On coc-coercive function

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Abstract :

In this paper , we use the concept of coc-closed and coc-compact sets to construct a new type of functions which is coc-coercive function and investigate the properties of this concept.

Keywords: coc-open , coc-closed , coc-compact space , coc-Hausdorff , coc-cluster point , coc'-continuous function , coc'-compact function and ccoc-coercive function .

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1. Introduction

The basic definitions that needed in this work are recalled . In this work , spaces always mean topological spaces on which no separation axioms are assumed unless explicitly stated , a topological space is denoted by (X, τ) (or simply X) . A subset A of a space X is called co-compact open set (for brief coc-open) , if for every $x \in A$, there is open set $U \subseteq$ X and a compact subset K $\subseteq X$ with $x \in U -$ K $\subseteq$ A **[2]** . The complement of coc-open set is called coc-closed set **[2]** . The coc-closure of A $\subseteq$ X is intersection of all coc-closed sets which contains A and it is denoted by $\bar{A}^{coc}$ **[2]** . By AL-Hussaini F. H. **[1]** , **[3]** give the definition of coc-Hausdorff and coc-compact spaces and study of it is properties. In **[1]** introduces the certain types of continuous (closed and compact) functions . Finally in **[4]** Hamzah S. H. and Hassan N. K. defined certain of a cluster points of a nets and shows relationship with the coc-compact space . We use T_{ind} to denote the indiscrete topology on a non-empty sets X.

1.1.Definition [1]:

i. A space X is called coc- Hausdorff space iff for each $x \neq y$ in X there exists disjoint coc- open sets U, V in X such that $x \in U, y \in V$, (every Hausdorff space is coc-Hausdorff) .

ii. A space X is called coc-compact if every coc-open cover of X has finite sub cover.

1.2.Theorem[1] , [2] , [3]:

i. Every open (closed) set is coc-open (coc-closed) set.

ii. If X be a space and Y be a nonempty closed set in X . If B be a coc-open (coc-closed) set in X , then $B \cap Y$ is coc-open (coc-closed) set in Y.

iii. Every finite subset of a space X is coc-compact.

iv. If Y be a coc-open in a space X and K $\subseteq$ Y , then K is a coc-compact in X iff it is coc- compact set in Y.

v. Every coc-compact subset of a Hausdorff space is coc-closed.

vi. Every coc-compact space is compact.

vii. The intersection of coc-closed set with a coc-compact set is coc-compact.

1.3. Definition[1] ,[2]:

A function $f: X \to Y$ is called:

i. coc'-continuous (coc'-compact), if $f^{-1}(A)$ is coc-open (or coc-closed) (coc-compact) set in X , for every coc-open (or coc-closed) (coc-compact) set A in Y .

ii. coc'- closed , if $f(A)$ is coc-closed set in Y , for every coc-closed set A in X .

iii. If f be a coc'-continuous onto and X be coc-compact, then Y is coc-compact.

iv. The coc'-continuous image for any coc-compact set is coc-compact.

1.4.Theorem [4]:

i. A net $(\chi_d)_{d\in D}$ in a space X is called to have $x \in X$ as coc- cluster point if $(\chi_d)_{d\in D}$ is frequently in every coc-open set contains x and it is denoted by $\chi_d \overset{coc}{\propto} x$.

ii. Let X be a space and $A \subseteq X$, then $x \in \bar{A}^{coc}$ iff there is a net $(\chi_d)_{d\in D}$ in A such that $\chi_d \overset{coc}{\propto} x$.

iii. X is coc-compact iff every net $(\chi_d)_{d\in D}$ in X has a coc-cluster point in X .

2.The main results:

This section is devoted to a new concept which is called coc- coercive function. Several various examples, theorems and remarks on the concept are proved . Furthermore are stated as well as the relationship between the concepts with the coc-compact function.

2.1. Definition:

A function $f: X \to Y$ is said to be coc-coercive , if for every coc-compact subset B of Y there is coc-compact subset A of X such that $f(X\backslash A) \subseteq (Y\backslash B)$.

2.2. Example:

i. The identity function for any space is coc-coercive.

ii. Let $X = \{1,2,3\}$, $Y = \{4,5\}$, $T_X = \{\emptyset , X , \{3\}\}$, $T_Y = T_{ind}$ and $f: X \to Y$ be a function with $f(1) = f(2) = 4$, $(3) = 5$, then f is coc-coercive .

iii. If X any finite space and $f: X \to Y$ be a function , then f is coc-coercive .

2.3. Theorem:

If $f: X \to Y$ be a function , such that X is coc-compact space, then f is coc-coercive.

Proof:

Let B be a coc-compact subset of Y .

Since X is coc-compact. Then$f(X\backslash X) = f(\emptyset) = \emptyset \subseteq f(Y\backslash B)$, so f is coc-coercive function.

2.4. Theorem:

Let $f: X \to Y$ be a coc'-continuous function with Y be a Hausdorff space , then f is coc-coercive if and only if it is coc'-compact.

Proof:

Suppose that f is coc-coercive and let B be a coc-compact subset of Y. To prove that f is coc'-compact function, since Y is T_2-space, then by Theorem (1.2.v), B is coc-closed . But f is coc'-continuous , then $f^{-1}(B)$ is coc-closed subset of X definition (1.3.i) . Since f be coc-coercive then by definition (2.1) , there is a coc-compact subset A of X such that $f(X\backslash A) \subseteq (Y\backslash B)$.

Since $f^{-1}(B)$ is a coc-closed, then by Theorem (1.4.ii) every net in $f^{-1}(B)$ has a coc-cluster in itself. By Theorem (1.4.iii), $f^{-1}(B)$ is coc-compact subset in X . Thus f is coc'-compact.

Conversely , suppose that B is coc-compact subset of Y . Since f is coc'-compact function, then $f^{-1}(B)$ is coc-compact subset of X . Put $A = f^{-1}(B)$, then $f(X\backslash A) \subseteq (Y\backslash B)$. Hence f is coc-coercive function.

2.5. Theorem:

For any closed and coc-open subset F of a space X , the inclusion function $i: F \to X$ is coc-coercive .

Proof:

Let A be a coc-compact subset of X , since F closed , then by Theorem (1.2.vii) , $F \cap A$ is coc-compact subset of X , by Theorem (1.2.iv) , $F \cap A$ is coc-compact of F. We have $i(F\backslash F\cap A) \subseteq X\backslash A$, then $i: F \to X$ is coc-coercive function.

2.6. Theorem:

Let X and Y be two spaces and $f: X \to Y$ be a function , if f be a coc-coercive with F be a closed and coc-open subset of X , the restriction function $f_{/F}: F \to Y$ is coc-coercive .

Proof:

Let B be a coc-compact subset of Y, since f be coc-coercive. Then there is a coc-compact subset A of X such that $f(X\backslash A) \subseteq (Y\backslash B)$. Since F be a closed subset of X, then by Theorem (1.2.vii) , $F\cap A$ is coc-compact subset of X , and hence $F\cap A$ is coc-compact subset of F. Since $f_{/F}(F\cap A) = f(F\backslash A)$ and $F\backslash A \subseteq X\backslash A \Rightarrow f(F\backslash A) \subseteq f(X\backslash A) \Rightarrow f_{/F}(F\backslash F\cap A) \subseteq Y\backslash B$, hence $f_{/F}: F \to Y$ is coc-coercive function .

2.7. Theorem:

Let X and Y be two spaces , $f: X \to Y$ be a coc-coercive , coc'-continuous function. If T closed and coc-open subset of Y,then $f_T: f^{-1}(T) \to T$ is coc-coercive function with $f^{-1}(T)$ is open in X.

<u>Proof:</u>

Let B be a coc-compact subset of T , since T is a closed subset of Y, then by Theorem (1.2.iv), B is coc-compact subset of Y. Since f be a coc-coercive, then there is a coc-compact subset A of X such that $f(X\backslash A) \subseteq (Y\backslash B)$.

Since f be coc'-continuous by (1.3.i), $f^{-1}(T)$ is coc-closed subset of X , by Theorem (1.2.vii) $f^{-1}(T)\cap A$ is coc-compact subset of $f^{-1}(T)$. Notice that:
$f_T(f^{-1}(T)\backslash f^{-1}(T)\cap A) = f_T(f^{-1}(T)\cap A^c) = f_T(f^{-1}(T)\backslash A)$. Since $f^{-1}(T)\backslash A \subseteq X\backslash A$,
then $f_T(f^{-1}(T)\backslash A) \subseteq f_T(X\backslash A)$ .So $f_T(X\backslash A) = T\cap f(X\backslash A)$ and $T\cap f(X\backslash A) \subseteq T\cap(Y\backslash B) = T\backslash B$.
Hence $f_T(f^{-1}(T)\backslash f^{-1}(T)\cap A) \subseteq T\backslash B$. Therefore f_T is coc-coercive function.

2.8. Theorem:

A composition of two coc-coercive functions is coc-coercive.

Proof:

Let $f: X \to Y$ and $h: Y \to Z$ be two coc-coercive functions . Let C is a coc-compact subset of Z , then there is a coc-compact subset B of Y such that $h(Y\backslash B) \subseteq Z\backslash C$. Since f is a coc-coercive , then there is a coc-compact subset A of X such that $f(X\backslash A) \subseteq Y\backslash B$.
So $h(f(X\backslash A)) \subseteq h(Y\backslash B)$, but $h(Y\backslash B) \subseteq Z\backslash C$. Hence $h\,(f(X\backslash A)) = hof\,(X\backslash A) \subseteq Z\backslash C$, therefore hof is coc-coercive function.

2.9. Corollary:

Let X and Y be two spaces , if $f: X \to Y$ is a function and X is a coc-compact with F closed and coc-open subset of X , then $f_{/F}: F \to Y$ is coc-coercive .

Proof: By using Theorems (2.3) and (2.8).

2.10. Theorem:

If $f: X \to Y$ is a bijective , coc'-compact and $g: Y \to Z$ is a coc-coercive function , then gof is coc-coercive function .

Proof:

Let C be a coc-compact subset of Z , then there is a coc-compact subset B of Y such that $g(Y\backslash B) \subseteq Z\backslash C$. Put $A = f^{-1}(B)$. Since f is coc'-compact , then A is a coc-compact subset of X. Thus $gof(X\backslash A) = g(f(X\cap A^c)) = g(f(X)\cap f(A^c))$.
Since f be a bijective, then $gof(X\backslash A) = g(Y\cap f(f^{-1}(B))^c = g(Y\cap B^c) = g(Y\backslash B) \subseteq Z\backslash C$.
Therefore gof is coc-coercive function.

2.11. Theorem:

Let $f: X \to Y$ and $g: Y \to Z$ be two functions such that:

i. If gof is coc-coercive and g is coc'-continuous and bijective , then f is coc-coercive.

ii. If gof is coc-coercive and f is coc'-continuous and onto, then g is coc-coercive.

Proof:

i. Let B be a coc-compact subset of Y, since g be a coc'-continuous ,by Theorem (1.3.iv) , $g(B)$ is coc-compact subset of Z . Since gof be a coc-coercive function, then there is a coc-compact subset A of X with $gof\,(X\backslash A) \subseteq Z\backslash\, g(B)$, since g be a bijective function then:
$g^{-1}(gof\,(X\backslash A)) \subseteq g^{-1}(Z\backslash g(B)) =$
$g^{-1}(Z\cap\big(g(B)\big)^c) = g^{-1}(Z)\cap g^{-1}(g\,(B^c))) = Y\backslash B$.
But $f(X\backslash A) = g^{-1}(gof(X\backslash A)) \subseteq Y\backslash B$, therefore f is coc-coercive function.

ii. Let C be a coc-compact subset of Z . Since gof is coc-coercive , then there is a coc-compact subset A of X such that $gof(X\backslash A) \subseteq Z\backslash C$, so $g(f(A^c)) \subseteq Z\backslash C$, since f is onto we get $g\,((f(A))^c)) \subseteq Z\backslash C$. Since f is coc'-continuous , then by Theorem (1.3.iv) , $f(A)$ is coc-compact subset of Y . Therefore h is coc-coercive function.

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الدوال الاضطرارية من النمط –COC

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