

On Solution of Two Point Second Order Linear Boundary Value Problems By using Semi-Analytic Method

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Abstract

In this paper a new method is proposed for the solution of two-point second order boundary-value problems (TPBVP) ,that is, we concerned with constructing polynomial solutions to two point second order boundary value problems for ordinary differential equation.

A semi-analytic technique using two-point osculatory interpolation with the fit equal numbers of derivatives at the end points of an interval $[0, 1]$ is compared with conventional methods via a series of examples and is shown to be that seems to converge faster and more accurately than the conventional methods.

1. Introduction

The most general form of the problem to be considered is :

$$y'' = f(x, y, y') , \quad x \in [a, b] ,$$

with boundary conditions : $y(a) = A$, $y(b) = B$

there is no loss in generality in taking $a = 0$ and $b = 1$, and we will sometimes employ this slight simplification. We view f as a generally nonlinear function of y and y' , but for the present, we will take $f = f(x)$ only. For such a problem to have a solution it is generally necessary either that $f(x) \neq 0$ hold, or that $A \neq 0$ at one or both ends of the interval. When $f(x) \equiv 0$, and $A = 0$, $B = 0$ the BVP is said to be homogeneous and will in general have only the trivial solution, $y(x) \equiv 0$ [1]. In this paper we introduce a new technique for the qualitative and quantitative analysis of non homogeneous linear TPBVP using two-point polynomial interpolation .

2. Approximation Theory

The primary aim of a general approximation is to represent non-arithmetic quantities by arithmetic quantities so that the accuracy can be ascertained to a desired degree. Secondly, we are also concerned with the amount of computation required to achieve this accuracy. A complicated function $f(x)$ usually is approximated by an easier function of the form $\varphi(x; a_0, \dots, a_n)$ where a_0, \dots, a_n are parameters to be determined so as to characterize the best approximation of f .

In this paper, we shall consider only the interpolatory approximation. From Weierstrass Approximation Theorem ,it follows that one can always find a polynomial that is arbitrarily close to a given function on some finite interval. This means that the approximation error is bounded and can be reduced by the choice of

the adequate polynomial. Unfortunately Weierstrass Approximation Theorem is not a constructive one, i.e. it does not present a way how to obtain such a polynomial. i.e. the interpolation problem can also be formulated in another way, viz. as the answer to the following question: How to find a .good. representative of a function that is not known explicitly, but only at some points of the domain of interest .In this paper we use Osculatory Interpolation since has high order with the same given points in the domain .

2.1. Osculatory Interpolation[2]

Given $\{x_i\}$, $i = 1, \dots, k$ and values $f_i^{(0)}, \dots, f_i^{(r_i)}$, where r_i are nonnegative integers and $f_i = f(x_i)$. We want to construct a polynomial $P(x)$ such that $P^{(j)}(x_i) = f_i^{(j)}$ (1), for $i = 1, \dots, k$ and $j = 0, \dots, r_i$.

Such a polynomial is said to be an osculatory interpolating polynomial of a function f

Remark

The degree of $P(x)$ is at most $\sum_{i=1}^k (r_i + 1) - 1$.

In this paper we use two-point osculatory interpolation [2]. Essentially this is a generalization of interpolation using Taylor polynomials and for that reason osculatory interpolation is sometimes referred to as two-point Taylor interpolation. The idea is to approximate a function $y(x)$ by a polynomial $P(x)$ in which values of $y(x)$ and any number of its derivatives at given points are fitted by the corresponding function values and derivatives of $P(x)$.

In this paper we are particularly concerned with fitting function values and derivatives at the two end points of a finite interval, say $[0,1]$, wherein a useful and succinct way of writing a osculatory interpolant $P_{2n+1}(x)$ of degree $2n + 1$ was given for example by Phillips [3] as :

$$P_{2n+1}(x) = \sum_{j=0}^n \{y^{(j)}(0) q_j(x) + (-1)^j y^{(j)}(1) q_j(1-x)\} \dots \dots \dots (2)$$

$$q_j(x) = (x^j/j!)(1-x)^{n+1} \sum_{s=0}^{n-j} \binom{n+s}{s} x^s = Q_j(x)/j! \dots \dots \dots (3)$$

so that (2) with (3) satisfies :

$$y^{(r)}(0) = (1)^{(r)} P_{2n+1}^{(r)}(0) , \quad r=0,1,2,\dots,n. \quad P_{2n+1}^{(r)}(1) = (-1)^{(r)} y^{(r)}(1) , \quad y^{(r)} P_{2n+1}^{(r)}$$

implying that $P_{2n+1}(x)$ agrees with the appropriately truncated Taylor series for $y(x)$ about $x = 0$ and $x = 1$. The error on $[0, 1]$ is given by :

$$R_{2n+1} = y(x) - P_{2n+1}(x) = \frac{(-1)^{n+1} x^{(n+1)} (1-x)^{n+1} y^{(2n+2)}(\varepsilon)}{(2n+2)!} \quad \text{where } 0 < \varepsilon < 1 \text{ and}$$

$y^{(2n+2)}$ is assumed to be continuous.

The osculatory interpolant for $P_{2n+1}(x)$ may converge to $y(x)$ in $[0,1]$ irrespective of whether the intervals of convergence of the constituent series intersect or are disjoint. The important consideration here is whether $R_{2n+1} \rightarrow 0$ as $n \rightarrow \infty$ for all x in $[0,1]$. In the application to the boundary value problems in this paper such convergence with n is always confirmed numerically. We observe that (2) fits an equal number of derivatives at each end point but it is possible and indeed sometimes desirable to use polynomials which fit different numbers of derivatives at the end points of an interval. As an example of a two-point osculatory interpolant we may take $n = 2$ so that (2) with (3) becomes the quintic :

$$P_5(x) = (1-x)^3(1+3x+6x^2)y(0) + x^3(10-15x+6x^2)y(1) + x(1-x)^3(1+3x)y'(0) - x^3(1-x)(4-3x)y'(1) + 1/2x^2(1-x)^3y''(0) + 1/2x^3(1-x)^2y''(1)$$

Satisfying :

$$P_5(0)=y(0), P'_5(0)=y'(0), P''_5(0)=y''(0).$$

$$P_5(1)=y(1), P'_5(1)=y'(1), P''_5(1)=y''(1).$$

Finally we observe that (2) can be written directly in terms of the Taylor coefficients a_i and b_i about $x = 0$ and $x = 1$ respectively, as :

$$P_{2n+1}(x) = \sum_{j=0}^n \{ a_j Q_j(x) + (-1)^j b_j Q_j(1-x) \}. \dots (4)$$

3. Solution Of Two Point Second-Order Boundary Value Problems

We consider the boundary value problem

$$y'' + f(x, y, y') = 0 \dots\dots\dots (5)$$

$$g_i(y(0), y(1), y'(0), y'(1)) = 0, \quad i = 1, 2 \dots\dots\dots (6)$$

where f, g_1, g_2 are linear functions of their arguments and g_1 and g_2 are given in three kinds [4] :

- 1- $y(0) = a_0, y(1) = b_0 \dots\dots (6a)$, and we say this kind Dirichlet condition (value specified).
- 2- $y'(0) = a_1, y'(1) = b_1 \dots (6b)$, and we say this kind Neumann condition (Derivative specified).
- 3- $c_0y'(0)+c_1y(0) = a, d_0y'(1) + d_1y(1) = b \dots (6c)$, where c_0, c_1, d_0, d_1 are all positive constants not all are zero but c_1, d_0 are equal to zero or c_0, d_1 are equal to zero and we say this kind Mixed condition (Gradient & value) .

The simple idea behind the use of two-point polynomials is to replace $y(x)$ in problem (5)–(6), or an alternative formulation of it, by a P_{2n+1} which enables any unknown boundary values or derivatives of $y(x)$ to be computed. The first step therefore is to construct the P_{2n+1} . To do this we need the Taylor coefficients of $y(x)$ at $x = 0$:

$$y = a_0 + a_1 x + \sum_{i=2}^{\infty} a_i x^i \dots\dots\dots (7a)$$

into (5) and equate coefficients of powers of x . The resulting system of equations can be solved to obtain a_i (a_0, a_1) for all $i \geq 2$. Also we need the Taylor coefficients of $y(x)$ at $x = 1$. Using MATLAB throughout we simply insert the series forms :

$$y = b_0 + b_1(x-1) + \sum_{i=2}^{\infty} b_i(x-1)^i \quad \dots\dots\dots(7b)$$

into (5) and equate coefficients of powers of $(x-1)$. The resulting system of equations can be solved to obtain b_i (b_0, b_1) for all $i \geq 2$. The notation implies that the coefficients depend only on the indicated unknowns a_0, a_1, b_0, b_1 . The algebraic manipulations needed for this process. We are now in a position to construct a $P_{2n+1}(x)$ from (7) of the form (2) and use it as a replacement in the problem (5)–(6). Since we have only the four unknowns to compute for any n we only need to generate two equations from this procedure as two equations are already supplied by the boundary conditions (6). An obvious way to do this would be to satisfy the equation (5) itself at two selected points $x = c_1, x = c_2$ in $[0,1]$ so that the two required equations become :

$$P''_{2n+1}(c_i) + f\{P_{2n+1}(c_i), P'_{2n+1}(c_i), c_i\} = 0, \quad i=1,2. \quad \dots\dots\dots(8)$$

An alternative approach is to recast the problem in an integral form before doing the replacement. Extensive computations have shown that this generally provides a more accurate polynomial representation for a given n . We therefore use this alternative formulation throughout this thesis although we should keep in mind that the procedure based on (8) is a viable option and shares many common features with the approach outlined below. Of the many ways we could provide an integral formulation we adopt the following. We first integrate (5) to obtain :

$$y'(x) - a_1 + \int_0^x f(y(s), y'(s), s) ds = 0 \quad \dots\dots\dots(9)$$

and again to find :

$$y(x) - a_0 - x a_1 + \int_0^x (x-s) f(y(s), y'(s), s) ds = 0 \quad \dots\dots\dots(10)$$

where $a_0 = y(0)$ and $a_1 = y'(0)$. Putting $x = 1$ in (9) and (10) then gives :

$$b_1 - a_1 + \int_0^1 f(y(s), y'(s), s) ds = 0 \quad \dots\dots\dots(11)$$

and

$$b_0 - a_0 - a_1 + \int_0^1 (1-s) f(y(s), y'(s), s) ds = 0 \quad \dots\dots\dots(12)$$

where $b_0 = y(1)$ and $b_1 = y'(1)$.

The precise way we make the replacement of $y(x)$ with a $P_{2n+1}(x)$ in (11) and (12) depends on the nature of $f(y, y', x)$ and will be explained in the examples which follow. In any event the important point to note is that once this replacement has been made, the equations (6), (11) and (12) constitute the four equations we require to determine the set $\{a_0, b_0, a_1, b_1\}$. As we shall see the fact that the number of

unknowns is independent of the number of derivatives fitted represents perhaps the most important feature of the method.

We make the following points at this stage :

(i) In the majority of cases where the boundary conditions are simple enough the system of algebraic equations may be reduced a priori to a system in two unknowns, since the boundary condition can be substituted directly into the integral formulations (11) and (12), which MATLAB can be utilized to solve. That is, if we have the BC(6a), then we have only the unknown pair $\{a_1, b_1\}$ and is known the required polynomial can be constructed. For the benefit of the reader the entire procedure for Examples in section 4 .And if, we have the BC(6b), then we have only the unknown pair $\{a_0, b_0\}$ and is known the required polynomial can be constructed. Also if, we have the BC(6c), then we have only the unknown pair $\{a_0, b_1\}$ or $\{a_1, b_0\}$ and is known the required polynomial can be constructed .

(ii) The method offers a certain amount of flexibility. For example we could choose to satisfy (9) and (10) at two internal points or we could use alternative integral formulations. The fact remains that whatever strategy we adopt produces a quickly convergent sequence of values of the set $\{a_0, a_1, b_0, b_1\}$ as n increases.

(iii) Throughout we assess the accuracy of the procedure by examining the convergence with n . Using a symbolic computational facility such as MATLAB, computing the required convergent is not an issue. Where possible we can also run checks on our solutions using shooting with MATLAB codes.

(iv) We compare our method with the other method. We now consider a number of examples designed to illustrate the convergence, accuracy, implementation and utility of the method. In what follows the use of bold digits in the tables is intended to give a rough visual indication of the convergence.

Remark

1- All computations in the following examples were performed in the MATLAB environment, Version 7, running on a Microsoft Windows 2003 Professional operating system .

2- In the following examples when analytical solutions are known so that we can measure the error of a solution.

4. Examples

In this section we introduce some examples illustrates suggested method :

Linear boundary value problems (BVPs) can be used to model several physical phenomena. For example, a common problem in civil engineering concerns the deflection of a beam of rectangular cross section subject to uniform loading, while the ends of the beam are supported so that they undergo no deflection. This problem is linear second-order TPBVP[5] .Now, we give many other examples, we first consider the linear problem with Dirichlet BC :

Example 1

$$y'' - 4(y - x) = 0, \quad y(0) = 0, \quad y(1) = 2 \quad \dots (13)$$

has exact solution [6] : $e^2 (e^4 - 1)^{-1} (e^{2x} - e^{-2x}) + x$

Here (11) and (12) become :

$$b_1 - a_1 + 2 - 4 \int_0^1 y(s) ds = 0 \quad (14)$$

$$8/3 - a_1 - 4 \int_0^1 y(s) ds = 0 - (1 \int_0^1 y(s) ds) \quad (15)$$

and the coefficients : $a_2, b_2, a_3, b_3, \dots$ can be found from (7a) and (7b) .

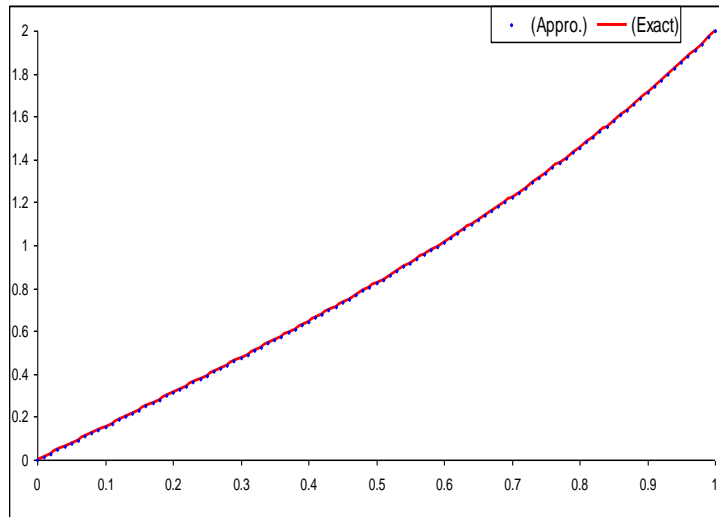
Abinitio inclusion of the boundary conditions in (13) has reduced the number of unknowns to two, namely $\{a_1, b_1\}$, which are computed by solving (14) and (15) with $y(s)$ replaced by a $P_{2n+1}(s)$. The results for $n = 2, 3, 4$ are displayed in Table 1. We can see that there is clear convergence with n to the 'exact' values which are obtained using MATLAB boundary value software. Table 2 gives the compare between the suggested method and other methods and figure 1 gives the accuracy of the method.

TABLE 1: The result of the methods for $n= 2, 3, 4$ of example1

		P5	P7	P9	
a1		1.5511387164	1.5514458006	1.5514410832	
b1		3.0749482402	3.0746246085	3.0746294890	
X	Y:exact	P5	P7	P9	Y-P9
0.25	0.3936766919	0.3937912461	0.3936753464	0.3936767011	0.000000009170200
0.5	0.8240271368	0.8244047619	0.8240204194	0.8240272117	0.000000074822647
0.75	1.3370861339	1.3372355396	1.3370844322	1.3370861455	0.000000011556981
S.S.E = 5.81608482230229E-15					

Then from table 1 and the relation (2)and (3) in the previous section we have :

$$\begin{aligned}
 P_5 &= .121739 x^5 - .662526e^{-1} x^4 + .393375 x^3 + 1.55114 x \\
 P_7 &= .114177e^{-1} x^7 - .905686 e^{-2} x^6 + .804532 e^{-1} x^5 - .189035 e^{-2} x^4 + .367631 x^3 + 1.55145 x \\
 P_9 &= .628069e^{-3} x^9 - .652397e^{-3} x^8 + .774834 e^{-2} x^7 - .403156 e^{-3} x^6 + 0.0736107 x^5 + \\
 &\quad 0.367627 x^3 + 1.55144 x
 \end{aligned}$$



**Figure1: Comparison between the exact solution
and semi-analytic method P₉**

Now we give the comparison between the solution of suggested method and solution of other methods in the following table :

**TABLE 2: A Comparison between P₉ and other methods of example
1**

x	Exact solution	Φ_1 by using linear shooting method	Φ_2 by using linear Finite- Difference method	P ₉ by using Oscillatory interpolation	Y-P ₉
0.25	0.393676692	0.3936767	0.39367669	0.3936767011	0.0000000092
0.5	0.824027137	0.8240271	0.82402714	0.8240272117	0.0000000748
0.75	1.337086134	1.337086	1.33708613	1.3370861455	0.0000000116

Example 2

$$y'' = \{-y' + (x+2)y - (2 - (x+1)^2)e^1 \ln(2) + 2e^x\} / (x+1)$$

with BC : $y(0) = 0$, $y(1) = 0$

Exact solution [6] $y(x) = e^x \ln(x+1) - (e^1 \ln(2))x$.

The results of solution given in the following table :

Table 3: The result of the methods for n = 2, 3, 4 of example2

		P5	P7	P9	
a1		-0.88416918	-0.88416925	-0.88416940	
b1		1.35913519	1.35914462	1.35914055	
x	Y	P5	P7	P9	Y-P9
0.25	-0.1845203549	-0.1845324937	-0.1845173602	-0.1845205972	0.000000242273476
0.5	-0.2735857444	-0.2735962538	-0.2735782173	-0.2735865577	0.000000813249308
0.75	-0.2284204067	-0.2284181840	-0.2284189827	-0.2284205035	0.000000096793373
S.S.E=7.29439830 658099E-13					

Therefore: $P_9 = 0.0015 x^9 - 0.0091 x^8 + 0.0257 x^7 - .0410 x^6 + 0.737 x^5 + 0.3333 x^3 + 0.5000 x^2 - 0.8842 x$

The accuracy of the solution given in the following figure :

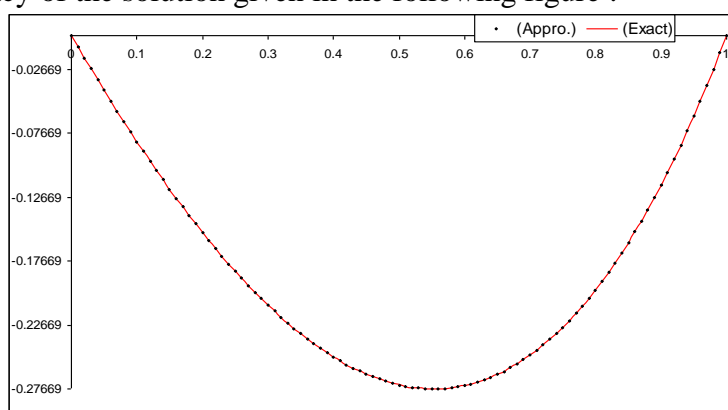


Figure2: A comparison between exact and approximate solution of example2

Now we give the comparison between the solution of suggested method and solution of other methods in the following table :

Table4: A Comparison between P₉ and other methods of example 2

xi	Y:exact	Φ_1 by using cubic splines	Φ_2 by another numerical solution	P ₉ by using Osculatory interpolation	Y-P ₉
0.25	- 0.1845203549	-0.1845191	-0.1846134	- 0.1845205970	0.0000002423
0.5	- 0.2735857444	-0.2735833	-0.2737099	- 0.2735865580	0.0000008132
0.75	- 0.2284204067	-0.2284186	-0.2285169	- 0.2284205030	0.0000000968

Now we give the linear problem with Neumann BC :

Example 3

$$y'' - \pi^2 y + 2\pi^2 \sin(\pi x) = 0 \text{ with BC : } y'(0) = \pi, y'(1) = -\pi$$

With Exact solution [6] : $\sin(\pi x)$

In this case equation (7) gives $\{a_i, b_i\}$ where $i = 2, 3, \dots$, so we have four unknown $\{a_0, a_1, b_0, b_1\}$ then BC give a_1, b_1 , thus solving (11) and (12) to obtain a_0, b_0 . The result of method given in the following table :

Table 5 :The result of the methods for n = 2, 3, 4 of example 3

		P5	P7	P9	
a0		0.0071286292	0.0002701079	0.0000066077	
b0		0.0071286292	0.0002701079	0.0000066077	
X	Y	P5	P7	P9	Y-P ₉
0	0.0000000000	0.0071286292	0.0002701079	0.0000066077	0.000006607700484
0.1	0.3090169943	0.3156038133	0.3092862669	0.3090237892	0.000006794799366
0.2	0.5877852522	0.5911087921	0.5879637222	0.5877906400	0.000005387754140
0.3	0.8090169943	0.8069586888	0.8089626630	0.8090164488	0.000000545560411
0.4	0.9510565162	0.9440928773	0.9507420034	0.9510478432	0.000008673077361
0.5	1.0000000000	0.9910749818	0.9995720325	0.9999874662	0.000012533840896
0.6	0.9510565162	0.9440928773	0.9507420034	0.9510478432	0.000008673077361
0.7	0.8090169943	0.8069586888	0.8089626630	0.8090164488	0.000000545560411
0.8	0.5877852522	0.5911087921	0.5879637222	0.5877906400	0.000005387754140
0.9	0.3090169943	0.3156038133	0.3092862669	0.3090237892	0.000006794799367
1	0.0000000000	0.0071286292	0.0002701079	0.0000066077	0.0000066077005

S.S.E = 5.45854779363699E-10

Then from table 5 and the relation (2) and (3) in the previous section we have :

$$P_5 = 3.17677 x^4 - 6.35354 x^3 + .351784 e^{-1} x^2 + 3.14159 x + .712863 e^{-2}$$

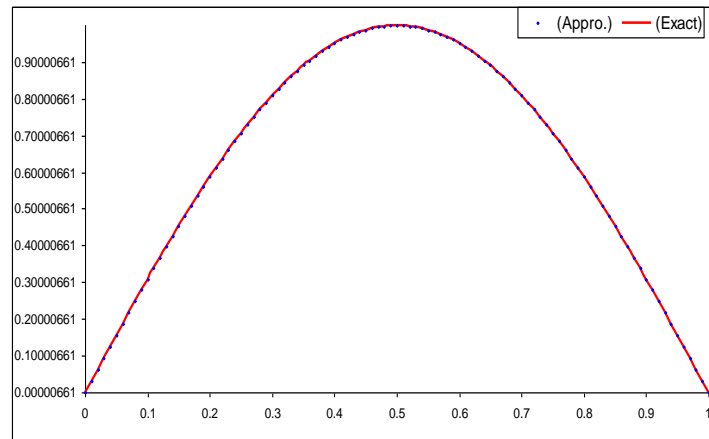
$$P_7 = -.900000 e^{-18} x^7 - 1.11814 x^6 + 3.35442 x^5 - .211490 x^4 - .16771 x^3 + .133293 e^{-2} x^2 +$$

$$3.14159 x + .270108 e^{-3}$$

$$P_9 = -.406544 e^{-17} x^9 + .205015 x^8 - 20059 x^7 + .114551 x^6 + 2.52655 x^5 + .268188 e^{-4} x^4 -$$

$$5.16771 x^3 + .326077 e^{-5} x^2 + 3.14159 x + .660770 e^{-5}$$

The accuracy of the solution given in the following figure :



**Figure3: A comparison between exact
and approximate solution of example3**

Now we give the comparison between the solution of suggested method and solution of other methods in the following table :

Table 6: A Comparison between P₉ and other methods of example 3

xi	Y exact	$\Phi_1(x_i)$ by using B-splines	$\Phi_2(x_i)$ by using piecewise linear Rayleigh-Ritz	P9 by using Osculatory interpolation	Y-P9
0	0.0000000000	0	0.0000000000	0.0000000000	0.0000000000
0.1	0.3090169943	0.30901644	0.3102866742	0.3090187881	0.0000017937
0.2	0.5877852522	0.58778549	0.5902003271	0.5877867479	0.0000014956
0.3	0.8090169943	0.80901687	0.8123410598	0.8090132782	0.0000037161
0.4	0.9510565162	0.95105667	0.9549641896	0.9510450787	0.0000114376
0.5	1.0000000000	1.00000002	1.0041087710	0.9999848327	0.0000151673
0.6	0.9510565162	0.95105713	0.9549641893	0.9510450787	0.0000114376
0.7	0.8090169943	0.80901773	0.8123410398	0.8090132782	0.0000037161
0.8	0.5877852522	0.5877869	0.5902003271	0.5877867479	0.0000014956
0.9	0.3090169943	0.3090181	0.3102866742	0.3090187881	0.0000017937
1	0.0000000000	0	0.0000000000	0.0000000000	0.0000000000

Now we give the linear problem with Mixed BC :

Example 4

$y'' - y' - 2y - \cos(x) = 0$ with BC : $y(0) = -0.3$, $y'(1) = -(1/10)(\cos(x) - 3\sin(x))$

With Exact solution [7] : $-(1/10)(\sin(x) + 3\cos(x))$.

In this case equation (7) gives $\{a_i, b_i\}$ where $i = 2, 3, \dots$, so we have four unknown $\{a_0, a_1, b_0, b_1\}$ then BC give a_0, b_1 , thus solving (11) and (12) to obtain a_1, b_0 . The result of method given in the following table :

The result of method given in the following table :

Table 7 :The result of the methods for n = 2, 3, 4 of example 4

Pa		P5	P7	P9	
a1		-0.1000049158	-0.1000000191	-0.1000000000	
b0		-0.2462378437	-0.2462377905	-0.2462377902	
X	Y: exact	P5	P7	P9	Y-P9
0	-0.3000000000	-0.3000000000	-0.3000000000	-0.3000000000	0.0000000000000000
0.1	-0.3084845912	-0.3084847623	-0.3084845927	-0.3084845913	0.000000000004490
0.3	-0.3161529674	-0.3161500465	-0.3161529573	-0.3161529674	0.000000000021532
0.5	-0.3112173224	-0.3112115318	-0.3112172966	-0.3112173224	0.000000000072229
0.7	-0.2938744249	-0.2938708653	-0.2938744115	-0.2938744249	0.000000000031538
0.9	-0.2648156814	-0.2648154464	-0.2648156812	-0.2648156814	0.000000000000232
1	-0.2462377902	-0.2462378437	-0.2462377905	-0.2462377902	0.000000000000700
S.S.E=6.69599140934226E-21					

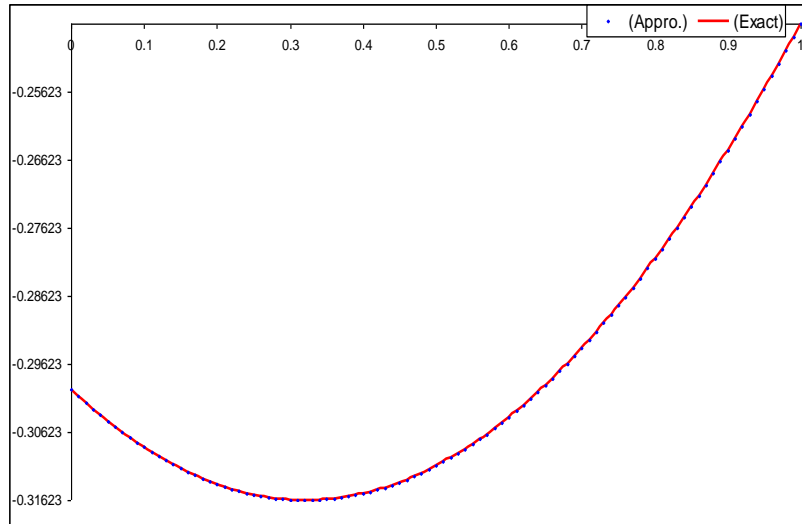
Then from table 7 and the relation (2) and (3) in the previous section we have :

$$P_5 = .475790e^{-3} x^5 - .138393e^{-1} x^4 + .171330 e^{-1} x^3 + .149998 x^2 - .100005 x - .300000$$

$$P_7 = -.112040e^{-4} x^7 + .463829 e^{-3} x^6 - .865238 e^{-3} x^5 - .124918 e^{-1} x^4 + .166667 e^{-1} x^3 + .150000 x^2 - .100000 x - .300000$$

$$P_9 = .154860 e^{-6} x^9 - .830918 e^{-5} x^8 + .207163 e^{-4} x^7 + .416225 e^{-3} x^6 - .833243 e^{-3} x^5 - .125000 e^{-1} x^4 + .166667 e^{-1} x^3 + .150000 x^2 - .100000 x - .300000$$

The accuracy of the solution given in the following figure :



**Figure4: A comparison between exact
and approximate solution of example4**

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حول حل مسائل القيم الحدودية الخطية من الرتبة الثانية ذات نقطتين باستخدام الطريقة شبه التحليلية

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الخلاصة:

يتضمن البحث طريقة جديدة لحل مسائل القيم الحدودية الخطية بنقطتين ذات الرتبة الثانية (TPBVP) ، حيث نقترح الحل كمتعددة حلول لحل مسائل القيم الحدودية الخطية بنقطتين ذات الرتبة الثانية (TPBVP) للمعادلات التفاضلية الاعتيادية. أستخدمنا الإستراتيجية شبه التحليلية باستخدام نوع من الاندراج (two-point osculatory interpolation) لعدد من مشتقات نقطتي نهاية الفترة $[0, 1]$ وقورنت النتائج مع الحل باستخدام الطرق التقليدية الأخرى وأثبتنا من خلال الأمثلة بان الطريقة المقترحة هي الأسرع والأدق .