On Coc − Dimension Theory

Raad Aziz Hussain **WroodHasanHweidee**

Raad-64@hotmail.com **wroodhasan1212@gmail.com**

Department of Mathematics College of Computer science and Information Technology University of AL-Qadisiya

Abstract :

In this paper we introduce and define a new type on coc−dimension theory. The concept of $indX, IndX, dimX$, for a topological space X have been studied. In this work, these concepts will be extended by using coc − $open$ sets.

Keywords: coc − $open$,$coc - ind\ X$, $coc - Ind\ X$,and coc− dim X.

1. Introduction

The "Dimension function" in Dimension theory is function defined on the class of topological spaces such that d(X) is an integer or ∞. Then d(X) = d(Y) if and only if X and Y are homeomorphic and d(ℝ) = n for each positive integer n. The domain of dimension function is topological space and the codomain is the set$\{-1,0,1,\dots\}$. In [A.P.Pears ,1975], The dimension functions ind, Ind, dim, Actually the dimension functions, S − indX, S − IndX, S − dimX by using S − open sets were studied in [Raad Aziz Hussain AL-Abdulla,1992], also the dimension functions, b − indX, b − IndX, b − dimX, by using b − open sets were studied in [Sama Kadhim Gabar,2010], also the dimension functions, f − indX, f − IndX, f − dimX, by using f − open sets were studied in [NedaaHasanHajee,2011]. and the dimension functions, N − ind, N − Ind, N − dim are by using N − open sets[Enas Ridha Ali,2014] also the dimension functions S_β − ind, S_β − Ind, S_β − dim, by using S_β − open sets[Sanaa Noor Mohammed, 2015]. In this paper actually the dimension function ,coc-ind , coc-Ind and coc-dim by using coc-open sets. Finally some relations between them are studied and some results relating to these concepts are proved.

2. Preliminaries

Definition (2.1): A subset A of a topological space (X, $\mathcal{T}$) is called co-compact open set (notation : coc-open set

) if for every x∈A , there exists an open set U in X and a compact set K ∈ C(X , $\mathcal{T}$) such that x ∈U-K⊆A, the complement of coc-open set is called coc-closed set. The family of all coc-open subsets of a space (X, $\mathcal{T}$) will be denoted $\mathcal{T}^K$ and the family {U – K : U ∈ $\mathcal{T}$ and K ∈ C(X , $\mathcal{T}$) } of coc – open sets will denoted by $B^K(\mathcal{T})$**[S.Al Ghour and S. Samarah, 2012]**.

Theorem(2.2)[S.Al Ghour and S. Samarah, 2012] : Let X be a topological space. Then the collection $\mathcal{T}^K$ form a topology on X .

Example(2.3): It is clear that every $coc -$ open set is open set and every $coc -$ closed set is closed, but the converse is not true in general, see the following example. Let $X = \{a, b\}$ and $T = \{\emptyset, X\}$, then $\mathcal{T}^K$=$\{\{a\}, \{b\}, \emptyset, X\}$.

Proposition(2.4)[Farah HausainJasim,2014]: Let X be a topological space and Y any non –empty closed set in X. If B is a coc-open set in X, then B∩Y a coc-open set in Y.

Corollary(2.5)[Farah HausainJasim,2014]: Let X be a topological space and Y any non –empty closed set in X. If B is a coc-closed set in X, then B∩Y a coc-closed set in Y.

Definition (2.6)[Farah Hausain Jasim, 2014]: Intersection of all coc $-$ $closed\ sets\ containing$ F is called the coc $-$ $closure$ of F is denoted by $\overline{F}^{\,coc}$.

Theorem(2.7)[Farah Hausain Jasim, 2014]: For any subset F of a topological space X, the following statements are true .

1- $\overline{F}^{\,\mathrm{coc}}$ is the intersection of all coc $-$ $closed\ sets$ in X containing F.

2- $\overline{F}^{\,\mathrm{coc}}$ is the smallest coc $-$ $closed\ set$ in X containing F.

3- F iscoc $-$ $closed$ if and only if $F = \overline{F}^{\,\mathrm{coc}}$.

4- If F and E are any subsets of a topological space X and $F \subseteq E$, then $\overline{F}^{\,\mathrm{coc}} \subseteq \overline{E}^{\,\mathrm{coc}}$.

Definitions(2.8)[Farah Hausain Jasim, 2014]: Let X be a topological space, Then :

1- $A^{\circ coc}$ = ∪{ B : B is coc-open in X and B ⊆ A}.

2- The coc – neighborhood of B is any subset of X which contains an coc-open set containing B. The coc – neighborhood of a subset {x} is also called coc-neighborho- od of the point x.

3- Let A subset of X. For each x in X is said to be coc-boudary point of A if and only if each coc – neighborhood U_x of x, we have $U_x \cap A \neq \emptyset$,$U_x \cap A^C \neq \emptyset$. The set of all coc – boundary points of A is denoted by $b_{coc}(A)$.

Proposition (2.9)[Farah Hausain Jasim, 2014]: Let A be any subset of a topological space X. If a point x is in the coc- interior of A, then there exists a coc-open set U of X containing x such that U ⊆ A.

Remark(2.10):

i. $b_{coc}(A) = \overline{A}^{coc} \cap \left(A^{\circ coc}\right)^c$.

ii. Let A be a subset of a topological space X, then $b_{coc}(A) = \emptyset$ if and only if A is both coc-open and coc-closed set.

Definition (2.11)[Farah Hausain Jasim, 2014]: A space X is called $coc - \mathcal{T}_1\ space$ if and only if, for each x ≠ y in X, there exist $coc - open$ sets U and V such that $x \in U$, $y \notin U$ and$y \in V$, x $\notin V$.

Proposition (2.12)[S.Al Ghour and S. Samarah, 2012]: If a space X is $\mathcal{T}_2$ – space, then $\mathcal{T}^K$= $\mathcal{T}$.

Proposition (2.13)[Farah Hausain Jasim, 2014]: Let X be a topological space. Then X is $coc-\mathcal{T}_1\ space$ if and only if {x} is $coc-closed$ set for each x ∈ X.

Definition(2.14)[Farah Hausain Jasim, 2014]: A space X is called coc – regular space if and if for each x in X and a closed set F such that $x \notin F$, there exist disjoint coc-open sets U, V such that $x \in U, F \subseteq V$.

Definition (2.15): A space X is said to be coc*−regular space if and only if ,for each x ∈ X and F is an coc −closed sub such that x ∉ F, there exist U,V disjoint open sets in X such that $x \in U\ and\ F \subseteq V$.

Proposition(2.16)[N.J.Pervin, 1964]: A space X is regular space if and only if for every x ∈ X and each open set U in X such that x ∈ U there exist an open set w such that x ∈ W ⊆ $\overline{W}$ ⊆ U.

Proposition(2.17)[Farah Hausain Jasim, 2014]: A space X is coc- regular space if and only if for every x ∈ X and each open set U in X such that x ∈ U there exist a coc- open set W such that $x \in W \subseteq \overline{W}^{coc} \subseteq U$.

Proposition(2.18): A topological space X is coc*-regular topological space if and only if, for every x ∈ X and each coc-open set U in X such that x ∈ U then there exists an open set W such that $x \in W \subseteq \overline{W} \subseteq U$.

Proof: Assume that X is coc*-regular topological space and let x ∈ X and U is a coc-open set in X such that x ∈ U.then U^c is an coc-closed in X, $x \notin U^c$. Since X is a coc*- regular topological space then there exist disjoint open sets W and V such that $x \in W$ and $U^c \subseteq V$.then it is means $x \in W \subseteq \overline{W} \subseteq \overline{V^c} = V^c \subseteq U$.

Conversely; Let x ∈ X and C be a coc-closed set such that x ∈ C^c. thus there exists an open set W such that $x \in W \subseteq \overline{W} \subseteq C^c$ .Then x ∈ W , $C \subseteq (\overline{W})^c$, then we get to W, $(\overline{W})^c$is an open sets, $W \cap (\overline{W})^c = \emptyset$.Hence X is coc*-regular topological space.

Definition (2.19)[Farah Hausain Jasim, 2014]: A space X is said to be $coc-$ normal space if and only if for every disjoint coc- closed sets F_1, F_2 there exist disjoint $coc-$ open sets V_1, V_2 such that $F_1 \subset V_1, F_2 \subset V_2$.

Definition (2.20)[Farah Hausain Jasim, 2014]: A space X is said to be $coc'-$ normal space if and only if for every disjoint coc-closed sets F_1, F_2 there exist disjoint open sets V_1, V_2 such that $F_1 \subset V_1$, $F_2 \subset V_2$.

Example(2.21): This example show that coc-normal space is not normal space in general. Let $x = \{a,b,c,d\}$, $\mathcal{T} = \{X, \emptyset, \{a,b,d\}, \{a,b,c\}\}$, $\mathcal{T}^K = \{\emptyset, X, \{a,b\},$ $\{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}\{d,b,c\}, \{a\}, \{b\}, \{d\},$ $\}$ It is clear that X is coc- normal space and not normal space, since there exists disjoint closed sets {d}, {c} but not there exist disjoint open sets contain them.

Proposition (2.22)[Farah Hausain Jasim, 2014]: A topological space X is coc−normal topological space if and only if for every closed set F ⊆ X and each open set U in X such that F ⊆ U then there exists an coc − open set W such that $F \subset W \subset \overline{W}^{coc} \subset U$.

Proposition (2.23)[Farah Hausain Jasim, 2014]: A topological space X is $coc'-$ normal topological space if and only if for every $coc-$ closed set F ⊆ X and each coc − open set U in X such that F ⊆ U then there exists an coc − open set W such that $F \subset W \subset \overline{W}^{coc} \subset U$.

3. On small Coc− Inductive Dimension Function

Definition (3.1): The $coc-$ small inductive dimension of a space X, $coc-ind\ X$, is defined inductively as follows. topological space X, $coc-ind\ X=-1$, and only if, X is empty. If n is a non-negative integer, then$coc-ind\ X\ \leq n$ means that for each point $x\in X$ and each open set G such that $x\in G$ there exists an $coc-$ open set U such that $x\in U\subseteq G$ and $coc-ind\ b(U)\leq n-1$. We put $coc-\text{ind}X=n$ if it is true that $coc-\text{ind}X\leq n$, but it is not true that $coc-\text{ind}X\leq n-1$. If there exists no integer n for which$coc-\text{ind}X\leq n$, then we put $coc-\text{ind}X=\infty$.

Definition (3.2): The $coc*-$small inductive dimension of a space X, $coc*-ind\ X$, is defined inductively as follows. topological space X , $coc-ind\ X=-1$, and only if, X is empty. If n is a non-negative integer, then coc*-$ind\ X\leq n$ means that for each point $x\in X$ and each open set G such that $x\in G$ there exists an $coc-$ open set U such that $x\in U\subseteq G$ and $coc-ind\ b_{coc}(U)\leq n-1$. We put $coc*-\text{ind}X=n$ if it is true that $coc*-\text{ind}X\leq n$, but it is not true that $coc*-\text{ind}X\leq n-1$. If there exists no integer n for which$coc*-\text{ind}X\leq n$, then we put $coc*-\text{ind}X=\infty$.

Theorem (3.4): Let X be a topological space, if $coc-\text{ind}X=0$ then X is regular space.

Proof: Let$x\in X$and$G$ an open set such that $x\in G$, since $coc-\text{ind}X=0$, then there exists an $coc-$ open set V such that $x\in V\subseteq G$ and $coc-\text{ind}\ b(V)=-1$. Thus $b(V)=\emptyset$, hence V is an open and closed set.

Therefore $x\in V\subseteq\overline{V}\subseteq G\ by$ proposition (2.16), hence X is regular space.

Theorem (3.5): Let X be a topological space, if $coc*-\text{ind}X=0$, then X is $coc-$regular space.

Proof: Let$x\in X$ and$G$ an open set such that $x\in G$, since $coc*-\text{ind}X=0$, then there exists an $coc-$ open set V such that $x\in V\subseteq G$ and $coc-\text{ind}\ b_{coc}(V)=-1$. Thus $b_{coc}(V)=\emptyset$ by Remark (2.9. ii.), hence$V$ is an $coc-$open and $coc-$closed set. Therefore $x\in V\subseteq\bar{V}^{coc}\subseteq G$ by proposition (2.18), hence X is $coc-$regular space.

Proposition(3.6): Let X be a topological space, if $coc-\text{ind}X$ is exists then $coc-\text{ind}X\leq\text{ind}X$.

Proof: By induction on n . If $n=-1$, then $\text{ind}X=-1$ and $X=\emptyset$, so that $coc-\text{ind}X=-1$. Suppose the statements is true for $n-1$. Now, suppose that $\text{ind}X\leq n$, to prove $coc-\text{ind}X\leq n$, let $x\in X$ and G is an open set in X such that $x\in G$ since$\text{ind}X\leq n$, then there exist anopen set V in X such that $x\in V\subseteq G$ and ind $b(V)\leq n$-1 and since each open set is coc-open set. Then V is a coc-open set such that $x\in V\subseteq G$ and $coc-$ ind $b(V)\leq n$-1. Hence coc-ind $X\leq n$.

Theorem (3.7): Let X be a topological space, then $coc-\text{ind}X=0$ if and only if $\text{ind}X=0$.

Proof: By proposition (3.6) if $\text{ind}X=0$, then $coc-\text{ind}X\leq 0$ and since $X\neq\emptyset$ then $coc-\text{ind}X=0$. Now let $coc-\text{ind}\ X=0$, let $x\in X$ and G an open set in X such that $x\in G$, since $\text{ind}X=0$ then there exists a coc-open set V such that $x\in V\subseteq G$ and coc-ind$b(V)$ = -1, then$b(V)=\emptyset$, hence ind$b(V)$ = -1, thus ind X≤ 0, but X ≠ ∅. Therefore ind X = 0.

The following example a space X with coc-ind$X=\text{ind}X=0$.

Example (3.8): Let $X=\{a,b\}$ and $T=\{\emptyset,X\}$ be a topology on X, then $\mathcal{T}^K=\{\{a\},\{b\},\emptyset,X\}$, and coc-ind X = ind X = 0.

Theorem (3.9) : If A is a clopen (closed and open) subspace of a space X , then coc-ind A ≤ coc-ind X.

Proof : By induction on n. It is clear if n = -1. Suppose that it is true for n-1. Let coc-ind X≤n, to prove that coc-ind A $\le n$, let x ∈ A and G an open set in A such that x ∈ G, thus there exist V an open set in X such that V = G∩A , then x ∈ V and since coc-ind X $\le n$, then there exist W a coc-open set in X , such that x ∈ W ⊆ V and coc-ind b(W) $\le n$-1. , let U = W ∩A ,then U coc-open set in A (since A closed in X) . b_A(U)⊆ b(U)∩A = $\overline{U}$ ∩ U^{oc} ∩A ⊆ $\overline{W}$ ∩A∩ U^{0c}= $\overline{W}$ ∩A∩ $(W \cap A)^{0c}$ =$\overline{W}$ ∩A∩ (W^{0^c} ∪ A^c) = b(W) ∩A⊆ b(W) . We to prove that $b_A(U)$ is closed set in b(W) , since $b_A(U)$ a closed set in A and A closed set in X , then $b_A(U)$a closed set in X . Now; $\overline{b_A(U)}^{b(W)} = \overline{b_A(U)} \cap b(W) = b_A(U) \cap b(W)$=$b_A(U)$. Since coc-ind b(W) ≤ n-1 , then coc-ind b(U) ≤ n-1 , therefore coc-ind A ≤ n .

Proposition (2.24):A topological space X is $coc*$ −normal topological space if and only if for every coc − closed set F ⊆ X and each coc − open set U in X such that F ⊆ U then there exists an open set W such that F ⊂ W ⊂ $\overline{W}$ ⊂ U.

Proof: Assume that X is coc*-normal space and let F is coc-closed set in X and ea- ch U is a coc-open in X such that F ⊆ U, then F,U^c are a disjoint coc-closed sets in X. Since X is a coc*- normal space, then there exist disjoint open sets W and V such that $F \subseteq$ W and $U^c \subseteq V$. then it is means $F \subseteq W \subseteq \overline{W} \subseteq \overline{V^c} = V^c \subseteq U$. Conversely; Let F and C a disjoint coc-closed sets in X ,then C^c is coc-open set such that F⊆ C^c. Thus there exists an open set W such that $F \subseteq W \subseteq \overline{W} \subseteq C^c$ .Then $F \subseteq W$, $C \subseteq (\overline{W})^c$, then we get to W, $(\overline{W})^c$is an open sets, $W \cap (\overline{W})^c = \emptyset$. Hence X is coc*- normal space.

4. On Large coc − Inductive Dimension Function

Definition(4.1): The coc-large inductive dimension of a space$X$,coc-IndX, is defined large inductively as follows. A space X satisfies coc- Ind$X$ = -1 if and only if X is empty. If n is a non-negative integer, then coc-Ind$X \le n$ means that for each closed set F and each open set G of X such that $F \subset G$ there exists acoc-open set U such that $F \subset U \subset G$ and Ind $b(U) \le n$-1. We put coc-Ind$X = n$, its true that coc- Ind$X \le n$, but it is not true that coc-Ind$X \le n - 1$. If there exists no integer n for which Ind$X \le n$ then we put coc-Ind $X = \infty$.

Definition (4.2): Thecoc $*$ - large inductive dimension of a space X,coc-Ind$X$, is defined large inductively as follows: A space Xsatisfies coc*-Ind$X$ = -1 if and only if X isempty. If $n$is a non-negative integer, thencoc $*$ -Ind$X \le n$ means that for each closedset F and each open set G of X such that $F \subset G$ there exists open set U such that $F \subset U \subset G$ and coc-Ind$b_{coc}(U) \le n$-1. We put coc $*$ -Ind$X = n$ if it is true that coc $*$ -Ind$X \le n$, but it is not true that coc-Ind X ≤ n-1. If there exists no integer n for which coc-Ind$X \le n$ then we put coc-Ind$X = \infty$.

Proposition(4.3): Let X be a topological space, if coc-Ind$X = 0$, then X is normal.

Proof: Let F be a closed set in X and G is an open set such that $F \subseteq$G. Since coc-Ind$X = 0$, then there exist coc-open W such that coc-Ind $b(W)$ = -1, hence W is open and closed set therefore $F \subseteq W \subseteq \overline{W} \subseteq U$, then X is normal.

Proposition(4.4): Let$X$be a topological space, if $coc*$ -Ind$X = 0$, then X coc-normal space.

Proof: Let F be a closed set in X and U is a open set such that $F \subseteq U$. Since coc*- Ind$X = 0$, then there exist coc-open W such that coc-Ind$b_{coc}(W) =$ -1, hence W is coc-open and coc-closed set therefore $F \subseteq W \subseteq \overline{W}^{coc} \subseteq U$ by proposition (2.23) X iscoc $-$normal space.

Proposition(4.5): Let Xbe a topological space.$coc -$ Ind$X$ is exists, then coc-Ind$X \leq$ Ind$X$.

Proof: By induction on n . It is clear that $n = -1$. Suppose that it is true for n-1. Now , suppose thatInd$X \leq n$, to prove thatcoc-Ind$X \leq n$, let F be a closed set in X and G is an open set in X such that $F \subseteq G$, since Ind$X \leq n$, then there is open set U in X such that $F \subseteq U \subseteq G$ and Ind $b(U) \leq n - 1$, since each open set is coc-open set. Then U is coc-open set such that $F \subseteq U \subseteq G$ and coc-Ind $b(U) \leq n - 1$. Hence coc -Ind $X \leq n$.

Theorem (4.6): Let X be a topological space, then Ind$X = 0$, if and only if $coc -$ Ind$X = 0$.

Proof: By proposition (4.5*)* If Ind$X = 0$, then coc-Ind$X \leq 0$, and since$X \neq \emptyset$, then coc-Ind$X = 0$.

Now, Let coc-Ind $X = 0$ and Let F is closed set in X and each open set G in X such that $F \subseteq G$. Since coc-Ind $X = 0$, then there exists a coc-open set U in X such that$F \subseteq U \subseteq G$ and coc-Ind $b(U) \leq$ -1.Then $b(U) = \emptyset$, therefore U is both open and closed set, then U open such that $F \subseteq U \subseteq G$, and Ind b(U) = -1, hence Ind $X \leq 0$ and since $X \neq \emptyset$, then Ind $X = 0$.

Theorem (4.7): If A is a clopen (closed and open) subspace of a space X, then coc-Ind A ≤ coc-Ind X.

Proof : By induction on n. It is clear if n = -1. Suppose that it is true for n-1. Let coc-ind X ≤ n, to prove that coc-Ind A ≤ n, let F is closed set in A and G an open set in A such that F ⊆ G, thus there exists V an open set in X such that V = G ∩ A, then F ⊆ V and since coc-Ind X ≤n, then there exist W a coc-open set in X, such that F ⊆ W ⊆ V and coc-Ind b(W) ≤ n-1, let U = W ∩A, then U coc-open set in A (since A closed in X). b_A(U) ⊆ b(U) ∩ A = $\overline{U} \cap U^{oc} \cap A \subseteq \overline{W} \cap A \cap U^{\circ c} = \overline{W} \cap A \cap (W \cap A)^{oc} = \overline{W} \cap A \cap (W^{\circ c} \cup A^c)$ = b(W) ∩ A⊆ b(W). We to prove that $b_A(U)$ is closed set in b(W), since $b_A(U)$ a closed set in A and A closed set in X, then $b_A(U)$a closed set in X. Now; $\overline{b_A(U)}^{b(W)} = \overline{b_A(U)} \cap b(W) = b_A(U) \cap b(W) = b_A(U)$. Since coc-Ind b(W) ≤ n-1, then coc-Ind b(U) ≤ n-1, therefore coc-Ind A ≤ n.

5. On Coc- Covering Dimensiaton Function

Definition(5.1) [**A.P. Pears, 1975]:** Let X be a set and $\mathcal{A}$ a family of subsets of X, by an order of the family $\mathcal{A}$ we mean the largest integer n such that the family $\mathcal{A}$ contains $n + 1$ sets with a non- empty intersection, if no such integer exists, we say that the family $\mathcal{A}$ has order ∞. The order of a family $\mathcal{A}$ is denoted by $ord\ \mathcal{A}$.

Definition(5.2) [Stephen Willard, 1970]: I f $\mathcal{U}$ and $\mathcal{V}$ are covers of X, we say $\mathcal{U}$ refines $\mathcal{V}$, iff each $U \in \mathcal{U}$ is contained in some $V \in \mathcal{V}$.

Definition (5.3): The coc-covering dimension (coc-dim$X$) of a topological X is the least integer n such that every finiteopen covering of X has open refinem-ent of order not exceeding n or ∞ if there is no such integer. Thus coc-dim$X =$ -1 if and only if X is empty, and coc-dim $X \leq n$ if each finite open covering of X has coc- open refinement of order not exceeding n that coc-dim X ≤ $n - 1$. Finally coc-dim X $= \infty$ if for every integer n it is false that coc-dim X≤ n.

Definition (5.4): The coc′-covering dimension ($coc' -$ dim$X$) of a topological space X is the least integer n such that every finite coc-open covering of X has coc-open refinement of order not exceeding n or ∞ if there is no such integer. Thus coc'-dim$X =$ -1 if and only if X is empty, and coc′-dim$X \leq n$ if each finite open cover- ing of X has coc-open refinement of order not exceeding n . We have coc′-dim$X = n$ if it is true that coc′-dim$X \leq n$, but it is not true that coc′-dim$X \leq n-1$. Finally coc′-dim$X = \infty$ if for every integer n it is false thatcoc′-dim$X \leq n$.

proposition(5.5) : Let X a topological space, then coc-dim$X \leq$ dim$X$.

proof : Suppose that dim X ≤ n . Let a finite open cover of X, then there exist $\mathcal{V}$ an open refinement of order ≤ n . Since every open set is coc-open set, then $\mathcal{U}$ has $\mathcal{V}$ a coc-open refinement of order ≤ n, therefore coc-dim X ≤ n.

proposition(5.6) : Let X a topological space , then coc-dim$X \leq coc' -$ dim$X$.

proof : suppose that coc′-dim X ≤ n. Let $\mathcal{U}$ a finite open cover of X, then $\mathcal{U}$ a finite coc-open cover of X, then$\mathcal{U}$ a coc-open cover. Since coc′-dim X ≤ n, then there exists $\mathcal{V}$ a coc-open refinement of order ≤ n. Hence has $\mathcal{V}$ a coc-open refinement of order ≤ n, therefore coc-dim X ≤ n.

Lemma (5.7) : If (X , $\mathcal{T}$) is a CC space , then coc-dim X = coc′-dim X = dim X.

Proof : By**[S.Al Ghour and S. Samarah, 2012]** the statement are equivalent :(i) (X , $\mathcal{T}$) is a CC space . (ii) $\mathcal{T} = \mathcal{T}^K$.

Lemma(5.8): If(X , $\mathcal{T}$) is a coc$\mathcal{T}_2$–space, then coc-dim X = coc′-dim X = dim X.

Proof : By **[S.Al Ghour and S. Samarah, 2012]** if (X , $\mathcal{T}$) is a coc$\mathcal{T}_2$ –space, then $\mathcal{T} = \mathcal{T}^K$.

Definition (5.9) : A coc-base for the topology of a space X is a set β of coc-open sets such that for every point x ∈ X and every neighborhood N of x , there exists B such that x ∈ B ⊆N . Equivalently β is a coc-base if each open set is a union of some members of β.

Definition(5.10): A coc*-base for the topology of a space X is a set β of coc-open sets such that for every point x ∈ X and every coc-neighborhood N of x , there exists B such that x ∈ B ⊆N . Equivalently β is a coc*-base if each open set is a union of some members of β.

Theorem(5.11): Let X be a topological space. If X has a coc-base of sets which are both coc-open and coc-closed , then coc-dim$X = 0$ for an $\mathcal{T}_1 -$ space, the converse is true .

Proof: Suppose that X has coc-base of sets which are both coc-open and coc-closed. Let $\{U_i\}_{i=1}^{k}$ be a finite open cover of X, it has a coc-open and coc-closed refinement $\mathcal{W}$, if $w \in \mathcal{W}$, then $w \subset U_i$ for some i. Let each $w \in \mathcal{W}$ be associated with one of the U_i sets containing it and let U_i be the union of these members of $\mathcal{W}$, thus associated with U_i. Thus $V_i = W_j \setminus \cup_{r<j} W_r$ is coc-open set and hence $\{V_i\}_{i=1}^{k}$ forms disjoint coc-open refinement of $\{U_i\}_{i=1}^{k}$, then coc-dim$X = 0$

Conversely; Suppose that X is coc-$\mathcal{T}_1$space such thatcoc-dim$X = 0$.Let $x \in X$ and G be anopen set in X such that $x \in G$. Then $\{x\}$is closed set , then $\{G\ ,\ X\text{-}\{x\}\}$ is finite open cover of X. Since coc-dim$X = 0$, then there exists coc-open refinement $\{V\ , W\}$ of order 0 such that $V \cap W = \emptyset$, $V \cup W = X$, $V \subset G$ and $W \subset X\text{-}\{x\}$. Then V iscoc-open and coc-closed set in X such that $x \in W^c \subseteq V \subseteq G$ and hence X has coc-base of coc-open and coc-closed .

Remark(5.14) [A.P. Pears, 1975]: Let X be a topological space with dim $X = 0$. Then X is normal space.

Theorem (5.15): Let X be a topological space. If coc-dim $X = 0$, then X is coc-normal space .

Proof: Let F_1 and F_2 are disjoint closed sets of X . Then $\{X\text{-}F_1\ , X\text{-}\ F_2\}$ is open cover of X. Since coc-dim $X = 0$, then it is has coc-open refinement of order 0, hence So that coc-open sets H and G such that $H \cap G = \emptyset$, $H \cup G = X$, therefore $H \subset X$-F_1 and $G \subset X$-F_2. Thus $F_1 \subset H^c = G$, $F_2 \subset G^c = H$ and since $H \cap G = \emptyset$, then is X is coc-normal space.

Theorem (5.16): Let X be a topological space. If coc′-dim $X = 0$, then X is coc′-normal space.

Proof: Let F_1 and F_2 are disjoint coc-closed sets of X. Then $\{X$-F_1 , X- $F_2\}$ is coc-open covering of X. Since coc′-dim $X = 0$, then it is has coc-open refinement of order 0, hence there exists coc-open sets H and G such that $H \cap G = \emptyset$, $H \cup G = X$, so that $H \subset X$-F_1 and $G \subset X$-F_2. Thus $F_1 \subset H^c = G$, $F_2 \subset G^c = H$ and since H $\cap G = \emptyset$, then is X is coc′-normal space.

Theorem (5.18): If A closed subset of a topological space X, then coc-dim $A \leq$ coc-dim X.

Proof: Suppose that dim $X \leq n$, let $\{U_1, U_2, \dots, U_k\}$be an open cover of A, then for each i , $U_i = A \cap V_i$, where $V_i$is an open set in X. The finite open covering $\{ V_1 , V_2, \dots , V_k , X$-$A \}$ of X has coc-open refinement $\mathcal{W}$in X of order $\leq n$. Let $\mathcal{V} = \{ W \cap A : W \in \mathcal{W}\}$, by proposition(2.4), then $\mathcal{V}$ is coc-open refinement of $\{U_1, U_2, \dots, U_k\}$of order $\leq n$. Thus coc-dim $A \leq n$.

6. Relation between the dimensions Coc-ind and Coc-Ind

Theorem (6.1): Let X be a topological space, if X is regular, thencoc-ind $X \leq coc$-Ind X .

Proof: By induction on n , if$n = -$, then the statement is true.

Suppose that the statement is true for $\mathrm{n} - 1$.

Now ; Suppose thatcoc-Ind$X \leq n$, to provecoc-ind$X \leq n$. Let$x \in X$ and G be an open set such that x ∈ G. Since X is regular space, then there exists an open set U such that $x \in U \subseteq \overline{U} \subseteq G$ by proposition (2.17) . Also since coc-Ind $X \leq n$ and $\overline{U}$ is closed, $\overline{U} \subseteq G$ then there exists a coc- open set V such that $\overline{U} \subseteq V \subseteq G$ and coc-Ind $b(V) \leq n - 1$, then coc-ind $b(V) \leq n - 1$ and coc-ind$X \leq n$, then coc-ind $X \leq coc$-Ind X.

Proposition (6.3): Let Xbe a topological space , if X $\mathcal{T}_1 - space$,then coc^*-ind$X \leq coc^*$-Ind$X$.

Proof: By induction on n , if $n = -1$, then the statement is true.

Suppose that the statement is true for $\mathrm{n} - 1$.

Now, Suppose thatcoc^*-Ind$X \leq n$,to provecoc^*-ind$X \leq n$. Let $x \in X$ and each open set G ⊆ X of the point x, since X is $\mathcal{T}_1$ − space, then{x} is closed set and $\{x\} \subseteq$ G. s Since coc^*-Ind$X \leq n$, then there exists an coc -open set V in X such that $\{x\} \subseteq V \subseteq G$ and coc^*-Ind $b_{coc}(V) \leq n - 1$. Hence coc^*-ind$b_{coc}(V) \leq n - 1$and $x \in V \subseteq G$.Thus coc^*-ind$X \leq n$.

Proposition (6.4): Let Xbe a topological space, if$X$iscoc-regularspace , thencoc^*-ind$X \leq coc^*$-Ind$X$.

Proof: By the same away in proposition(6.1).

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حول نظرية البعد باستخدام المجموعات المفتوحة من النمط -coc

رعد عزيز حسين العبد الله،   ورود حسن هويدي شروم

قسم الرياضيات، كلية علوم الحاسوب وتكنولوجيا المعلومات ، جامعة القادسية

ملخص :

في هذا البحث نقدم ونعرف نوع جديد وعام حول نظرية البعد من النمط - coc لقد درسنا المفاهيم ind ,Ind,dim وسوف نعرف في هذا البحث تلك المفاهيم باستخدام المجموعات المفتوحة من النمط - coc وايجاد بعض العلاقات فيما بينها .