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Fuzzy b-Compact Topological Spaces

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Abstract:-

The purpose of this paper is to introduce [6] and discuss the concept of b-compactness for fuzzy topological spaces .Also we introduce and discuss the concept of b-continuous function and b-irresolute function [6].

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1- Introduction:-

The concept of fuzzy set introduced by L .A. Zadeh in [1].The discovery of fuzzy subsets lead to the Introduction of the basic concept of fuzzy topology which was introduced by C.L.Chang [2].D. Andrijevic[3] introduced the concept of b-open set .

In this paper, we introduce and discuss the concept of fuzzy b-open set, and fuzzy b-compact topological space.

2-Preliminaries

In this section, we recall the basic concepts needed in this work.

Definition 2-1 [2]

Let X be a non empty set, and let τ be collection of fuzzy sets in X satisfying:

1- $0,1 \in \tau$,
2- If A and B belongs to τ, then $A \wedge B \in \tau$, and
3- If A_i belongs to τ, for each $i \in I$ then $\bigvee_{i\in I} A_i \in \tau$.

Definition.2-2 [ ]

For a set X we define a fuzzy set in X to be a function $A(x): X \to [0,1]$here $A(x)$ "represents the degree of membership of x in the fuzzy set A."

Then we say that τ is a fuzzy topology on X , and the pair (X, τ_f) is called a fuzzy topological space.

Members of τ are called fuzzy open set. Fuzzy sets of the form $1-A$, where A is fuzzy open set are called fuzzy closed.

Definition 2-3 [4]

A fuzzy set A in a fuzzy topological space X is called Fuzzy semi-open set if $A \le \mathrm{cl}\,(\mathrm{int}\, A)$.

Definition 2-4 [5]

A fuzzy set A in a fuzzy topological space X is called Fuzzy pr-open set if $A \le \mathrm{int}\,(\mathrm{cl}\, A)$.

Definition 2-5[5]

A fuzzy set A in a fuzzy topological space X is called Fuzzy b-open set if $A \le [\mathrm{int}\,(\mathrm{cl}\, A) \vee \mathrm{cl}\,(\,\mathrm{int}\, A\,)$.

The set of all fuzzy b-open subsets of X will be denoted by FB (X) ,The complement of fuzzy b-open set is called fuzzy b-closed set, A will be fuzzy b-closed set if $[\mathrm{int}\,(\mathrm{cl}\, A) \wedge \mathrm{cl}\,(\,\mathrm{int}\, A\,) \le A$

Definition 2-6 [5]

Let (X, τ_f) be a fuzzy topological space, let u be a fuzzy set in X .A fuzzy set A in X is said to be a fuzzy b-neighborhood of u iff there exist a fuzzy b-open set v such that $u \le v \le A$.

Definition 2-7 [6]

The union (resp. intersection) of the fuzzy sets $\bigvee_{i\in I} A_i$, $i \in I$ ($\bigwedge_{i\in I} A_i(x)$, $i \in I$) is defined by $\bigvee_{i\in I} A_i(x) = \sup\{A_i(x) : i \in I\}$ (resp. $\bigwedge_{i\in I} A_i(x) = \inf\{A_i(x) : i \in I\}$

Definition 2-8 [6]

Let (X, τ_f) be a fuzzy topological space:

1- A fuzzy set B in X is fuzzy b-compact if whenever $\bigvee_{i\in I} B_i \ge A$ where $B_i \in \tau$ for all $i \in I$ and $\varepsilon > 0$, then there are finitely many B_i 's, say $B_{i_1},\ldots,B_{i_n}$ such that $\bigvee_{i\in I} B_{i_j} \ge A - \varepsilon$.

2- Is (X, τ_f) fuzzy b-compact iff each constant fuzzy set in X is fuzzy b-compact.

Definition 2-9 [3]

A fuzzy topological space (X,τ_f) is called fuzzy b-compact iff fuzzy b-open cover has a finite fuzzy sub cover.

Definition 2-10 [8]

Let $(X,\tau_f),(Y,\sigma_f)$ be fuzzy topological space, Let $f:(X,\tau_f)\to(Y,\sigma_f)$ be a function for a fuzzy set B in Y, the inverse image of B under f is the fuzzy set $f^{-1}(B)$ in X defined by $f^{-1}(B)(x)=B(f(x))=(B\circ f)(x)$ for $x\in X$.

For fuzzy set A in X , the image of A under f is the fuzzy set $f(A)$ in Y defined for $y\in Y$ by

$$f(A)y=\begin{cases}\sup\{A(z): z=f^{-1}(y)\} & \text{if } f^{-1}(y)\neq\phi \\ 0 & \text{if } f^{-1}(y)=\phi\end{cases}$$

Definition 2-11 [7]

Let $(X,\tau_f),(Y,\sigma_f)$ be fuzzy topological spaces, a function $f:X\to Y$ is fuzzy b-continuous if the inverse image under of any fuzzy b-open set in Y is fuzzy b-open set in X . than:

$$\text{If } \quad f^{-1}(B)\in\tau_f \quad \text{whenever} \quad B\in\sigma_f$$

Definition 2-12 [8]

Let $(X,\tau_f),(Y,\sigma_f)$ be fuzzy topological spaces, and let $\tau_{f\lambda}$ be fuzzy topology on X which has $\mathrm{FB(X)}$ as a subbase. Then:

1- A function $f:X\to Y$ is called fuzzy λ- b-continuous if $f:(X,\tau_{f\lambda})\to(Y,\sigma_f)$ is fuzzy b-continuous.
2- A function $f:X\to Y$ is said to be fuzzy λ'- b-continuous if $f:(X,\tau_{f\lambda})\to(Y,\sigma_{f\lambda})$ is fuzzy b- continuous.

Definition 2-13 [16]

A function $f:X\to Y$ is said to be fuzzy b-irresolute if the inverse image of every fuzzy b-open set in Y is fuzzy b-open in X .

3- Results:-

In this section we recalled and introduce new proportions.

Lemma 3-1

Let $f:(X,\tau_f)\to(Y,\sigma_f)$ be a fuzzy b-continuous function of fuzzy topological space:

1- If A is fuzzy b- compact fuzzy set in (X,τ_f), then $f(A)$ is fuzzy b- compact fuzzy set (Y,σ_f).

2- If f a surjection and (X,τ_f) is fuzzy b-compact, then (Y,σ_f) is fuzzy b-compact.

Lemma 3-2

For a fuzzy b-open set A_i belongs to τ, for each $i \in I$

1. $B \wedge \left(\vee_{i\in I} A_i \right) = \vee_{i\in I} (B \wedge A_i)$;
2. $B \vee \left(\wedge_{i\in I} A_i \right) = \wedge_{i\in I} (B \vee A_i)$;
3. $1 - \left(\vee_{i\in I} A_i \right) = \wedge_{i\in I} (1 - A_i)$; and
4. $1 - \left(\wedge_{i\in I} A_i \right) = \vee_{i\in I} (1 - A_i)$.

Remark 3-3

The identity mapping $\mathrm{id}_X:(X,\tau_f)\to(X,\tau_f)$ on fuzzy topological space is fuzzy b – continuous .

Proposition 3-4

A compotation of fuzzy b-continuous functions is fuzzy b-continuous.

Proof

Let $(X,\tau_f),(Y,\sigma_f),(Z,\delta_f)$ and v fuzzy b-open set $f:(X,\tau_f)\to(Y,\sigma_f)$ And $g:(Y,\sigma_f)\to(Z,\delta_f)$ be fuzzy b-continuous. For $v\in\delta$

$$\begin{aligned}(g\circ f)^{-1}(v) &= v\circ(g\circ f)\\ &=(v\circ g)\circ f\\ &= f^{-1}(v\circ g)\\ &= f^{-1}\left(g^{-1}(v)\right)\end{aligned}$$

$g^{-1}(v)\in\sigma$, since g is fuzzy b-continuous, and so $(g\circ f)^{-1}(v) = f^{-1}\left(g^{-1}(v)\right)\in\tau$ since g is fuzzy b-continuous. □

If A_i belongs to τ, for each $i \in I$ then $\underset{i\in I}{\vee} A_i = 1$.then there are a finite many indices' $i_1, i_2, ..., i_n \in I$ such that $\overset{n}{\underset{j=1}{\vee}} A_{i_j} = 1$ (see,[2]).

Proposition 3-5

Let $(X,\tau_f),(Y,\sigma_f)$ be fuzzy topological space, with (X,τ_f) be Fuzzy b-compact, and $f : X \to Y$ be a fuzzy b-continuous surjection. Then (Y,σ_f) is fuzzy b-compact.

Proof

Let $A_i \in \sigma_f$ for each $i \in I$, and suppose that $\overset{n}{\underset{j=1}{\vee}} A_{i_j} = 1$ for each $x \in X$,

$$\underset{i\in I}{\vee} f^{-1}(A_i)(x) = \underset{i\in I}{\vee} A_i\left(f(x)\right) = 1$$

So the fuzzy τ -b- open set $f^{-1}(A_i)$, $i \in I$ cover of X .

Thus a finitely many indices' $i_1, i_2, ..., i_n \in I$ such that $\overset{n}{\underset{j=1}{\vee}} f^{-1}\left(A_{i_j}\right) = 1$.

If v is fuzzy set in Y, and since f surjective function onto Y, implies that.

For any $y \in Y$

$$f\left(f^{-1}(A_i)\right)(y) = \sup\left\{f^{-1}(A)(z)\right\}$$
$$= \sup\{v(f(z)) : f(z) = y\} = A(y)$$

So that $f\left(f^{-1}(A)\right) = A$,Thus, as a fuzzy set in Y

$$1 = f(1) = f\left(\overset{n}{\underset{J=1}{\vee}} f^{-1}\left(A_{i_j}\right)\right)$$
$$= \overset{n}{\underset{J=i}{\vee}} f\left(f^{-1}\left(A_{i_J}\right)\right)$$
$$= \overset{n}{\underset{J=1}{\vee}} A_{i_J}$$

Therefore (Y,σ_f) is fuzzy b-compact □

Proposition 3-6

Let $(X,\tau_f),(Y,\sigma_f)$ be fuzzy topological spaces and let τ_{f_λ} be fuzzy topology on X which has FB(X) as a subbase .let $f:(X,\tau_f) \to (Y,\sigma_f)$ is fuzzy b-continuous, then f is fuzzy λ -b-continuous.

Proof

Let f be fuzzy b-continuous and let $V \in \sigma_f$. Then $f^{-1}(V) \in \mathrm{FB}(X)$ and so $f^{-1}(V) \in \tau_{f_\lambda}$. Thus f is fuzzy λ-continuous □

Proposition 3-7

Let $(X,\tau_f),(Y,\sigma_f)$ be fuzzy topological spaces. Let $\tau_{f\lambda}$ and $\sigma_{f\lambda}$ be the fuzzy topologies on X and Y respectively which have FB(X) and FB(Y) as a subbase. If $f:(X,\tau_f)\to(Y,\sigma_f)$ is fuzzy b-irresolute then f is fuzzy λ'-continuous.

Proof

Let f be fuzzy b- irresolute and $V \in \sigma_{f\lambda}$. Then

$V = \underset{i}{\vee}(\overset{n}{\underset{J=1}{\wedge}}\sigma_{i_{n_J}})$ Where $\sigma_{i_{n_J}} \in \mathrm{FB}(Y,\sigma_f)$ and

$$f^{-1}(V) = f^{-1}\left(\underset{\mathrm{i}}{\vee}(\overset{n}{\underset{J=1}{\wedge}}\sigma_{i_{n_J}})\right)$$

$$= \underset{\mathrm{i}}{\vee}\left(\overset{n}{\underset{J=1}{\wedge}} f^{-1}(\sigma_{i_{n_J}})\right)$$

Since f is fuzzy b-irresolute. $f^{-1}(\sigma_{i_{n_J}}) \in \mathrm{FB}(X,\tau_f)$.

The implies that $f^{-1}(V) \in \tau_{f\lambda}$ and that f is fuzzy λ'-continuous □

Proposition 3-8

A fuzzy topological space X is b-compact if and only if every family of fuzzy b-closed subsets of X with finite intersection property has non empty intersection.

Remark 3-9

Let (X,τ_f) be a fuzzy topological space and τ_λ a fuzzy topology on X which has $\mathrm{FB}(X)$ as a subbase .Then (X,τ_f) is b-compact iff $(X,\tau_{f\lambda})$ is b-compact .

Proof

$(X,\tau_{f\lambda})$ be compact Then, since $\mathrm{FB}(X) \subset \tau_{f\lambda}$. It follows that (X,τ_f) is b-compact □

Proposition 3-10

Let (X,τ_f) be a fuzzy topological space which is b-compact .Then each $\tau_{f\lambda}$-closed fuzzy set in X is b-compact.

Proof

Let U be any $\tau_{f\lambda}$-closed fuzzy set in X . Let $\{V_{\beta_i} : \beta_i \in I\}$ be $\tau_{f\lambda}$ an open cover of U . Since $X - U$ is $\tau_{f\lambda}$-open $\{V_{\beta_i} : \beta_i \in I\} \vee (X - U)$ is $\tau_{f\lambda}$ an open cover of X .
Since X is $\tau_{f\lambda}$-compact, by proposition 3-9, there exists a finite subset $I_0 \subseteq I$ such that
$X = \{V_{\beta_i} : \beta_i \in I\} \vee (X - U)$
This implies that $U \leq \{V_{\beta_i} : \beta_i \in I_0\}$.
Hence U is b-compact relative to X .

Proposition 3-11

Let (X, τ_f) be a fuzzy b-compact topological space. Then every family of $\tau_{f\lambda}$-closed fuzzy subsets of X with finite intersection property has non-empty intersection.

Proof

Let X be b-compact. Let $U = \{B_{\beta_i} : \beta_i \in I\}$ be any family of $\tau_{f\lambda}$-closed fuzzy subsets of X with finite intersection property. Suppose $\wedge \{B_{\beta_i} : \beta_i \in I\} = \phi$. Then $\{X - B_{\beta_i} : \beta_i \in I\}$ is a $\tau_{f\lambda}$-open cover of X . Hence it must contain a finite subcover $\{X - B_{\beta_{ij}} : j = 1,2,3,...,n\}$ for X . This implies that $\wedge \{B_{\beta_{ij}} : j = 1,2,3,...,n\} = \phi$ and contradicts with hypothesis that U has finite intersection property. □

Proposition 3-12

Let $(X, \tau_f), (Y, \sigma_f)$ be fuzzy topological spaces and let $f : X \to Y$ be fuzzy λ'-b-continuous. If a fuzzy subset G of X is b-compact relative to X , then $f(G)$ is b-compact relative to Y .

Proof

Let $\{V_{\beta_i} : \beta_i \in I\}$ be a cover of $f(G)$ by $\sigma_{f\lambda}$-open fuzzy sets in Y . Then $f^{-1}\{(V_{\beta_i}) : \beta_i \in I\}$
is a cover of G by $\sigma_{f\lambda}$-open fuzzy sets in X . G is b-compact relative to X . Hence by remark 3-9. G is $\tau_{f\lambda}$-compact .So there exists a finite subset $I_0 \subseteq I$ such that
$\mathrm{G} \leq \cup \{f^{-1}(V_{\beta_i}) : \beta_i \in I_0\}$ and so $f^{-1}(\mathrm{G}) \leq \cup \{V_{\beta_i} : \beta_i \in I_0\}$.
Hence $f^{-1}(\mathrm{G})$ is $\tau_{f\lambda}$-compact relative to Y .Thus $f^{-1}(\mathrm{G})$ is b-compact relative to Y □

Corollary 3-13

If $f : (X, \tau_f) \to (Y, \sigma_f)$ is a fuzzy λ'-b-continuous surjective function and X is b-compact, then Y is b-compact.

Corollary 3-14

If $f:(X,\tau_f)\to(Y,\sigma_f)$ is a fuzzy b-irresolute surjective function and X is b-compact then Y is b-compact.

Proposition 3-15

Let A and B be fuzzy subsets of a fuzzy topological space X such that A is b-compact relative to X and B is $\tau_{f\lambda}$-closed in X. Then $A\wedge B$ is b-compact relative to X.

Proof

Let $\{V_{\beta_i}:\beta_i\in I\}$ be a cover of $A\wedge B$ by $\tau_{f\lambda}$-open fuzzy subsets of X.
Since $X-B$ is a $\tau_{f\lambda}$-open fuzzy set, $\{V_{\beta_i}:\beta_i\in I\}\vee(\mathrm{X-B})$ is a cover of A. A is b-compact and thus $\tau_{f\lambda}$-compact relative to X. Hence there exists a finite subset $I_0\subseteq I$ such that

$A\leq \vee\{V_{\beta_i}:\beta_i\in I\}\vee(\mathrm{X-B})$

Therefore

$A\wedge B\leq \vee\{V_{\beta_i}:\beta_i\in I\}$

Hence $A\wedge B$ is $\tau_{f\lambda}$-compact. Therefore $A\wedge B$ is b-compact □

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الفضاءات التبولوجية الضبابية المرصوص من النوع bٍ

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الخلاصة:

الهدف من هذا البحث هو تقديم ودراسة مفهوم التراص من النوع b في الفضاءات التبولوجية الضبابية ، وكذلك قدمنا ودرسنا مفهوم الدوال المستمرة من النوع b والدوال المحيرة من النوع b .