

Symmetry for Initial Boundary Value Problems of PDEs

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Abstract

In this paper, we discuss the reduction of IBVP by using the definition of Bluman or Ibragimov. Moreover, some examples that explain this definition for linear and nonlinear heat equation, are given. Next, we will give some restrictions on the Lie symmetry, which make the initial-boundary conditions are invariant. Finally, we find an initial condition which need not to be left invariant by a Lie symmetry in order to find an invariant solution, which satisfies that initial condition.

Keywords : Symmetry, Invariant, Reduction, Lie point symmetry, Exact solution, BBM equation.

1 Introduction

Most of the engineering and physical problems require PDEs to solve subject to suitable initial and/or boundary conditions. Group-theoretic methods are powerful and fundamental to the development of systematic procedures that leads to solutions of IBVPs. In the literature of engineering and applied sciences, some authors [12, 13, 17] investigated similarity methods by considering the governing equation first and only examining the boundary Hansen, that made the key contributions to the area of similarity analysis pertaining to engineering IBVP. One may note that, little attention, in the literature, has been devoted to use Lie symmetries to solve IBVP. We mention [18], since three criteria must be satisfied in order to guarantee that the equation is integrability or reduction, namely:

1. A symmetry of the governing differential equation.
2. A smooth bijective mapping of the domain to itself.
3. A mapping of the set of boundary data to itself.

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The procedure for finding Lie point symmetries of a given differential equations is well known [3, 5, 6, 10, 14, 15, 18, 20, 23, 24, 26]. The difficulty that the extra conditions (2) and (3) may not be satisfied by any of these symmetries; in this case, it seems that IBVP has no point symmetries. In fact, there exist several approaches exploiting Lie symmetries of IBVP for PDEs. The new purpose of this paper is to study these several approaches in two directions:-

The first direction, which is a direct implementation of the rigorous definitions of Bluman and Ibragimov. This leads to a reduction of the IBVP for PDE into a IBVP, which depends on less of independent variables. This means that, we have determined all the Lie point symmetries (by Lie's algorithm), that leave the differential equation invariant, then we determine which of those Lie point symmetries (subgroups) can also leave the domain and boundary-initial conditions invariant. The Lie point symmetries that leaves the differential equation and the boundary condition invariant is used to determine the invariant solution of the IBVP [1, 3, 7, 19].

The other direction is to make the definition above more tractable, which is called the classification approach, this an approach in which the group classification of the PDE and associated initial and boundary conditions are carried out simultaneously, where classifying possible initial and boundary conditions that are consistent with symmetries of a particular differential equations, that is, the form of any boundary-initial conditions invariant under some subalgebra, may be found [22].

In [4, 25, 27] it has been studied the conversion in the BVP of the ODE into IVP by using scaling group and/or translation group and then solving IVP numerically.

Habiballin [16] discussed the problem of boundary conditions for evolution PDE, which considered compatible with their higher (Lie Backlund) symmetries . Zhdanov and Andreitser [2, 28] solved the IVP for evolutionary PDE by using higher-order conditional symmetries though the reduction of IVP to cauchy problem for a system of ODEs. In [7, 8, 9] it was given a new definition of Lie invariance for IBVPs (Standard and moving boundary conditions) and a new definition of conditional invariance for IBVP.

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$$G(y, v, v_{(1)}, \dots, v_{(k)}) = 0 \quad (8)$$

which defined on some domain Ω_y in t-space with the boundary conditions

$$C_\alpha(y, v, v_{(1)}, \dots, v_{(k-1)}) = 0 \quad (9)$$

prescribed on the boundary surfaces $V_\alpha(y) = 0$ (10)
for some G, C_α, V_α

Remark 2.1 [5] Note that the surfaces $y_j(x) = 0, j = 1, 2, \dots, n - 1$ are invariant surfaces of the group. The condition (6) means that each boundary surface $w_\alpha(x)$ is an invariant surface $V_\alpha(y) = 0$ of the infinitesimal generator $\sum \zeta_i(x) \partial_{x_i}$ which is the restriction of y to x -space. From invariance under (4) the number of independent variables in BVP (1), (2) and (3) is reduced by one. The solution of IBVP (1), (2) and (3) is an invariant solution

$$v = \phi(y_1, y_2, \dots, y_{n-1}) \quad (11)$$

of PDE (1) corresponding to its invariance under (4). The invariant solution $u = \theta(x)$, defined by (1), satisfies

$$X(u - \theta(x)) = 0 \quad \text{when } u = \theta(x)$$

this is

$$\sum \zeta_i \frac{\partial \theta}{\partial x_i} = \eta(x, \theta(x)).$$

Ibragimov stated the previous definition in the following sense:

Definition 2.2 (Invariance principle)[21].

If a boundary-value problem is invariant with respect to a group, then one should look for a solution in the class of functions invariant with respect to that group.

Remark 2.2 Invariance of boundary value problem in the sense Ibragimov presupposes invariance of the differential equation, as well as of the manifold containing the data, and of the data given on that manifold. Clear that there exists equivalence of the definitions (2.1) and (2.2).

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The above definition is used in many articles to find invariant (similarity) solutions [1, 3, 5, 11].

3 Some Applications

We gave some applications for a first direct, that is, we find all the Lie point symmetries, that leaves the DE invariant, then we determine which of these Lie point symmetries also leaves the domain and boundary-initial conditions invariant. Also, we find the exact solution and reduction equation for the linear and nonlinear heat equation.

3.1 Exact Solution of Heat Equation[1]

As an illustration, we will consider the following IBVP commonly known as heat problem, this problem related to transfer of heat by conduction. The analysis of such problems is required in many physical engineering problems. For example, the cooling of electronic equivalent, the design of themal-fluid, and so on. In particular, a major objective of the solution of such problems is to determine the temperature field in a medium. The problem studied here for the transient condition in semi-infinite solid with constant surface temperature conditions precisely, we investigate the following IBVPs

$$u_t = \alpha u_{xx} \quad t > 0, x > 0 \quad (12)$$

with initial boundary conditions

$$u(x, 0) = T_0 \quad x > 0 \quad (13)$$

$$u(0, t) = T_1 \quad t > 0 \quad (14)$$

$$u(\infty, t) = T_0 \quad t > 0. \quad (15)$$

The Lie symmetry of (12) is

$$X = (c_1 + c_2 t + c_3 x + c_4 xt) \partial_x + (c_5 + 2c_3 t + c_4 t^2) \partial_t + \left\{ \tilde{u}(x, t) + \left[c_6 - c_4 \left(\frac{x^2}{4} + \frac{t}{2} \right) - \frac{c_2}{2} x \right] u \right\} \partial_u$$

we can write the above Lie symmetry X by

$$X: = \tilde{X} + X_7 = \sum_{i=1}^6 c_i X_i + X_7$$

where

$$X_1 = \partial_x, \quad X_2 = t \partial_x - \frac{xu}{2} \partial_u, \quad X_3 = x \partial_x + 2t \partial_t \\ X_4 = xt \partial_x + t^2 \partial_t - \left(\frac{x^2}{4} + \frac{t}{2} \right) u \partial_u$$

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$$X_5 = \partial_t \quad , \quad X_6 = u\partial_u \quad , \quad X_7 = \tilde{u}\partial_u$$

where \tilde{u} is the particular solution of (12).

We consider the symmetry operator \tilde{X} of PDE (12) and search for the operator that preserves the boundary and boundary conditions. The invariance of the boundaries $x=0, t=0$, that is,

$$\begin{aligned} [\tilde{X}(x-0)]|_{x=0} &= 0 \\ [\tilde{X}(t-0)]|_{t=0} &= 0 \end{aligned}$$

which implies

$$c_1 = c_2 = c_4 = 0$$

hence, \tilde{X} must be

$$\tilde{X} = c_3x_3 + c_5x_5 + c_6x_6.$$

The invariance of the boundary condition, which are:

$$\begin{aligned} [\tilde{X}(u(x,t) - T_0)]|_{t=0} &= 0 \quad \text{on} \quad u(x,0) = T_0 \\ [\tilde{X}(u(x,t) - T_1)]|_{x=0} &= 0 \quad \text{on} \quad u(0,t) = T_1 \end{aligned}$$

This implies, we must have

$$c_5 = 0 = c_6$$

hence the IBVP is invariant under the symmetry

$$\tilde{X} = 2t\partial_t + x\partial_x$$

where we have chosen $c_3 = 1$.

The invariant solution of the problem is constructed by utilizing the transformation through similarity variables for \tilde{X} . Solving the characteristic system for $\tilde{X}I = 0$ gives $I_1 = \frac{x^2}{t}$ and $I_2 = u$ as the differential invariants of $\tilde{X} = 2t\partial_t + x\partial_x$. Hence, the similarity variables for \tilde{X} are

$$u = v(z) \quad \text{where} \quad z(x,t) = \frac{x^2}{t} \tag{16}$$

substitution (16) and its derivative in (12), implies the ODE,

$$uz \frac{d^2v}{dz^2} + \left(2 + \frac{z}{\alpha}\right) \frac{dv}{dz} = 0$$

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the above equation can be integrated, using the substitution

$$W = \frac{dv}{dz}$$

to obtain

$$v(z) = k_1 \int \frac{e^{-\frac{z}{\sqrt{z}}}}{\sqrt{z}} dz + k_2$$

making the change of variable $y^2 = \frac{z}{4\alpha}$ in the above solution yields

$$v = 4k_1\sqrt{\alpha} \int e^{-y^2} dy + k_2$$

that is,

$$v = 2k_1\sqrt{\alpha\pi}(\operatorname{erf}(y)) + k_2$$

which implies the exact solution of IBVP that is invariant under $\tilde{X} = 2t\frac{\partial}{\partial t} + x\frac{\partial}{\partial x}$ is

$$u(x, t) = 2k_1\sqrt{\alpha\pi}[\operatorname{erf}(\frac{x/\sqrt{t}}{2\sqrt{\alpha}})] + k_2$$

imposing the boundary conditions determines

$$k_1 = \frac{T_0 - T_1}{2\sqrt{\alpha\pi}}, k_2 = T_1$$

we get the solution

$$u(x, t) = (T_0 - T_1)\operatorname{erf}(\frac{x}{2\sqrt{\alpha\pi}}) + T_1$$

of the IBVP (12), (13), (14) and (15).

3.2 The Reduction of Nonlinear Heat Equation

Consider the nonlinear equation

$$u_t = (uu_x)_x \quad 0 < x < \infty, \quad t > 0 \quad (17)$$

with initial-boundary value conditions

$$u(0, t) = 1, \quad t > 0 \quad (18)$$

$$u(\infty, t) = 0, \quad t > 0 \quad (19)$$

$$u(x, 0) = 0, \quad x \geq 0. \quad (20)$$

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By using PDE tools package, we get the Lie symmetries of the equation (17),

$$X_1 = \partial_t, \quad X_2 = \partial_x,$$

$$X_3 = \frac{1}{2}x\partial_x + t\partial_y,$$

$$X_4 = \frac{1}{2}x\partial_x + u\partial_u.$$

We can write the above Lie symmetry, as

$$X = \sum_{i=1}^4 C_i X_i$$

the invariance of the boundaries $x=0$ and $t=0$ implies $c_1 = c_2 = 0$ and the invariance of the boundary condition (18), (19) and (20) implies $c_4 = 0$ hence the IBVP is invariant under the symmetry,

$$X_3 = \frac{1}{2}x\partial_x + t\partial_y$$

solving the characteristic system for,

$$X_3 u = 0$$

we get, the similarity variables,

$$u = f(z), \quad z = \frac{x}{\sqrt{t}} \quad (21)$$

substituting (21) and its derivative in (17), (18), (19) and (20), we get the BVP for ODE,

$$2(f''f + f'^2) + Zf' = 0$$

with boundary conditions

$$f(0) = 1$$

$$f(\infty) = 0.$$

4 Classification Technique

Although the previous approach, which consists of three steps is conventional to solving IBVP, sometimes it is difficult to proceed beyond the first step. This is because the PDE admits two or three symmetries, which are not suitable to satisfy second and/or third step or, there are infinite numbers of transformation groups that leave the PDE invariant. Alternatively, the form of any boundary condition invariant under some subalgebra may be found, that is how to find the initial-boundary conditions, which are invariant under the Lie symmetries (subgroup) of the PDE.

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4.1 Restrictions on Lie Symmetry X

In this section, we consider the conditions on the initial or boundary data (are determined) which guarantee the invariance under the Lie symmetry X.

Suppose that we have (1+1) dimensional PDE, which is invariant under Lie point symmetry $X = \zeta(x, t, u)\partial_x + \tau(x, t, u)\partial_t + \eta(x, t, u)\partial_u$

Proposition 4.1 *The initial condition $u(x, t_0) = f(x)$ is invariant under X, if and only if $\eta(x, t_0, f(x)) = \zeta(x, t_0, f(x))f'(x)$*

The boundary $(t = t_0)$ is invariant under X, if and only if $\tau(x, t_0, u(x, t_0)) = 0$

Proof.

$u(x, t_0) = f(x)$ is invariant, that means

$$[X(u(x, t) - f(x))] \Big|_{\substack{u(x, t_0)=f(x) \\ t=t_0}} = 0$$

which implies

$$\eta(x, t, u) - \zeta(x, t, u)f'(x) \Big|_{\substack{u(x, t_0)=f(x) \\ t=t_0}} = 0$$

i.e.

$$\eta(x, t_0, f(x)) = \zeta(x, t_0, f(x))f'(x)$$

and vice versa

The boundary $t - t_0 = 0$ is invariant, this means,

$$X(t - t_0) \Big|_{t=t_0} = 0$$

Hence $\tau(x, t_0, u(x, t_0)) = 0$ and vice versa

In general, if we assume that the boundaries and the boundary condition are invariant, we get

Proposition 4.2 *The boundary $B(x, t) = 0$ and the boundary condition $u = M(x, t)$ on $B(x, t) = 0$ are invariant under X, if and only if*

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$$\begin{aligned} \zeta(x, t, M)M_x + \tau(x, t, M)M_t &= \zeta(x, t, M) && \text{and} \\ \zeta(x, t, M)B_x + \tau(x, t, M)B_t &= 0 \end{aligned}$$

Proof. By the same way, as in prop. (4.2).

Corollary 4.1 *The boundary ($x = x_0$) and boundary condition $u(x_0, t) = g(t)$ are invariant under X , if and only if $\eta(x_0, t, g(t)) = \tau(x_0, t, g(t))g'(t)$ and $\xi(x_0, t, u) = 0$*

Corollary 4.2 *The boundary ($x=h(t)$) and boundary condition $u(x, t) = u_0$, where u_0 is constant at $x = h(t)$ are invariant under X , if and only if $\eta(h(t), t, u_0) = 0$ and $\zeta(h(t), t, u_0) = \tau(h(t), t, u_0)h'(t)$*

Proposition 4.3 *The boundary ($x = x_0$) and boundary condition $u_x(x_0, t) = f(t)$ are invariant under X , if and only if $\xi(x_0, t, u) = 0$, $\tau_x(x_0, t, u(x_0, t)) = \tau_u(x_0, t, u(x_0, t))f(t)$ and $\eta_x(x_0, t, u) - \eta_u(x_0, t, u)f(t) = f(t)[\xi_x(x_0, t, u) + \xi_u(x_0, t, u)f(t)] + \tau(x_0, t, u)f'(t)$*

Proof.

From

$$X[x - x_0]|_{x=x_0} = 0$$

which implies

$$\xi(x_0, t, u) = 0$$

and from

$$X^{[1]}[u_x(x, t) - f(t)]|_{\substack{u_x=f(t) \\ x=x_0}} = 0$$

where $X^{[1]}$ is the prolongation of X , that implies

$$[\eta^{(1)} - \tau f'(t)]|_{\substack{u_x=f(t) \\ x=x_0}} = 0$$

that is

$$\eta_x + \eta_u \cdot u_x - u_t(\tau_x + \tau_u \cdot u_x) - u_x(\xi_x + \xi_u \cdot u_x) - \tau f'(t)|_{\substack{u_x=f(t) \\ x=x_0}} = 0$$

we get

$$\eta_x + \eta_u f - u_t(\tau_x + \tau_u \cdot f) - f(\xi_x + \xi_u f) - \tau f'(t) = 0 \tag{22}$$

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since $\eta_x, \eta_u, \tau_x, \tau_u, \xi_x, \xi_u, \tau, f$ are functions of x, t, u , we get

$$\tau_x(x_0, t, u(x_0, t)) = \tau_u(x_0, t, u(x_0, t))f(t)$$

and

$$\eta_x(x_0, t, u) - \eta_u(x_0, t, u)f(t) = f(t)[\xi_x(x_0, t, u) + \xi_u(x_0, t, u)f(t)] + \tau(x_0, t, u)f'(t)$$

and vice versa.

Remark 4.1 We note the restriction on X are differential equations. These equations may be solved to obtain the admissible initial-boundary condition which are compatible with Lie point symmetries for PDE. Which will be clarified in the following application.

4.2 Application the IBVP of the BBM Equation

The BBM equation

$$u_t + u_x + uu_x - u_{xxt} = 0 \quad x > 0, t > 0 \quad (23)$$

with initial-boundary conditions

$$u(0, t) = g(t) \quad t \geq 0 \quad (24)$$

$$u(x, 0) = f(x) \quad x \geq 0 \quad (25)$$

with compatibility condition $u(0, 0) = g(0) = f(0)$, has a unique classical solution $u(x, t) \in C^2$, if $f(x) \in C^2$ and $g(t) \in C^1$ [?].

Proposition 4.4 The IBVP (23), (24) and (25) are invariant under

$$X = c_1 \partial_x + (c_2 + c_3 t) \partial_t - c_3 (1 + u) \partial_u, \quad (26)$$

if and only if $f(t) = -1, g(t) = -1 + \frac{c}{t}$ and $c_1 = c_2 = 0$.

Proof.

To find a classification of all initial and boundary conditions that invariant under Lie symmetry (26), this is analogous to classification problem for differential equation with arbitrary parameters, identifying special values of arbitrary parameters, allowing a larger symmetry group.

By using proposition (4.1) and corollary (4.1), we get the restrictions:

i. $\tau(x, 0, f(x)) = 0$

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ii. $\xi(0, t, g(t)) = 0$

iii. $\eta(x, 0, f(x)) = \xi(x, 0, f(x))f'(x)$

iv. $\eta(0, t, g(t)) = \tau(0, t, g(t))g'(t)$

clearly, (i) implies $c_2 = 0$ and (ii) implies $c_1 = 0$.

Hence X becomes

$$X = t\partial_t - (1 + u)\partial_u \quad (27)$$

(iii) gives $-(1 + f(x)) = 0$, that is $f(x) = -1$, while (iv) gives

$$-(1 + g(t)) = tg'(t) \quad (28)$$

i.e. we get $g(t) = -1 + \frac{c}{t}$, where c is arbitrary constant

Hence the IBVP (23), (24) and (25) with $g(t) = -1$ and $f(x) = -1$ are invariant under Lie symmetry (27).

Now, we are trying to find a solution to the IBVP (23), (24) and (25) as follows from $Xu = 0$, where X defined in (27), we find that the similarity variables

$$u = \frac{\varphi(x)}{t} - 1$$

substituting this variable and their derivative in (23), we get the reduced equation

$$\varphi''(x) - \varphi(x)\varphi'(x) - \varphi(x) = 0 \quad (29)$$

with the boundary condition

$$\varphi(0) = 0.$$

The equation (29) has only one Lie symmetry (hidden symmetry)

$$X = \partial_x \quad (30)$$

by using invariant differential method, we can transform (29) into the first order of ODE as follows:

From (30), we find

$$u = \varphi \quad \text{and} \quad v = \varphi', \quad (31)$$

substitute (31) into (29), we get

$$v'v - uv - u = 0, \quad (32)$$

where $v' = \frac{dv}{du}$, and from integrating (32), we get

$$v + \ln(v + 1) = \frac{u^2}{2} + A$$

, where A is constant, from (31), we get

$$\ln(\varphi' + 1) + \varphi' - \frac{x^2}{2} = A, \quad (33)$$

with $\varphi(0) = 0$.

This problem may be solved numerically.

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The remark below explains that, an initial condition may not be left invariant under a symmetry, in order to find invariant solutions, which satisfies initial condition.

Remark 4.2 Consider the initial condition (25). By using proposition (4.1), if $\tau(x, 0, f(x)) \neq 0$ then the domain $t = 0$ is not an invariant domain under X . By using invariant surface condition for the invariant solutions, that is,

$$\zeta(x, t, u)u_x + \tau(x, t, u)u_t = \eta(x, t, u), \quad (34)$$

we get

$$u_t(x, 0) = \frac{\eta(x, 0, f(x)) - \zeta(x, 0, f(x))f'(x)}{\tau(x, 0, f(x))}, \quad (35)$$

consider evolution scalar PDE

$$u_t = F(t, u, u_{(1)}, \dots, u_{(n)}), \quad (36)$$

which implies

$$u_t(x, 0) = F(0, x, u(x, 0), u_x(x, 0), u_{(n)}(x, 0)),$$

that is,

$$u_t(x, 0) = F(0, x, f, f', \dots, f^{(n)}), \quad (37)$$

substituting (37) into (35) we get n_{th} ODE

$$F(0, x, f, f', \dots, f^{(n)}) = \frac{\eta(x, 0, f(x)) - \zeta(x, 0, f(x))f'(x)}{\tau(x, 0, f(x))},$$

If we could solve this equation, we get that the initial condition is not invariant.

Example 4.1

$$u_t = u_{xx} + tu \quad -\infty < x < \infty, \quad t > 0, \quad (38)$$

$$u(x, t)|_{t=0} = f(x) \quad -\infty < x < \infty,$$

The (38) is admitted by symmetry operator.

$$X = \frac{\partial}{\partial t} - \frac{\partial}{\partial x} + (2+t)u\partial_u$$

To find $f(x)$ such that the initial condition is not invariant under X , we must solve the following equation

$$f''(x) = \frac{2f(x) + f'(x)}{1}$$

that is

$$f''(x) - f'(x) - 2f(x) = 0 \quad (39)$$

solving (39), we get

$$f(x) = e^{2x} \quad \text{or} \quad e^{-x}$$

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this implies

$$\begin{aligned} u_t &= u_{xx} + tu & -\infty < x < \infty, t > 0 \\ \text{if } u(x, 0) &= e^{2x} & -\infty < x < \infty \end{aligned} \quad (40)$$

solving the corresponding ISC to this symmetry operator

$$\frac{dx}{-1} = \frac{dt}{1} = \frac{du}{(2+t)u'}$$

we get

$$u(x, t) = e^{\frac{t^2}{2} + 2t} \varphi(x + t)$$

comparing at $t = 0$ with I.C.

$$\varphi(x) = e^{2x}$$

so that

$$\varphi(x + t) = e^{2(x+t)}$$

hence, the solution to the IVP (40) is

$$u(x, t) = e^{\frac{t^2}{2} + 4t + 2x}$$

5 Conclusion

1. We note that, once we find a suitable functional form for a similarity solution, we need not substitute it into the governing equation to find the arbitrary function. Rather that we compare the functional form at the initial value with the initial condition to find the arbitrary function.

2. This method can be applied not only to the equation of the type of evolution, but for any scalar partial differential equation. Difficulty lies in finding the solution of the resulting ordinary differential equation and often with nonlinear equation.

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التماثل لمسائل القيم الحدودية الابتدائية للمعادلات التفاضلية الجزئية

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المستخلص

في هذا البحث ناقشنا تخفيض مسائل القيم الحدودية الابتدائية باستخدام تعريف بلمان (Blumen) أو أبرغيموف (Ibragimov) وأكثر من ذلك تناولنا بعض الأمثلة لتوضيح التعريف لمعادلة الحرارة الخطية-الغير الخطية وأخيراً أعطينا بعض القيود على تماثل لي للشروط الحدية-الابتدائية لكي تكون لامتغيرة ووجدنا الشرط الابتدائي الذي لا يحتاج أن يكون لامتغير بواسطة تماثل لي من أجل إيجاد الحلول اللامتغيرة والذي يحقق الشرط الابتدائي.

الكلمات الدالة : التماثل, اللامتغيرة, تخفيض, تماثل لي النقطي, الحل المضبوط, معادلة

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