DERIVATION OF BAYESIAN RISK APPROXIMATION TO SELECTING THE ODD MEDIAN

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Abstract: We introduce in this paper, approximation functional for $S_i(n''_1, n''_2, .., n''_o, .., n''_k; m'')$ stopping risk by Bayesian procedure(prior and posterior) distribution for selecting odd median multi- category multinomial distribution using functional analysis

Key word: approximate, ranking and selection approach, prior and posterior dis., k-nomial dis

1. INTRODUCTION

Selecting the alternative with the largest or smallest mean performance from a finite number of alternatives is a common problem in many areas of operations research and management science. For instance, in designing a multi-stage manufacturing line one may need to determine the best allocation of the buffer space to maximize the average throughput; in controlling an inventory system one may need to identify the best reorder point to minimize the average cost; and in managing an ambulance service one may need to select the optimal vehicle dispatching policy to minimize the average response time. In all of these examples the mean performances of the alternatives may be evaluated by running simulation experiments. This type of optimization problem is known as a ranking-and-selection (R&S) problem in the simulation literature [1],[5],[6]. So in everyday life it is often important to collect data, perhaps from a survey, or a questionnaire. Once we have collected the data it is then important to arrange it in a way which allows someone to analyze the information, so that conclusions or decisions can be made. Mathematicians have named this study of "data" three of the most important statistics we use when we analyze data are called: The mean, the median and the mode[2],[3]. In this paper we choose the location is median. Approximation theory is a branch of mathematics, a quantitative part of functional analysis . Approximation usually occurs when an exact form or an exact numerical number is unknown or difficult to obtain. However some known form may exist and may be able to represent the real form so that no significant deviation can be found. It also is used when a number is not rational , such as the number π, which often is shortened to 3.14159, or √2 to 1.414.

In this paper, we will use functional analysis to approximate Bayesian stopping risk for selection median category where the population is distribute multinomial and prior Dirchlet distribution and sample of size is odd.

2. BAYESIAN DESCION APPROACH

1. Prior and Posterior Distributions[4]

Bayesian statistics, (Named for the Revd Thomas Bayes (1702- 1761), and a mateur 18th century mathematician), represents a different approach to statistical inference. Data are still assumed to come from a distribution belonging to a known parametric family. However, whereas classical statistics considers the parameters to be fixed but unknown, the Bayesian approach treats them as random variables in their own right. Prior beliefs about θ are represented by the prior distribution, with a prior probability density (or mass) function, $\pi(\theta)$. The posterior distribution has posterior density (or mass) function, $\pi(\theta \mid x_1, x_2, \ldots, x_n)$, and captures our beliefs about θ after they have been modified in the light of the observed data.

By Baye's formula, for the density function

$$\pi(\theta \mid x_1, x_2, ..., x_n) = \frac{f(x_1, x_2, ...x_n \mid \theta)\pi(\theta)}{\int_{\Omega} f(x_1, x_2, ...x_n \mid \theta)\pi(\theta)d\theta}$$

The denominator of the above equation does not involve θ and so in practice is usually not calculated. Bayes rule is written as,

$$\pi(\theta \mid x_1, \ldots, x_n) \propto f(x_1, \ldots, x_n \mid \theta)\, \pi(\theta)$$

In fact the actual choice of $\pi(\theta)$ depends upon the experimenter and the information and experience available to him at the time of doing the experiment. A prior which contains no information about θ is called non-informative prior or (vague prior). Mathematical and computational difficulties may arise from using some prior distributions.

A reasonable method of overcoming these difficulties is to use a particular class of prior distributions. This class of prior has been termed as natural conjugate priors in, for example the Beta distribution is a natural conjugate prior to Binomial distribution.

2. Risk[4]

For given decision function *d*, the loss function may be written as $L\{\underline{\theta}, d(\underline{x})\}$,

Since the action *a* depends on the particular sample data $\underline{x}$ that we observe. Thus, we see that the loss is a r.v. and depends on the sample outcome. Therefore, let us define the *risk* to be the expected value of the loss function. That is, the risk $R(\underline{\theta}, d)$ is a function of $\underline{\theta}, d$, and the loss function L such that

$$R(\underline{\theta}, d) = E\left[L\{\underline{\theta}, d(\underline{x})\} \mid \underline{\theta}\right]$$
$$= \int_{\chi} L\{\underline{\theta}, d(\underline{x})\} f(\underline{x} \mid \underline{\theta}) d\underline{x}.$$

3. Bayes Risk[4]

The Basyes risk of a decision d is the expected value of the risk $R(\underline{\theta}, d)$ with respect to the prior distribution π on *Ω;*

Namely,

$$r(\pi, d) = E\left[R(\Theta, d)\right]$$
$$= \int_{\Omega}\int_{\chi} L\{\underline{\theta}, d(\underline{x})\} f(\underline{x} \mid \underline{\theta}) \pi(\underline{\theta}) d\underline{x} d\underline{\theta}.$$

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3.THE STOPPING RISKS OF THE MEDIAN PROCEDURE

Consider a multinomial distribution which is characterized by *k* events (cells) with probability vector $\underline{p} = (p_1, p_2, ..., p_k)$, where p_i is the probability of the event E_i $(1 \le i \le k)$ with $\sum_{i=1}^{k} p_i = 1$. Let $n_1, n_2, .., n_m, ..., n_k$ be respective frequencies in *k* cells of the distribution with $\sum_{i=1}^{k} n_i = m$. Further, let $p_{[1]} \le p_{[2]} \le ... \le p_{[o]} \le ...... \le p_{[k]}$ denote the ordered values of the $p_i (1 \le i \le k)$. It is assumed that the values of p_i and of the $p_{[j]} (1 \le i, j \le k)$ is completely unknown. When select the median is to select bigger than half probable event, that, is the event associated with $p_{[o]}$, also called the median cell. The conjugate family in this case is the family of Dirichlet distribution. Accordingly, let $\underline{p}$ is assigned Dirichlet prior distribution with parameters $m', n'_1, ..., n'_o, ..., n'_k$. The normalized density function is given by

$$\pi(\underline{p}) = \frac{\Gamma m'}{\prod_{i=1}^{k} \Gamma(n'_i)} \prod_{i=1}^{k} p_i^{n'_i - 1}, \text{ where } \quad m' = \sum_{i=1}^{k} n'_i \text{ and}$$

$$\Gamma m' = (m' - 1)!$$

so the posterior distribution has density function

$$\pi(\underline{p} \mid \underline{n}) = \frac{(m'' - 1)!}{(n''_1 - 1)! ... (n''_o - 1)! .... (n''_k - 1)!} p_1^{n''_1 - 1} ..... p_o^{n''_o - 1} .... p_k^{n''_k - 1}$$

,

The stopping risk (the posterior expected loss) of the terminal decision d_i when the posterior distribution for $\underline{p}$ has parameters $(n''_1, n''_2, .., n''_o, ..., n''_k; m'')$, that is when the sample path has reached $(n''_1, n''_2, .., n''_o, ..., n''_k; m'')$ from the origin $(n'_1, n'_2, .., n'_o, .., n'_k; m'')$, denoted by $S_i(n''_1, n''_2, .., n''_o, .., n''_k; m''),$ can be found as follows.

$$S_i(n''_1, n''_2, .., n''_o, .., n''_k; m'') = \underset{\pi(\underline{p}|\underline{n})}{E}[L(d_i, \underline{p}^*)]$$
$$= k^* \left[\underset{\pi(\underline{p}|\underline{n})}{E}(\text{þ}_{[o]}) - \frac{n''_i}{m''}\right] \quad (5.1)$$

The value of $\underset{\pi(\underline{p}|\underline{n})}{E}[\text{þ}_{[o]}]$ is derived as follows.

$$\underset{\pi(\underline{p}|\underline{n})}{E}[\text{þ}_{[o]}] = \int_0^1 p_{[o]} . g(p_{[o]}) dp_{[o]},$$

So, if the number of observations is odd then

$$g(p_{[o]}) = k \binom{\text{k - 1}}{\text{k} + 1/2} \left[F(p_{[o]})\right]^{k + 1/2 - 1} \left[1 - F(p_{[o]})\right]^{k - k + 1/2} . f(p_{[o]})$$

$$= k \binom{k - 1}{k + 1/2} \frac{(m'' - 1)! \, p_{[o]}^{n''_{[o]} - 1} (1 - p_{[o]})^{m''' - n''_{[o]} - 1}}{(n''_{[o]} - 1)!(m'' - n''_{[o]} - 1)!}$$

$$\left[\sum_{j = n''_{[o]}}^{m''' - 1} \frac{(m'' - 1)!}{j!(m'' - n''_{[o]} - j)!} p_{[o]}{}^{j} (1 - p_{[o]})^{m''' - 1 - j}\right]^{k - 1/2}$$

$$\left[1 - \sum_{j = n''_{[o]}}^{m''' - 1} \frac{(m'' - 1)!}{j!(m'' - 1 - j)!} . p_{[o]}{}^{j} (1 - p_{[o]})^{m''' - 1 - j}\right]^{k - 1/2}.$$

Let,

$$w_1 = \left[\sum_{j=n''_{[o]}}^{m'''-1} \frac{(m''-1)!}{j!(m''-n''_{[o]}-j)!} p_{[o]}{}^{j} (1-p_{[o]})^{m'''-1-j} \right]^{k-1/2}$$

$$w_2 = \left[1 - \sum_{j=n''_{[o]}}^{m'''-1} \frac{(m''-1)!}{j!(m''-1-j)!} . p_{[o]}{}^{j} (1-p_{[o]})^{m'''-1-j} \right]^{k-1/2}$$

Now, to approximate w1 & w2 by the theorem in functional analysis(if a and b are two number then

$$(a+b)^2 \le 2^{n-1}(a^n + b^n) \text{ [7]}$$

And take this formula for generals,

$$w_1 \le 2^{(m''-2)(k-1/2)} \left[\sum_{j=n''_{[o]}+1}^{m'''-1} \frac{(\frac{p_{[o]}}{(1-p_{[o]})})^{j-1} (1-p_{[o]})^{m'''-1}}{(j-1)!(m''-n''_{[o]}-j-1)!} \right]^{k-1/2}$$

$$\frac{((m''-1)!)^{k-1/2}}{2^{k-1/2(m''-n_{[o]}-j)}} + \left[\frac{(\frac{p_{[o]}}{(1-p_{[o]})})^{m''-1} (1-p_{[o]})^{m'''-1}}{(n''_{[o]}-1)!} \right]^{k-1/2}$$

$$w_2 = \left[1 - \sum_{j=n''_{[o]}}^{m'''-1} \frac{(m'''-1)!}{j!(m'''-1-j)!} . p_{[o]}{}^{j} (1-p_{[o]})^{m'''-1-j} \right]^{k-1/2}$$

$$= \sum_{l=0}^{k-1/2} \binom{k-1/2}{l} \left[\sum_{j_2=n''_{[o]}}^{m'''-1} \frac{(m''-1)!\, p_{[o]}{}^{j_2} (1-p_{[o]})^{m'''-1-j_2}}{j_2!(m''-1-j_2)!} \right]^{l}$$

$$.(-1)^{l}$$

then,

$$w_2 \le \sum_{l=0}^{k-1/2} \binom{k-1/2}{l} (-1)^{l} \Big\{ 2^{(m''-2)(l-1)}$$

$$\left[\sum_{j_2=n''_{[o]}+1}^{m'''-1} \frac{(m''-1)! \left(\frac{p_{[o]}}{1-p_{[o]}}\right)^{j_2-1} \left(1-p_{[o]}\right)^{m'''-1}}{(j_2-1)!(m''-j_2)!} \right]^{l}$$

$$\frac{1}{2^{(l-1)(m''-1-j_2)}} + \left[\left(\frac{p_{[o]}}{1-p_{[o]}} \right)^{m''-1} \left(1-p_{[o]}\right)^{m''-1} \right]^{l} \Big\}$$

Such that the ordered values of $n''_1, n''_2, .., n''_o, .., n''_k$ is $n''_{[1]} \le n''_{[2]} \le, .., \le n''_{[o]}, ., \le n''_{[k]}$.

Then,

$$\underset{\pi(\underline{p}|\underline{n})}{E}(\mathfrak{b}_{[o]}) \le \frac{k(m''-1)!}{(n''_{[o]}-1)!(m''-n''_{[o]}-1)!} \binom{k-1}{k+1/2}$$

$$\left[\int_0^1 p_{[o]} p_{[o]}{}^{n''_{[o]}-1} (1-p_{[o]})^{(m'''-n''_{[o]}-1)} \right].$$

$$\left\{ 2^{(m-2)(\frac{k-1}{2})} \sum_{j_1=n''_{[o]}}^{m''-1} \left[\frac{(m''-1)! \left(\frac{p_{[o]}}{1-p_{[o]}} \right)^{j_1-1} (1-p_{[o]})^{m''-1}}{(j_1-1)!(m''-n''_{[o]}-j_1-1)!} \right]^{\frac{k-1}{2}} \right.$$

$$\left. \frac{1}{2^{(k-1)(m-n''_{[o]}-j_1)/2}} + \left[\frac{\left(\frac{p_{[o]}}{1-p_{[o]}} \right)^{m''-1} (1-p_{[o]})^{m''-1}}{(n''_{[o]}-1)!} \right]^{\frac{k-1}{2}} \right\}$$

$$. \left\{ \sum_{l=0}^{\frac{k-1}{2}} \binom{\frac{k-1}{2}}{l} \sum_{j_2=n''_{[o]+1}}^{m'''-1} {}^{l} \left[\frac{\left(\frac{p_{[o]}}{1-p_{[o]}} \right)^{j_2-1} (1-p_{[o]})^{m''-1}}{(j_2-1)!(m''-j_2)!} \right]^{l} \right.$$

$$\left. 2^{(m-2)(l-1)} \frac{((-1)(m''-1)!)^{l}}{2^{(l-1)(m-1-j_2)}} + \left[(p_{[o]})^{m''-1} \right]^{l} \right\} dp_{[o]}$$

$$= \frac{k(m''-1)!}{(n''_{[o]}-1)!(m''-n''_{[o]}-1)!}\binom{k-1}{k+1/2}2^{(m-2)(\frac{k-1}{2})}\left[\int_0^1 p_{[o]}^{\;n''_{[o]}} \left\{\sum_{l=0}^{k-1/2}\binom{k-1/2}{l} (-1)^l\, 2^{(m-2)(l-1)} \left(p_{[o]}\right)^{(m''-1)l} + \right.\right.$$

$$(1-p_{[o]})^{(m'''-n''_{[o]}-1)} . \left\{\sum_{j_1=n''_{[o]}}^{m'''-1}\left[\frac{(m''-1)!}{(j_1-1)!(m''-n''_{[o]}-j_1-1)!}\right]^{\frac{k-1}{2}} \sum_{l=0}^{k-1/2}\binom{k-1/2}{l}\frac{(-1)^l\,2^{(m-2)(l-1)}}{2^{(l-1)(m-1-j_2)}}\left[\frac{(m''-1)!}{(j_2-1)!(m''-j_2)!}\right]^l \right.$$

$$\left.\sum_{j_2=n''_{[m]+1}}^{m'''-1} p_{[o]}^{\;l(j_2-1)}(1-p_{[o]})^{l(m''-j_2)}\right\} dp_{[o]}$$

$$\sum_{l=0}^{k-1/2}\binom{k-1/2}{l}\frac{(-1)^l\,2^{(m-2)(l-1)}}{2^{(l-1)(m-1-j_2)}}\left[\frac{(m''-1)!}{(j_2-1)!(m''-j_2)!}\right]^l$$

$$\left.\sum_{j_2=n''_{[m]+1}}^{m'''-1} p_{[o]}^{\;l(j_2-1)}(1-p_{[o]})^{l(m''-j_2)} + p_{[o]}^{\;l(m''-1)}\right\} dp_{[o]}$$

$$= \frac{k(m''-1)!}{(n''_{[o]}-1)!(m''-n''_{[o]}-1)!}\binom{k-1}{k+1/2}2^{(m-2)(\frac{k-1}{2})} .$$

$$\left[\int_0^1 p_{[o]}^{\;n''_{[o]}}(1-p_{[o]})^{m'''-n''_{[o]}-1}\left\{\left(\frac{(1-p_{[o]})^{m''-1}}{(n''_{[o]}-1)!}\right)^{\frac{k-1}{2}} + \sum_{j_1=n''_{[o]}}^{m'''-1}\right.\right.$$

$$\left(\frac{(m''-1)!}{(j_1-1)!(m''-n''_{[o]}-j_1-1)!}\right)^{k-1/2} \frac{p_{[o]}^{\;(j_1-1)((k-1/2)}}{2^{(k-1)(m-n''_{[o]}-j_1)/2}}$$

$$\left. .(1-p_{[o]})^{(m''-1)(k-1/2)}\right\}$$

$$\left.\frac{p_{[o]}^{\;(j_1-1)((k-1/2)}(1-p_{[o]})^{(m''-1)(k-1/2)}}{2^{(k-1)(m-n''_{[o]}-j_1)/2}} + \left(\frac{(1-p_{[o]})^{m''-1}}{(n''_{[o]}-1)!}\right)^{\frac{k-1}{2}}\right\}$$

$$= \frac{k(m''-1)!}{(n''_{[o]}-1)!(m''-n''_{[o]}-1)!}\binom{k-1}{k+1/2}2^{(m-2)(\frac{k-1}{2})}$$

$$\left[\int_0^1 p_{[o]}^{\;n''_{[o]}}(1-p_{[o]})^{m'''-n''_{[o]}-1}\left\{\left(\frac{1}{(n''_{[o]}-1)!}\right)^{\frac{k-1}{2}} \sum_{l=0}^{k-1/2}\binom{k-1/2}{l}\right.\right.$$

$$(-1)^l\,2^{(m-2)(l-1)}\left(p_{[o]}\right)^{(m''-1)(k-1/2)} + \left(\frac{1}{(n''_{[o]}-1)!}\right)^{\frac{k-1}{2}}$$

$$\left(p_{[o]}\right)^{(m''-1)(k-1/2)}\sum_{l=0}^{k-1/2}\binom{k-1/2}{l}(-1)^l\,2^{(m-2)(l-1)}$$

$$\sum_{j_2=n''_{[o]+1}}^{m'''-1} \frac{p_{[o]}^{\;l(j_2-1)}(1-p_{[o]})^{l(m''-j_2)}}{2^{(l-1)(m-1-j_2)}}\left[\frac{(m''-1)!}{(j_2-1)!(m''-j_2)!}\right]^l$$

$$+\sum_{j_1=n''_{[o]}}^{m'''-1}\left(\frac{(m'''-1)!}{(j_1-1)!(m'''-n''_{[o]}-j_1-1)!}\right)^{\frac{k-1}{2}} p_{[o]}^{\;(j_1-1)((k-1/2)}$$

$$\frac{(1-p_{[o]})^{(m''-j_1)(k-1/2)}}{2^{(k-1)(m-n''_{[o]}-j_1)/2}}\sum_{l=0}^{k-1/2}\binom{k-1/2}{l}(-1)^l\,2^{(m-2)(l-1)}\left(p_{[o]}\right)^{(m''-1)l}$$

$$+\sum_{j_1=n''_{[o]}}^{m'''-1}\left(\frac{(m''-1)!}{(j_1-1)!(m''-n''_{[o]}-j_1-1)!}\right)^{\frac{k-1}{2}} p_{[o]}^{\;(j_1-1)((k-1/2)}$$

$$\frac{(1-p_{[o]})^{(m''-j_1)(k-1/2)}}{2^{(k-1)(m-n''_{[o]}-j_1)/2}}\sum_{l=0}^{k-1/2}\binom{k-1/2}{l}\frac{(-1)^l\,2^{(m-2)(l-1)}}{2^{(l-1)(m-1-j_2)}}$$

$$\left.\left[\frac{(m'''-1)!}{(j_2-1)!(m'''-j_2)!}\right]^l \sum_{j_2=n''_{[o]+1}}^{m'''-1} p_{[o]}^{\;l(j_2-1)}(1-p_{[o]})^{l(m''-j_2)}\right\} dp_{[o]}$$

$$= \frac{k(m''-1)!}{(n''_{[o]}-1)!(m''-n''_{[o]}-1)!}\binom{k-1}{k+1/2}2^{(m-2)(\frac{k-1}{2})}$$

$$\left[\int_0^1\left(\frac{1}{(n''_{[o]}-1)!}\right)^{\frac{k-1}{2}}\sum_{l=0}^{k-1/2}\binom{k-1/2}{l}(-1)^l\,2^{(m-2)(l-1)} .\right.$$

$$p_{[m]}^{\;n''_{[o]}+(m''-1)(\frac{k-1}{2})+l(m''-1)}(1-p_{[o]})^{m'''-n''_{[m]}-1} + \left(\frac{1}{(n''_{[o]}-1)!}\right)^{\frac{k-1}{2}}$$

$$\sum_{l=0}^{k-1/2}\binom{k-1/2}{l}(-1)^l\,2^{(m-2)(l-1)}\sum_{j_2=n''_{[o]+1}}^{m'''-1}\frac{1}{2^{(l-1)(m-1-j_2)}}$$

$$\left[\frac{(m''-1)!}{(j_2-1)!(m''-j_2)!}\right]^l p_{[o]}^{\,n''_{[o]}+(m''-1)(\frac{k-1}{2})+l(j_2-1)} (1-p_{[o]})^{l(m''-j_2)}$$

$$(1-p_{[o]})^{m''-n''_{[o]}-1} + \sum_{j_1=n''_{[o]}}^{m'''-1} (\frac{(m''-1)!}{(j_1-1)!(m''-n''_{[o]}-j_1-1)!})^{k-1/2}$$

$$\frac{1}{2^{(k-1)(m-n''_{[o]}-j_1)/2}} \sum_{l=0}^{k-1/2} \binom{k-1/2}{l} (-1)^l 2^{(m-2)(l-1)}$$

$$p_{[o]}^{\,n''_{[o]}+l(m''-1)+(k-1/2)(j_1-1)} (1-p_{[o]})^{(m''-j_1)(k-1/2)+m''-n''_{[o]}-1} +$$

$$\sum_{j_1=n''_{[o]}}^{m'''-1} (\frac{(m''-1)!}{(j_1-1)!(m''-n''_{[o]}-j_1-1)!})^{\frac{k-1}{2}} \frac{1}{2^{(k-1)(m-n''_{[o]}-j_1)/2}}$$

$$\sum_{l=0}^{k-1/2} \binom{k-1/2}{l} (-1)^l 2^{(m-2)(l-1)} \sum_{j_2=n''_{[o]+1}}^{m'''-1} \frac{1}{2^{(l-1)(m-1-j_2)}}$$

$$\left[\frac{(m''-1)!}{(j_2-1)!(m''-j_2)!}\right]^l p_{[o]}^{\,n''_{[m]}+(\frac{k-1}{2})(j_1-1)+l(j_2-1)}$$

$$\left((1-p_{[o]})^{(m''-j_1)(\frac{k-1}{2})+l(m''-j_2)+m''-n''_{[o]}-1}\right] dp_{[o]}$$

Then,

$$\underset{\pi(\underline{p}|\underline{n})}{E}(\mathrm{p}_{[o]}) \le \frac{k(m'''-1)!}{(n''_{[o]}-1)!(m'''-n''_{[o]}-1)!} \binom{k-1}{k+1/2}$$

$$2^{\frac{m-2)k-1}{2}} \left(\frac{1}{(n''_{[o]}-1)!}\right)^{\frac{k-1}{2}} \sum_{l=0}^{k-1/2} \binom{k-1/2}{l} (-1)^l 2^{(m-2)(l-1)}$$

$$.\frac{\Gamma(n''_{[o]}+\frac{k-1}{2}(m''-1)+l(m''-1)+1)\Gamma(m''-n''_{[o]})}{\Gamma(\frac{k-1}{2}(m''-1)+l(m''-1)+m''+1)}$$

$$+\left(\frac{1}{(n''_{[o]}-1)!}\right)^{\frac{k-1}{2}} \sum_{l=0}^{k-1/2} \binom{k-1/2}{l} (-1)^l 2^{(m-2)(l-1)}$$

$$\sum_{j_2=n''_{[o]}}^{m'''-1} \frac{1}{2^{(l-1)(m-1-j_2)}} \left[\frac{(m''-1)!}{(j_2-1)!(m''-j_2)!}\right]^l$$

$$\frac{\Gamma(\frac{km''-m''-1+n''_{[o]}+l(j_2-1}{2})\Gamma(l(m''-j_2+m''-n''_{[o]})}{\Gamma(\frac{k-1}{2}(m''-1)+l(j_2-1)+l(m''-j_2)+m''+1}$$

$$+\sum_{j_1=n''_{[o]}}^{m'''-1} (\frac{(m''-1)!}{(j_1-1)!(m''-n''_{[o]}-j_1-1)!})^{\frac{k-1}{2}} \frac{1}{2^{(k-1)(m-n''_{[o]}-j_1)/2}}$$

$$\sum_{l=0}^{k-1/2} \binom{k-1/2}{l} (-1)^l 2^{(m-2)(l-1)}$$

$$\frac{\Gamma(\frac{2n''_{[o]}+2l(m''-1)+kj-j+3}{2})}{\Gamma(\frac{k-1}{2}(m''-1)+l(j_2-1)+l(m''-j_2)+m''+1)}$$

$$\frac{\Gamma(\frac{k-1)(m''-j_1)+2m''-2n''_{[o]}}{2})}{\Gamma(\frac{k-1}{2}(m''-1)+l(j_2-1)+l(m''-j_2)+m''+1)}$$

$$+\sum_{j_1=n''_{[o]}}^{m'''-1} (\frac{(m''-1)!}{(j_1-1)!(m''-n''_{[o]}-j_1-1)!})^{\frac{k-1}{2}} \frac{1}{2^{(k-1)(m-n''_{[o]}-j_1)/2}}$$

$$\sum_{l=0}^{k-1/2} \binom{k-1/2}{l} (-1)^l 2^{(m-2)(l-1)} \sum_{j_2=n''_{[o]+1}}^{m'''-1} \frac{1}{2^{(l-1)(m-1-j_2)}}$$

$$\left(\left[\frac{(m''-1)!}{(j_2-1)!(m''-j_2)!}\right]^l\right]$$

$$\frac{\Gamma(n''_{[o]}+l(j_2-1)+\frac{k-1}{2}(j_1-1)+1))}{\Gamma(-l)+\frac{kj_1-j_1+1+(m''-j_1)k-1}{2}+lm''+m'')}$$

$$\frac{\Gamma(\frac{k-1}{2}(m''-j_1)+l(m''-j_2)+m''-n''_{[o]}}{\Gamma(-l)+\frac{kj_1-j_1+1+(m''-j_1)k-1}{2}+lm''+m'')}.$$

Hence

$$S_i(n''_1, n''_2, .., n''_o, .., n''_k; m'') = k^* \left\langle \frac{k(m''' - 1)!}{(n''_{[o]} - 1)!(m''' - n''_{[o]} - 1)!} \right.$$

$$\binom{k-1}{k+1/2} 2^{(m-2)(\frac{k-1}{2})} \left(\frac{1}{(n''_{[o]} - 1)!} \right)^{\frac{k-1}{2}} \sum_{l=0}^{k-1/2} \binom{k-1/2}{l}$$

$$\frac{\Gamma(n''_{[o]} + \frac{k-1}{2}(m''-1) + l(m''-1) + 1)\Gamma(m'' - n''_{[o]})}{\Gamma(\frac{k-1}{2}(m''-1) + l(m''-1) + m'' + 1)}$$

$$(-1)^l 2^{(m-2)(l-1)} + \left(\frac{1}{(n''_{[o]} - 1)!} \right)^{\frac{k-1}{2}} \sum_{l=0}^{k-1/2} \binom{k-1/2}{l} (-1)^l$$

$$2^{(m-2)(l-1)} \sum_{j_2 = n''_{[o]}}^{m'''-1} \frac{1}{2^{(l-1)(m-1-j_2)}} \left[\frac{(m''-1)!}{(j_2-1)!(m''-j_2)!} \right]^l$$

$$\frac{\Gamma(n''_{[o]} + \frac{k-1}{2}(m''-1) + l(j_2-1) + 1)\Gamma(l(m''-j_2) + m'' - n''_{[o]})}{\Gamma(\frac{k-1}{2}(m''-1) + l(j_2-1) + l(m''-j_2) + m'' + 1}$$

$$+ \sum_{j_1 = n''_{[o]}}^{m'''-1} \left(\frac{(m''-1)!}{(j_1-1)!(m'' - n''_{[o]} - j_1 - 1)!} \right)^{\frac{k-1}{2}} \frac{(-1)^l 2^{(m-2)(l-1)}}{2^{(k-1)(m - n''_{[o]} - j_1)/2}}$$

$$\sum_{l=0}^{k-1/2} \binom{k-1/2}{l} \frac{\Gamma(\frac{k-1}{2}(m''-j_1) + m'' - n''_{[o]})}{\Gamma(\frac{k-1}{2}(m''-1) - l + lm'' + m'' + 1)}$$

$$\frac{\Gamma(n''_{[o]} + l(m''-1) + \frac{k-1}{2}(j_1-1) + 1)}{\Gamma(\frac{k-1}{2}(m''-1) + l(j_2-1) + l(m''-j_2) + m'' + 1)}$$

$$+ \sum_{j_1 = n''_{[o]}}^{m'''-1} \left(\frac{(m''-1)!}{(j_1-1)!(m'' - n''_{[o]} - j_1 - 1)!} \right)^{\frac{k-1}{2}}$$

$$\frac{1}{2^{(k-1)(m - n''_{[o]} - j_1)/2}} \sum_{l=0}^{k-1/2} \binom{k-1/2}{l} (-1)^l 2^{(m-2)(l-1)}$$

$$\sum_{j_2 = n''_{[o]+1}}^{m'''-1} \frac{1}{2^{(l-1)(m-1-j_2)}} \left(\left[\frac{(m''-1)!}{(j_2-1)!(m''-j_2)!} \right]^l \right]$$

$$\frac{\Gamma(n''_{[o]} + l(j_2-1) + \frac{k-1}{2}(j_1-1) + 1)}{\Gamma(-l) + \frac{k-1}{2}(j_1-1) + \frac{k-1}{2}(m''-j_1) + lm'' + m'')}$$

$$\left. \frac{\Gamma(\frac{k-1}{2}(m''-j_1) + l(m''-j_2) + m'' - n''_{[o]})}{\Gamma(-l) + \frac{(k-1)[(j_1-1) + (m''-j_1)]}{2} + (lm'' + m'')} - \frac{n''_i}{m''} \right\rangle$$

FUTURE WORK

- We can use sterling's approximation to simplify $S_i(n''_1, n''_2, .., n''_o, .., n''_k; m'')$ for large factorials.
- Our plan in future is to produce some numerical results for this procedure and simulation.
- We can be developed this procedure fully and group Bayesian sequential scheme using dynamic programming to selecting the median multinomial problem.
- General loss functions may be used, where linear loss is considered as a special case.

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أشتقاق تقريب الخطورة البيزنية لاختيار المتوسط الفردي

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الملخص
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الكلمات الافتتاحية
التقريب , الاختيار والترتيب , التوزيع السابق والاحق , التوزيع المتعدد الحدود

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