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The relation ships Between contra t-continuous functions

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Abstract:-

In this paper we use the concept of (δ,gδ)-sets [4] ,α-sets [11],Semi-sets [7], pre-sets [1],β-sets [6] and regular sets[11] .

To study the relation ships of the functions contra t-continuous, contra continuous with anew class of functions called contra (δ,gδ)-continuous[12],where t meaning the type of these functions that depending on these sets.

1. Introduction

In 1996 Dontchev[3] introduced anew class of functions called contra-continuous functions, in 1999 Dontchev and Noiri introduced and studied ,among others anew weaker form of this class of functions called contra –semi continuous[2],also in the semi year Jafari and noiri introduced and studied anew class of functions called contra Super-continuous[8] ,in 2001 Jafari and noiri introduced and studied anew class of functions called contra-precontinuous[10] , contra α-continuous[9] and contra β-continuous [11],where the function contra α-continuous is weaker than contra continuous and stronger than contra –semi continuous and contra-precontinuous,in (2008)the researcher introduced and investigated strong form of this class of functions called contra (δ,gδ)- continuous [12] .

2.Preliminaries

Through the present note (x,t_x) and (y,t_y) (or simply x,y) alwayes topological space.

Aset Ais called regular open or(prif.R-O) (resp. regular closed or (prif.R-C) if A=into(cl(A))(resp. A=cl(int(A))).

Let A be asub set of the topological space (x,t_x),apoint $x\in X$ is called a δ–cluster point of A [4] if $A\cap u\neq \emptyset$,for every regular open set u containing x.The set of all δ–cluster points of A form the δ–closure of A and is denoted by $cl_\delta(A)$.

Aset A is called δ–closed or(prif.δ-C) if $A=cl_\delta(A)$. Aset A is called δ–open or(prif.δ-O) if it is a union of regular open sets, The complement of δ-open set is said to be δ-closed and aset A is called α–open or (prif.α-open) [1]if $A\subset int(cl(int(A)))$, The complement of α-open set is said to be α-closed .

Recall that the reguler open sets in agiven topological space (x,t_x) form abase for anew topology T_s on X called Semi-regularzation,the collection of all δ-open sets forms atopology on X.which is finer than any topology t denoted by T_δ [12] ,it well-known that $T_s= T_\delta$ and the collection of all α-open sets forms atopology on X ,Stronger than any topology t denoted by T^α (or α(X))(Nejastad,1965).It is abvious that $T\subset T^\alpha$ and $T_\delta \subset T$ as a consequence of definitions we have $T_\delta \subset T\subset T^\alpha$,also a $\alpha cl(A)\subset cl_\delta(A)$.

3.properties of Sets

Definitions 3.1

1- A sub Set A of aspace X is called pre-open or(prif.pre-O.) [1](resp. Semi-open or(prif. S-O.) [7],β-open or (prif. β-O.)) if $A\subset int(cl(A))$(resp. $A\subset cl(int(A))$, $A\subset cl(int(cl(A)))$). The complement of preopen (resp. Semi-open,β-open) set is said to be preclosed (resp.Semi-closed-closed).

2-Afunction f:X→Y is called contra-super continuous or(prif. C.-Su.C.)[8] (resp. contra α- continuous or(prif. C.α-C.) [9],contra-semi continuous or(prif. C.-S.)[2]) if f^{-1}(v)is regular-closed (resp.α-closed,Semi- closed) sub set in X,for every open sub set v of Y.

3- Afunction f:X→Y is called contra-precontinuous or(prif. C.-pre.C.)[10] (resp. contra continuous or(prif. C. -C.) [3],contra β-continuous or(prif. C. β-C.) [11], contra δ-continuous or(prif. C. δ-C.) [12]) if f^{-1}(v)is preclosed (resp.closed,β-closed,δ-closed) sub set in X,for every open sub set v of Y.

Lemma 3.2

Let Abe asub set of aspace X.

1- If A is closed ,then A is α-closed(resp. Semi-closed,preclosed).

2- If A is δ-closed,then A is closed (resp. α-closed, β-closed).

Proof

2-Let A be aδ-closed set ,$A=cl_\delta(A)$,by part (2) $A= cl_\delta(A)=cl(A)$,so by part(1) A is α-closed.

Or let A is δ-closed,$(A)^c=B$ is δ-open,$B=\cup_{i\in I}U_i$,where U_i are regular open sets for every $i\in I$,since $\cup_{i\in I}U_i$ open set ,so B is open set ,$B\subset cl(B)$,$int(B)\subset cl(int(B))$,$int(int(B))\subset int(cl(int(B)))$,$int(int(B))=B$, So $B\subset int(cl(int(B)))$ is α-open ,thus $(B^c)=(A^c)^c=A$ is α-closed.

To prove that A is β-closed, let A is δ-closed,$(A)^c=B$ is δ-open,$B=\cup_{i\in I}U_i$,where U_i are regular open sets for every $i\in I$,since $\cup_{i\in I}U_i$ open set ,so B is open set ,$B\subset cl(B)$,$int(B)\subset int(cl(B))$,$cl(int(B))\subset cl(int(cl(B)))$,since $B\subset cl(int(B))$,therefore $B\subset cl(int(cl(B)))$ is β-open, thus $(B^c)=(A^c)^c=A$ is β-closed.

Remark 3.3

The converse of the above Lemma is not true in general to see this ,we give the following counter Examples.

Examples 3.4

1- Let X={a,b,c,d} be aset and t_x={Ø,X,{a},{b},{a,b},{a,b,c},{a,b,d}}is atopologe defined on X.let A={d}⊂X,A is closed (resp. α-closed, β-closed).)but not δ-closed set.

2-Let R be the real line with the usuall topology ,the set A=[a,b) ⊂R is Semi-closed but not closed.

Lemma 3.5

Let A be asub set of aspace X.
1- If A is δ-closed,then A is Semi-closed(resp. preclosed).
2- If A is α-closed,then A is Semi-closed(resp.preclosed).
3- If A closed,then A is β-closed.
4- If A Semi-closed,then A is β-closed.

Proof

1- To prove that A is Semi-closed,let A is δ-closed,$(A)^c$=B is δ-open,B=$\cup_{i\in I}U_i$,where U_i are regular open sets for every i∈I,since $\cup_{i\in I}U_i$ open set ,so B is open set ,B⊂cl(B),B⊂cl(int(B)) is Semi-open,thus $(B^c)=(A^c)^c$=A is Semi-closed.

To prove that A is preclosed , let A is δ-closed,$(A)^c$=B is δ-open,B=$\cup_{i\in I}U_i$,where U_i are regular open sets for every i∈I,since $\cup_{i\in I}U_i$ open set ,so B is open set , B⊂cl(B), int(B)⊂int(cl(B)),since B=int(B),so B⊂int(cl(B)) is preopen ,thus $(B^c)=(A^c)^c$=A is preclosed.

2- Let A is α-closed set,A^c=B is α-open,so B⊂int(cl(int(B))),since int(cl(int(B))) ⊂cl(int(B)).Thus B⊂cl(int(B)),hence B is Semi-open ,so $B^c=(A^c)^c$=A is Semi-closed.

To prove that A is preclosed, Let A be an α-closed set,A^c=B is α-open,so B⊂int(cl(int(B))),since int(cl(int(B))) ⊂int(cl(B)),so B⊂int(cl(B)).Thus B is preopen , $B^c=(A^c)^c$=A is apreclosed.

Remark 3.6

The converse of the above Lemma is not true in general to see this ,we give the following counter Examples.

Examples 3.7

1- Let X={1,2,3} be aset and t_x={Ø,X,{1},{2},{1,2}} be atopology defined on X. Let A={3}⊂X is Smi-closed β-closed α-closed but not δ-closed .

2- Let X={1,2,3} be aset and t_x={Ø,X,{2},{1,2}} be atopology defined on X.Let A={1}⊂X is preclosed but not closed and not δ-closed .
3- See Example 3.5(2) ,A=[a,b) is Semi-closed but not α-closed.

Lemma 3.8

Let A be asub set of aspace X.
1- If A is regular closed, then A is δ-closed(resp. closed ,preclosed , β-closed).
2- If A is preclosed,then A is Semi-closed(resp. β-closed).

Proof

1-To see that A is δ-closed see[12],clearly that A is closed ,preclosed and β-closed.

To prove that A is preclosed,let A be aregular closed , A^c=B is regular open ,B=int(cl(B)),so B⊂int(cl(B)).Thus B is preopen, $B^c=(A^c)^c$=A is apreclosed.

2- To prove that A is Semi-closed Let A be apreclosed set, A^c=B is preopen ,B⊂int(cl(B)),since int(cl(B)) ⊂cl(int(B)),so B⊂cl(int(B)).Thus Bis Semi-open ,$B^c=(A^c)^c$=A is Semi-closed.

The interrelation of the Seventh Sets are decided the refore,we obtain the following diagram □

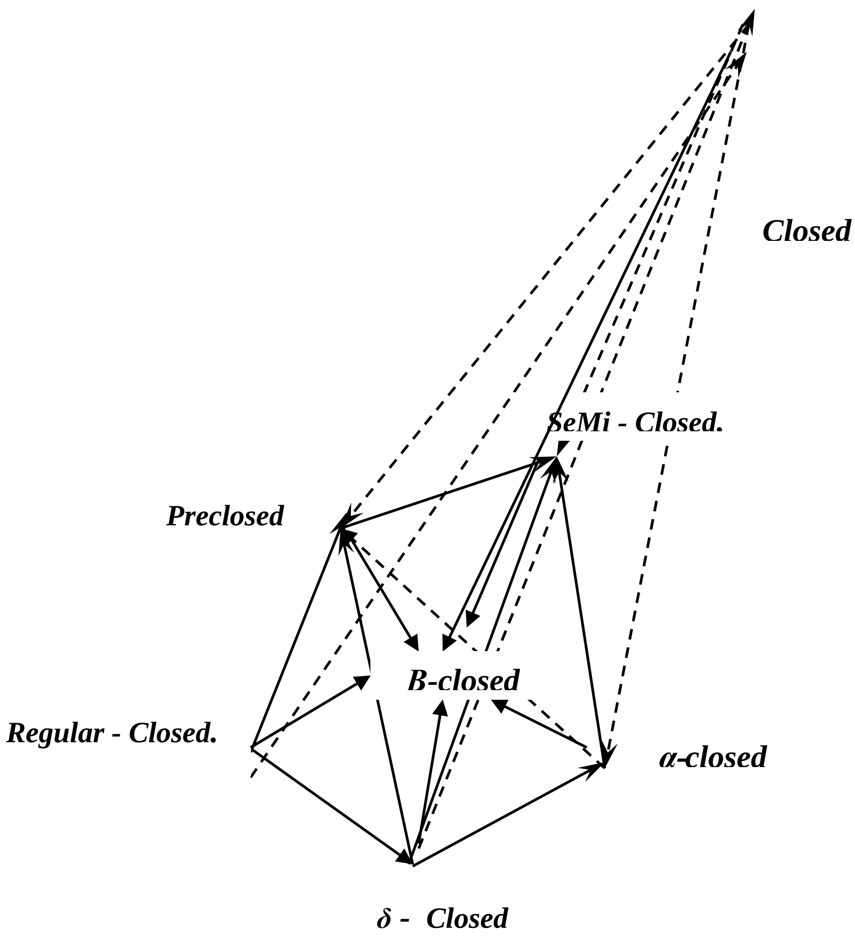

Diagram (1)

4- Relation ships

Proposition 4.1

If f:X→Y is contra δ-continuous,then f is contra t-continuous.

Proof

By definition 3.1, and Lemma 3.2 part(2),Lemma 3.5 part (1).
Where t meaning closed,α-closed,Semi-closed, preclosed and β- closed.

Remark 4.2

The converse of the above Proposition is not true in general,to see this we give the following counter Examples.

Examples 4.3

Let X=Y={a,b,c} be aset.

1-Let t_X={Ø,X,{a},{a,b}} and t_Y={Ø,y,{c},{b,c}}are topological spaces defined on X,Y respectively,let f:X→Y is the identity function.
Note that f is contra continuous and contra β-continuous but not contra δ-continuous.

2- Let t_X={Ø,X,{a}} and t_Y={Ø,Y,{b},{c},{b,c}} are topological spaces defined on X ,Y respectively,let f:X→Y is the identity function.
Note that f is contra α-continuous but not contra δ-continuous.

3- Let t_X={Ø,X,{a},{a,b},{a,c}}and t_Y={Ø,Y,{b},{c},{b,c}} are topological spaces defined on X,Y respectively,let f:X→Y is the identity function. Note that f is contra –Semi continuous but not contra δ-continuous.

4- Let t_X={Ø,X,{b,c}} and t_Y={Ø,Y,{b},{c},{b,c}} are topological spaces defined on X ,Y respectively,let f:X→Y is the identity function.
Note that f is contra –precontinuous but not contra δ-continuous.

Proposition 4.4

If f:X→Y is contra continuous,then f is contra t-continuous.

Proof

By definition 3.1 , Lemma 3.2 part(1) and Lemma 3.5 part(3).
Where t in this Proposition meaning α-closed,Semi-closed ,preclosed and β-closed.

Remark 4.5

The converse of the above Proposition is not true in general,to see this we give the following counter Examples.

Examples 4.6

1- See Example 4.3(2),Note that f is contra α-continuous but not contra continuous.
2- Let X= {a,b,c,d} be aset ,let
t_X={Ø,X,{b},{c},{a,b},{b,c},{a,b,c},{b,c,d}}be atopology defined on X,let f:X→X be afunction defined as follows :f(a)=c,f(b)=d,f(c)=b and f(d)=a.
Note that f is contra –Semi continuous and contra β–continuous but not contra continuous.

3- See Example 4.3(4),Note that f is contra –precontinuous but not contra continuous.

Proposition 4.7

If f:X→Y is contra α–continuous,then f is contra –Semi continuous.

Proof

By definition 3.1 and Lemma 3.5 part(2).

Proposition 4.8

Afunction f:(X,t)→(Y,σ) is contra α-continuous (resp. contra –precontinuous, contra Semi–continuous if and only if
f:(X,t^{α})→(Y,σ) is contra continuous.

Proof

→ By definition 3.1 and since X is t^{α}-space.
← By Lemma 3.5 part(2).
Where t in this Proposition meaning closed,Semi-closed , preclosed and β-closed set.

Proposition 4.9

If f:X→Y is contra –Super continuous,then f is contra t-continuous.

Proof

By definition 3.1 and Lemma 3.8 part(1).
Where t in this Proposition meaning δ-closed,closed , preclosed and β-closed set.

Remark 4.10

The converse of the above Proposition is not true in general,to see this we give the following counter Examples.

Examples 4.11

1- Let X=Y={a,b,c}be aset ,let t_X={Ø,X,{a},{b},{a,b},{b,c}} and t_Y={Ø,Y,{c},{a,c},{b,c}} are topological spaces defined on X,Y respectively,let f:X→Y is the identity function.Note that f is contra δ-continuous but not contra –Super continuous function.

2- See Example 4.3(4).Note that f is contra –continuous(resp. contra –precontinuous but not contra –Super continuous.

Remark 4.12

The converse of the above Proposition is not true in general,See Example 4.6(2),Note that f is contra –Semi continuous but not contra α–continuous function.

Proposition 4.13

If f:X→Y is contra–precontinuous,then f is contra –Semi continuous.

Proof

By definition 3.1 and Lemma 3.8 part(2).

Remark 4.14

The converse of the above Proposition is not true in general,See Example 4.6(2),Note that f is contra –Semi continuous but not contra–precontinuous function.
To make the converse of Proposition 4.1 and Proposition 4.8 for t=δ-closed set is true.we will give the sufficient condition.

Proposition 4.15

Afunction $f:(X,t)\rightarrow(Y,\sigma)$ is contra δ-continuous if and only if $f:(X,t_\delta)\rightarrow(Y,\sigma)$ is contra t-continuous.

Proof

→By definition 3.1 and Proposition 4.1.
←By definition 3.1 and since X is T_δ-space.
Where t in this proposition meaning closed,α-closed,Semi-closed,preclosed and regular closed β-closed.

The interrelation of the Seventh Functions are decided the refore, we obtain the following diagram

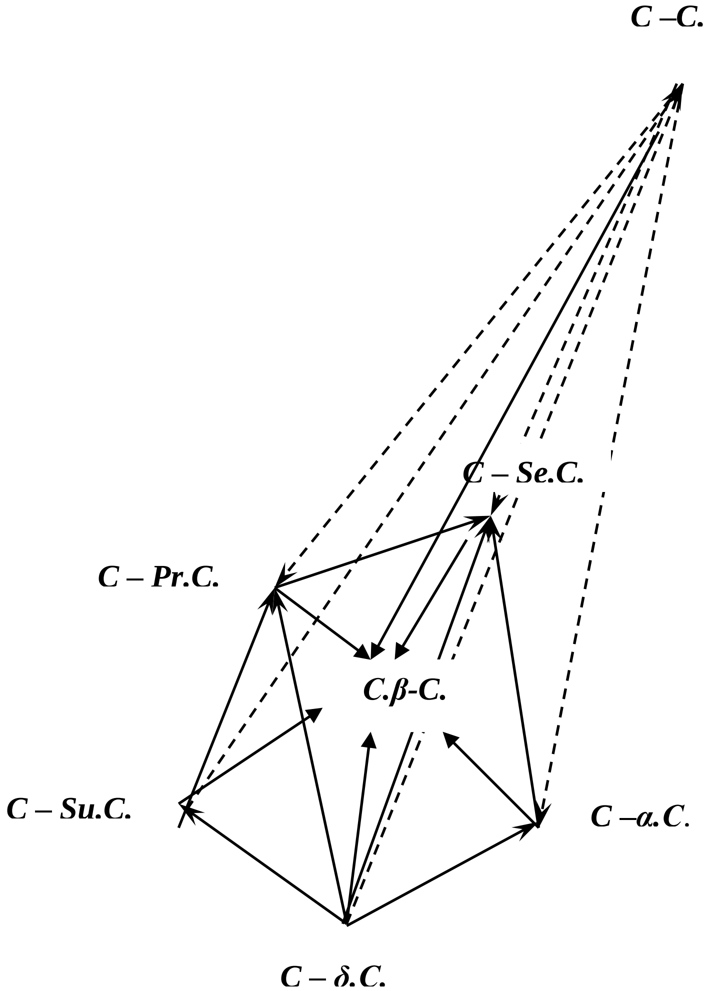

Diagram (2)

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العلاقة بين الدوال العكس مستمرة - t

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الملخص:-

في هذا البحث استخدمنا مفهوم المجموعات - (δ وgδ) [4] المجموعه- α [11] ، المجموعات شبه[7]، المجموعات الأولية[1] ،المجموعات المنتظمة[11] ،المجموعات-β [6] ،لدراسة العلاقة ببين تلك المجموعات كتمهيد لأيجاد العديد من العلاقات بين الدوال العكس مستمرة (او الضد مستمرة)، العكس مستمرة-t مع نوع جديد من الدوال تسمى العكس مستمرة-(δ,gδ) [12]حيث t تعني نوع هذه الدوال التي اعتمد تعريف كل واحدة منها عاى احدى انواع تلك المجموعات.