha$-closed map.*

Theorem 3.34.*Let (X, T) , (Z, β) be two intuitionistic topological spaces, and (Y, γ) be topological spaces where every $Irg\alpha$-closed subset is an intuitionistic closed .Then the composition $g \circ f$:$(X,T) \to (Z, \beta)$ of the $Irg\alpha$-closed maps f : $(X, T) \to (Y, \beta)$ and g :$(Y, \gamma)\to (Z, \beta)$ is $Irg\alpha$-closed .*

Proof.*Let A be a an intuitionistic closed set of (X, T) .Since f is $Irg\alpha$-closed, f(A) is $Irg\alpha$- closed in (Y, γ) . Then by hypothesis, $f(A)$ is an intuitionistic closed . Since g is $Irg\alpha$-closed, $g(f(A))$is $Irg\alpha$-closed in (Z, β) and $g(f(A)) = g\circ f(A)$.Therefore $g \circ f$ is $Irg\alpha$-closed .*

Theorem 3.35.*If a map f : $(X, T) \to (Y, \gamma)$ is $Irg\alpha$-closed and A isan intuitionistic closed of X, then f_A : $(A, \tau_{1A}, \tau_{2A})\to (Y, \sigma_1, \sigma_2)$ is $Irg\alpha$-closed.*

Proof.*Let F be an intuitionistic closed set of A. Then F = A ∩ E for some an intuitionistic closed setE of (X, τ_1, τ_2) and so F is an intuitionistic closed set of(X,T).Since f is $Irg\alpha$-closed, $f(F)$ is $Irg\alpha$-closed set in (Y, γ) . But $f(F) = f_A(F)$ and therefore f_A : $(A, \tau_{1A}, \tau_{2A})\to (Y, \sigma_1, \sigma_2)$ is $Irg\alpha$-closed.Analogous to $rg\alpha$-closed maps, we define $rg\alpha$-open map as follows.*

Definition 3.36.*A map f : $(X, T) \to (Y, \gamma)$ is called an$Irg\alpha$-open mapif the image f(A) is $Irg\alpha$-open in (Y, γ) for each an intuitionistic open set A in (X, T) .From the definitions we have the following results.*

Corollary 3.37.*(i)Every an intuitionistic open map is $Irg\alpha$-open but not conversely.*

(ii) Every Iw-open map is $Irg\alpha$-open but not conversely.

(iii) Every $Irg\alpha$-open map is Irg-open but not conversely.

(iv) Every $Ig\alpha$-open map is $Irwg$-open but not conversely.

(v) Every $Irg\alpha$-open map is $Igpr$-open but not conversely.

Theorem 3.38*.For any bijection map f : (X, T) → (Y, γ) ,the following statements are equivalent:*

(i) f^{-1}: $(X, T) \to (Y, \gamma)$ is $Irg\alpha$-continous.

(ii) f is $Irg\alpha$-open map.

(iii) f is $Irg\alpha$-closed map.

Proof.*(i) ⇒(ii) Let U be an intuitionistic open set of (X, T) . By assumption, $(f^{-1})(U)= f(U)$ is $Irg\alpha$-open in (Y, γ) and so f is $Irg\alpha$-open .*

(ii) ⇒(iii) Let F =$\langle x, F_2, F_1\rangle$be a an intuitionistic closed set of (X, T) . Then F^c is an intuitionistic open set in(X, T) . By assumption, $f(F^c)$ is $Irg\alpha$-open in (Y, γ) . That is$f(F^c) = f(F)^c$is $Irg\alpha$-open in $(Y, \gamma)$and therefore $f(F)$ is $Irg\alpha$-closed in(Y, γ).Hence f is $rg\alpha$-closed.

(iii) ⇒(i) Let F be an intuitionistic closed set of (X,T) . By assumption, $f(F)$is$Irg\alpha$-

closed in (Y,γ) *. But* $f(F) = (f^{-1})^{-1}(F)$ *and therefore* f^{-1}*is an intuitionistic continuous .*

Theorem 3.39.*If a map f :*$(X, T) \rightarrow (Y,\gamma)$ *is* $Irg\alpha$*-open, then* $(int(A)) \subset Irg\alpha$*-* $int(f(A)f$ *for every subset A of* (X, T) *.*

Proof .*Let f :* $(X, T) \rightarrow (Y,\gamma)$ *be an intuitionistic open map and A be any subset of* (X, T) *.Then* $int(A)$ *is an intuitionistic open in* (X, T) *and so* $f(int(A))$ *is* $Irg\alpha$*-open in* (Y,γ)*We have* $f(int(A)) \subset f(A)$*. Therefore*$f(int(A)) \subset Irg\alpha$*-* $int(f(A))$*.*

Remark 3.40.*The converse of the above Theorem need not be true in general as seen from the following example .*

Example 3.41.*Let X =* $\{a,b\}, T = \{\tilde{X}, \widetilde{\emptyset}, A, B\}$ *,where A =*$\langle x,\{a\},\{b\}\rangle$ *,B =*$\langle x,\{a\},\emptyset\rangle$*,* $Y = \{1,2\}$*with topology* $\gamma = \{\tilde{Y}, \widetilde{\emptyset}, C, D\}$*, where C =*$\langle x,\{2\},\{1\}\rangle$ *, Define a function f:* X→ Y *by* $f(a) = 2,\ f(b) = 1$*. In* (Y,γ) *,* $Irg\alpha$*-* $int(f(A))= f(A)$ *for every subset A of* (X, T) *. So* $f(int(A)) \subset f(A) = Irg\alpha$*-*$int(f(A))$ *for every subset A of X. But f is not* $Irg\alpha$*-open map, since for the an intuitionistic open set Aof* (X, T) *,* $f(A)$ *is not* $Irg\alpha$*-open in* (Y,γ) *.*

Theorem 3.42.*If a function f :*$(X, T) \rightarrow (Y,\gamma)$ *is* $Irg\alpha$*-open, then* $f^{-1}(Irg\alpha\text{-}\ cl(B)) \subset cl(f^{-1}(B))$ *for each subset B of* (Y,γ) *.*

Proof.*Let f :*$(X, T) \rightarrow (Y,\gamma)$ *be an*$Irg\alpha$*-open map and B be any subset of* (Y,γ) *.Then*$f^{-1}(B) \subset cl(f^{-1}(B))$ *and* $cl(f^{-1}(B))$ *is an intuitionistic closed set in* (X, T) *. So there exists a* $Irg\alpha$*-closed setK of* (Y,γ) *such that* $B \subset K$ *and* $f^{-1}(K) \subset cl(f^{-1}(B))$*.Now* $Irg\alpha$*-*$cl(B) \subset Irg\alpha$*-*$cl(K) = K$*, as K is* $Irg\alpha$*-closed set of* (Y,γ) *.Therefore* $f^{-1}(Irg\alpha\text{-}cl(B)) \subset f^{-1}$ *(K) and so* $f^{-1}(Irg\alpha\text{-}cl(B)) \subset f^{-1}(K) \subset cl(f^{-1}(B))$*.Thus* f^{-1} $(Irg\alpha\text{-}cl(B)) \subset cl(f^{-1}(B))$ *for each subset of Bof* (Y,γ) *.*

Remark 3.43.*The converse of the above Theorem need not be true in general as seen from the following example.*

Example 3.44.*Let X =* $\{a,b,c\}, with\ topology\ T = \{\tilde{X}, \widetilde{\emptyset}, A, B\}$ *, where A =*$\langle x,\{a,b\},\{c\}\rangle$ *,B =*$\langle x,\{b\},\emptyset\rangle$*,* $Y = \{1,2,3\}$*with topology* $\gamma = \{\tilde{Y}, \widetilde{\emptyset}, C, D\}$*, where C =*$\langle x,\{3,1\},\emptyset\rangle$ *, D =*$\langle x,\{3\},\emptyset\rangle$*. Let fbe the identity map from* (X, T) *to* (Y,γ) *. In* (Y,γ) *,* $Irg\alpha$*-cl(B) = Bfor every subset B of* (Y,γ) *.So* $f^{-1}(Irg\alpha\text{-}cl(B)) = f^{-1}(B) \subset cl(f^{-1}(B))$ *for every subset B of*(Y,γ) *. But f is not* $Irg\alpha$*-open map, since for the an intuitionistic open setA =*$\langle x,\{a,b\},\{c\}\rangle$*of* (X, T) *,* $f(A) = f(\langle x,\{a,b\},\{c\}\rangle)$*which is not* $Irg\alpha$*-open in* (Y,γ) *.We define another new class of maps called bi* $rg\alpha^{*}$*-closed maps which arestronger than bi* $rg\alpha$*-closed maps.*

Definition 3.45. *A map f :* $(X, T) \rightarrow (Y,\gamma)$ *is said to be* $Irg\alpha^{*}$*-closedmapif the image f(A) is* $Irg\alpha$*-closed in* (Y,γ) *for every* $Irg\alpha$*-closedset A in* (X, T) *.*

Theorem 3.46.*Every* $Irg\alpha^{*}$*-closed map is* $Irg\alpha$*-closed map but not conversely.*

Proof.*The proof follows from the definitions and fact that every an intuitionistic closed set is* $Irg\alpha$*-closed.*

The converse of the above Theorem is not true in general as seen from the following example.

Example 3.47.Recall Example3.41.*we see that f* $Irg\alpha$*-closedmap but not* $Irg\alpha^{*}$*-closed map.Since* $\{a\}$*is bi* $rg\alpha$*-closed set in*(X, T)*,but its image under f is* $\{a\}$*, which is not bi* $rg\alpha$*-closed in*(Y,γ) *.*

Theorem 3.48.*If* $f : (X, T) \rightarrow (Y, \gamma)$ *and* $g : (Y, \gamma) \rightarrow (Z, \beta)$*are* $Irg\alpha^*$-*closedmaps,then their composition* $g \circ f : (X, T) \rightarrow (Z, \beta)$*is also* $Irg\alpha^*$-*closed.*
Proof.*Let F be a* $rg\alpha$-*closed set in* (X, T) *.Since f is* $Irg\alpha^*$-*closed map ,* $f(F)$ *is* $Irg\alpha$-*closed set in* (Y, γ) *. Since* g *is* $Irg\alpha^*$-*closed map,* $g(f(F))$ *is* $Irg\alpha$-*closed set in* (Z, β) *.Therefore*$g \circ f$*is* $Irg\alpha^*$-*closedmap.*
We define another new class of maps called $Irg\alpha^*$-*open maps whichare stronger than*$Irg\alpha$-*open maps.*
Definition 3.49.*A map* $f : (X, T) \rightarrow (Y, \gamma)$ *is said to be* $Irg\alpha^*$-*openmap if the image f(A) is* $Irg\alpha$-*open set in* (Y, γ) *for every* $Irg\alpha$-*open setA in* (X, T) *.*
Remark 3.50.*Since every an intuitionistic open set is an* $Irg\alpha$-*open set, we have every* $Irg\alpha^*$*open map is* $Irg\alpha$-*open map. The converse is not true in general as seen from the following example.*
Example 3.51.*Let* $X = \{a, b\}, with\ topology\ T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$ *, where* $A = \langle x, \{a\}, \emptyset \rangle$ *,B* $= \langle x, \{b\}, \emptyset \rangle, C = \langle x, \emptyset, \emptyset \rangle$ *,* $Y = \{1,2,3\}$*with topology* $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$*, where D* $= \langle x, \{3\}, \emptyset \rangle$ *. Define* $f : (X, T) \rightarrow (Y, \gamma)$ *by f(a) = 1, f(b) = 3. Then f is* $Irg\alpha$-*open mapbut not* $Irg\alpha^*$-*open map, since for the* $Irg\alpha$-*open set Bin* $(X, T) f(B) = f(\langle x, \{b\}, \emptyset \rangle) = \langle x, \{1\}, \emptyset \rangle$*which is not* $Irg\alpha$-*open set in*(Y, γ) *.*
Theorem 3.52.*If* $f : (X, T) \rightarrow (Y, \gamma)$ *and* $g : (Y, \gamma) \rightarrow (Z, \beta)$ *are* $Irg\alpha^*$-*open maps, then theircomposition* $g \circ f : (X, T) \rightarrow (Z, \beta)$*is also* $Irg\alpha^*$-*open .*
Proof.*Proof is similar to the Theorem 3.48.*

<u>REFRENCES</u>

[1] –Atanassov, K .and Stoeva , S . (1983) " Intuitionistic fuzzy sets In : Polish Symp on Interval and fuzzy mathematics " Poznan pp.23-26 .
[2] –Atanassov, K .T. (1984) " Intuitionistic fuzzy sets " VII ITKR 's Session CenteralSci.and Tech.Library Academy of Sci. , Sofia .
[3] –Atanassov, K .T. (1986) " Intuitionistic fuzzy sets " result on intuitionistic fuzzy sets" Fuzzy set and system 20, No .1, pp.87- 96 .
[4] –Atanassov, K . (1988) " Review and new result on intuitionistic fuzzy sets" IM-MFAIS Sofia , pp.1-88 .
[5]– Bayhan , S .and Coker , D . (2001) " On separation axioms in intuitionistic topological spaces " IJMMS 27, NO.10 pp . 621- 630 .
[6] – Chang, C . (1968) " Fuzzy topological spaces " J .Math .Anal .APPl.24, pp. 182-190 .
[7]– Coker , D . (1996) " A note on intuitionistic sets and intuitionistic points " Turkish J .Math. 20, No.3 pp. 343-351.
[8] – Coker , D . (1997) " An introduction to intuitionistic fuzzy topological spaces " Fuzzy set and system 88, No .1, pp.81- 89 .
[9] – Coker , D . (2000) " An introduction to intuitionistic topological spaces " AkdenizUniv, Mathematics Dept. pp . 51-56 .
[10] –Malghan, R .S.and Benchalli ,S.S. (1981) " On fuzzy topological spaces " Glasnik Mathematics 16, No.36, pp. 313-325.

[11] Ozcelik ,A.z.andNarli,S.(2007)" On submaximalityintuitionistictopological spaces" I.J.of Math , and Math .Sci.Vol.1,No.1,pp.139-141 .
[12] Vadivel , A. and Vairamanickam, K.(2010)" rgα-Closed and rgα-Open Maps in Topological Spaces" J **.**Math **.**No. 10, pp.453 – 468.
[13] Yaseen S.R.(2008) " Weak form of separation axioms in intuitionistictopological spaces" M.Sc. thesis college of Education ,Tikrit Univ.
[14]Younis J. Yaseenand G. Raouf (2009**) "** On generalization closed set and generalized continuity on Intuitionistic Topological spaces" University of Tikrit- College of Computer Science and Mathematics.
[15] –Zadeh,L **.**A **.**(1965) " Fuzzy sets " Information and Control 8, pp **.**338–353 **.**

بعض تعميمات المجموعات والدوال في الفضاءات التبولوجيه الحدسية

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الملخص:

في هذا البحث قدمنا انواع جديدة من تعميم المجموعة المغلقة السوية الحدسية ، تعميم المجموعة المغلقة شبه السوية الحدسية وتعميم المجموعة المغلقة الضعيفة الحدسية وتعميم المجموعة شبه المغلقة القوية الحدسية والمجموعة الضعيفة المغلقة الحدسية وانواع اخرى كثيرة .
ومن خلال هذه المفاهيم عرفنا صفوف جديدة من الدوال الحدسية ودرسناها ، وتمر بنا بعض الخصائص والعلاقات فيما بينهما .

مفتاح الكلمات : تعميم المجموعة المغلقة السوية الحدسية ، تعميم المجموعة المغلقة شبه السوية الحدسية وتعميم المجموعة المغلقة الضعيفة الحدسية.