Al-Qadisiyah Journal of Pure Science

Volume 25 | Number 2 Article 6

4-7-2020

Some Results Related With Type Of Family Of Sets

Alaa M. Jasem
Department of Mathematics, College of Science, University of Al-Qadisiyah, Diwaniyah, Iraq,,
Nafm60@yahoo.com

Noori F. Al-Mayahi
Department of Mathematics, College of Science, University of Al-Qadisiyah, Diwaniyah, Iraq,,
sybzqoa@gmail.com

Al-Qadisiyah Journal of Pure Science Vol.(25) Issue (2) (2020) pp. Math. 51–59

Al-Qadisiyah Journal of Pure Science

ISSN(Printed): 1997-2490 ISSN(Online): 2411-3514

DOI: /10.29350/jops.

http://qu.edu.iq/journalsc/index.php/JOPS

Some Results Related With Type Of Family Of Sets

Authors Names

a. ALAA M. JASEM

b. NOORI F. AL-MAYAHI

Article History

Received on: 06/01/2020
Revised on: 20/01/2020
Accepted on:31/03/2020

Keywords:

fuzzy set, fuzzy field , fuzzy σ-field , fuzzy α-field ,fuzzy α-σ-field ,fuzzy β-field, fuzzy β-σ-field.

DOI: https://doi.org/10.29350/ jops.2020.25. 2.1018

ABSTRACT

In this paper, we will study new types of fuzzy families such as the fuzzy β-field ,fuzzy β-σ-field ,fuzzy α-field ,fuzzy α-σ-field ,and fuzzy σ-field and the relationship between them

MSC: 30C45, 30C50

1.Introduction

The concept of a fuzzy integral with respect to fuzzy measure was introduced by Sugeno [4] , and the definition of a fuzzy σ-field on fuzzy set provided by [3],[7] , in 2019 Ibrahim and Hassan introduced some concepts such as α-σ-field and β-σ-field which represent the generalizations of σ-field [1], in this paper we will introduced the concept of fuzzy α-field , fuzzy α-σ-field ,fuzzy β-field , fuzzy β-σ-field ,fuzzy σ-field and the relation between them .

2. FuzzySets

This section deals with the concepts of fuzzy set, complement , and operation on fuzzy sets.

[a] Department of Mathematics, College of Science, University of Al-Qadisiyah, Diwaniyah, Iraq, E-Mail: N afm60 @yahoo.com

[b] Department of Mathematics, College of Science, University of Al-Qadisiyah, Diwaniyah, Iraq, E-Mail: sybzqoa@gmail.com

Definition (2. 1): [5][8]. Let Ω be anon empty set, a fuzzy set A in Ω(or a fuzzy subset inΩ) is a function from Ω into I, $A \in I^{\Omega}$. $A(x)$is interpreted as the degree of membership of element x in a fuzzy set A for each $x \in \Omega$, a fuzzy set A in Ωcan be represented by the set of pairs:

$$A = \{(x, A(x)): x \in \Omega\}$$

Note that every ordinary set is a fuzzy set, i.e. $P(\Omega) \subseteq I^{\Omega}$.

Definition (2. 2) : [5][8][6]. Let A and B be a fuzzy sets in Ω .

(i) A and B are said to be equal (or$A$ equals B) , which written as $A = B$ if$A(x) = B(x)$ for all $x \in \Omega$.

(ii) Aisincludedin B and we write $A \subseteq B$, if $A(x) \le B(x)$ for all $x \in \Omega$. Hence $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

(iii) A is proper subset of B and write $A \subset B$ if and only if $A \subseteq B$ and $A \neq B$.

(iv) The union $A \cup B$ of A and B is defined by

$$(A \cup B)(x) = \max\{A(x), B(x)\} \text{ for all } x \in \Omega.$$

(v) The intersection $A \cap B$ of A and B is defined by

$$(A \cap B)(x) = \min\{A(x), B(x)\} \text{ for all } x \in \Omega.$$

Similar to operations on ordinary sets, one can generalize the union and the intersection for an arbitrary family of fuzzy sets: if $\{A_\lambda: \lambda \in \Lambda\}$ is a family of fuzzy sets, where Λan arbitrary of index set, the union is $\bigcup_{\lambda\in\Lambda} A_\lambda$ is the fuzzy set having membership function $\sup\{A_\lambda(x): \lambda \in \Lambda\}$, i.e.

$$(\bigcup_{\lambda\in\Lambda} A_\lambda)(x) = \sup\{A_\lambda(x): \lambda \in \Lambda\} \; for\ all\ x \in \Omega$$

and the intersection $\bigcap_{\lambda\in\Lambda} A_\lambda$ is the fuzzy set having membership function $\inf\{A_\lambda(x): \lambda \in \Lambda\}$, i.e.

$$\left(\bigcap_{\lambda\in\Lambda} A_\lambda\right)(x) = \inf\{A_\lambda(x): \lambda \in \Lambda\} \; for\ all\ x \in \Omega.$$

Definition (2. 3): [5][6]. Let A and B be fuzzy sets inΩ .

(i)The complement A^c , of A is defined by $A^c(x) = 1 - A(x)$ for all $x \in \Omega$.

(ii)The difference A/Bbetween A and B is defined by $A/B = A \cap B^c$.

(iii)The symmetric difference, $A \triangle B$, between A and B is defined by $A \triangle B = ({}^{A}/_{B}) \cup ({}^{B}/_{A})$.

3. Type of some family of sets and relation between them

In this section we will introduce and study new concepts such as fuzzy β-field ,fuzzy $\beta - \sigma$-field ,fuzzy α-field and fuzzy α-σ-field , and we give basic properties ,and examples of these concepts.

Definition (3.1) :[2].A non empty family $\mathcal{F}$ of a fuzzy sets of a set Ω is called fuzzy field on Ω if

1. $\emptyset, \Omega \in \mathcal{F}$

2. If A$\in \mathcal{F}$ $then\ A^c \in \mathcal{F}$
3. If $A_1, A_2, \dots\dots, A_n \in \mathcal{F}$ then $\bigcup_{i=1}^{n} A_i \in \mathcal{F}$

If (3) is replaced by the closure under countable union we get on the following definition

Definition (3.2) :[3] [7].A non empty family $\mathcal{F}$ of a fuzzy sets of a set Ω is called fuzzy σ- field on aset Ω if
1. $\emptyset$,Ω$\in \mathcal{F}$
2. If A$\in\mathcal{F}$ then $A^c\in\mathcal{F}$
3. If $A_n \in \mathcal{F}$ $n = 1,2,3, \dots .$ $then$ $\bigcup_{n=1}^{\infty} A_n \in \mathcal{F}$

A fuzzy measurable space is a pair (Ω,$\mathcal{F}$),where Ω is a nonempty set and $\mathcal{F}$ is a fuzzy σ-field on Ω. a fuzzy set A in Ω is called fuzzy measurable (fuzzy measurable with respect to the fuzzy σ-field) if A$\in \mathcal{F}$, i.e. any member of $\mathcal{F}$ is called a fuzzy measurable set .

Example (3.3) : The family $\mathcal{F}$ of all fuzzy sets of a set Ω is a fuzzy field on Ω.

Proof: Let $\mathcal{F} = \{A: A \in I^{\Omega}\}$

1. It is clear that Ø , Ω $\in \mathcal{F}$.
2. Let A$\in \mathcal{F}$,hence A$\in I^{\Omega}$, then $0\leq A(x) \leq 1$, $A^c(x) = 1 - A(x)$

Hence $0\leq 1 - A(x) \leq 1 \Longrightarrow 0 \leq A^c(x) \leq 1$

Therefore $A^c \in I^{\Omega}$, hence $A^c \in \mathcal{F}$.

3. Let $A_1, A_{2,\dots,} A_n \in \mathcal{F}$, then $A_1, A_2, \dots , A_n \in I^{\Omega}$

$0\leq A_i(x) \leq 1 \quad \forall\, i = 1,2, \dots , n$

$\cup_{i=1}^{n} A_i(x) = \max\{A_i(x):\ i = 1,2, \dots , n\} \Longrightarrow 0 \leq \max\{A_i(x): i = 1,2, \dots , n\} \leq 1$

Hence $\cup_{i=1}^{n} A_i \in \mathcal{F}$, therefore $\mathcal{F}$ is a fuzzy field

Example (3.4) :[2]. 1. The family $\mathcal{F}$ of all fuzzy sets on a set Ω is a fuzzy σ-field on Ω.

2.The family $\mathcal{F}$ ={Ø , Ω} is a fuzzy σ-field on Ω

■in the following theorem ,we can show the relationships between a fuzzy field and a fuzzy σ-field .

Theorem (3.5) :[2]. Any fuzzy σ-field is a fuzzy field

Proof: Suppose $\mathcal{F}$ is a fuzzy σ-field , hence

1. Ø, Ω $\in \mathcal{F}$.
2. If A $\in \mathcal{F}$, then $A^c \in \mathcal{F}$.
3. Let $A_1, A_2, \dots , A_n \in \mathcal{F}$,we put $A_k = \emptyset \forall\, k > n$

Since $\mathcal{F}$ is fuzzy σ-field , it is clear $\cup_{i=1}^{\infty} A_i \in \mathcal{F}$,

$$But\ \cup_{i=1}^{\infty} A_i = \cup_{i=1}^{n} A_i\ , when\ A_k = \emptyset\ \forall\, k > n$$

$\therefore \cup_{i=1}^{n} A_i \in \mathcal{F}$, Thuse $\mathcal{F}$ is fuzzy field

■ in the next theorem ,we will demonstrate that the intersection of fuzzy σ-fields is a fuzzy σ-field

Theorem (3.6):[2]. Let $\{\mathcal{F}_i\}_{i\in I}$be a family of fuzzy σ-field ,then $\cap_{i\in I} \mathcal{F}_i$is a fuzzy σ-field

Remark (3.7):[2]. The union of fuzzy σ-field does not to be fuzzy σ-field as in the following example :

Example (3.8):[2]. Let A, B, C, D are fuzzy sets and $\Omega = [0,1]$such that

$$A(x) = \begin{cases} 2x & 0 \le x \le \frac{1}{2} \\ 0 & \frac{1}{2} < x \le 1 \end{cases} \quad B(x) = \begin{cases} 0 & 0 \le x \le \frac{1}{4} \\ 2x & \frac{1}{4} < x \le \frac{1}{2} \\ 1 & \frac{1}{2} < x \le 1 \end{cases}$$

$$C(x) = \begin{cases} 1-2x & 0 \le x \le \frac{1}{2} \\ 1 & \frac{1}{2} < x \le 1 \end{cases} \quad D(x) = \begin{cases} 1 & 0 \le x \le \frac{1}{4} \\ 1-2x & \frac{1}{4} < x \le \frac{1}{2} \\ 0 & \frac{1}{2} < x \le 1 \end{cases}.$$

Let $\mathcal{F}_1 = \{\emptyset, A(x), C(x), \Omega\}$, $\mathcal{F}_2 = \{\emptyset, B(x), D(x), \Omega\}$ are two fuzzy $\sigma-$fields, but $\mathcal{F}_1 \cup \mathcal{F}_2$is not fuzzy$\sigma-$field.

Definition (3.9): Let Ω be a nonempty set and let $\mathcal{F}$ be a family of fuzzy sets on a set Ω ,then$\mathcal{F}$ is called fuzzy α-field if the following conditions satisfied :

1. $\Omega \in \mathcal{F}$.

2.if$A_1, A_2, \dots, A_n \in \mathcal{F}$, then $\cup_{i=1}^{n} A_i \in \mathcal{F}$.

Example (3.10): Let Ω=[0,1] and A be a fuzzy set on Ω, define as follows

$$A(x) = \begin{cases} 0 & 0 \le x \le \frac{1}{2} \\ 1 & \frac{1}{2} < x \le 1 \end{cases}$$ and let $\mathcal{F} = \{A , \Omega\}$, then $\mathcal{F}$ is fuzzy α-field

Theorem (3.11): Every fuzzy field is fuzzy α-field

Proof : Let $\mathcal{F}$ be fuzzy field (by definition of $\mathcal{F}$) we get

1. $\Omega \in \mathcal{F}$

2. If $A_1, A_2, \dots, A_n \in \mathcal{F}$, then $\cup_{i=1}^{n} A_i \in \mathcal{F}$

Hence $\mathcal{F}$ is a fuzzy α-field on a set Ω.

■ in general the covers of theorem (3.11) is not true and example (3.10) indicate that ,

$\mathcal{F} = \{A , \Omega\}$ iis fuzzy α-field , but not fuzzy field because A$\in \mathcal{F}$,but

$$A^c(x) = \begin{cases} 1 & 0 \le x \le \frac{1}{2} \\ 0 & \frac{1}{2} < x \le 1 \end{cases} \notin \mathcal{F}$$

Theorem (3.12): every fuzzy σ-field is fuzzy α-field

proof : Direct

■ in general the convers theorem (3.12) is not true and example (3.10) indicate that

Definition (3.13): Let Ω be a nonempty set and let $\mathcal{F}$ be a family of fuzzy sets on Ω , $\mathcal{F}$ is called fuzzy α-σ-field if the following condition satisfied :

1. $\emptyset$, $\Omega \in \mathcal{F}$.
2. If $A_1, A_2, \dots \in \mathcal{F}$, then $\cup_{i=1}^{\infty} A_i \in \mathcal{F}$.

Definition (3.14): Let Ω be a nonempty set and $\mathcal{F}$ be a fuzzy α-σ-field of a set Ω, then a pair $(\Omega, \mathcal{F})$ is called α-fuzzy measurable space and the member of $\mathcal{F}$ are called fuzzyα-measurable sets

Example (3.15): Let Ω=[0,1] and A, fuzzy set define on Ω as follows

$$A(x) = \begin{cases} 0 & 0 \le x \le \frac{1}{2} \\ 1 & \frac{1}{2} < x \le 1 \end{cases}$$ and let $\mathcal{F} = \{\emptyset, A, \Omega\}$, then $\mathcal{F}$ is fuzzy α-σ-field .

Proposition (3.16): Every fuzzy α-σ-field is a fuzzy α-field

Proof : Let $\mathcal{F}$ be a fuzzy α-σ-field on a set Ω, (by definition of $\mathcal{F}$) we get

1. $\emptyset$, $\Omega \in \mathcal{F}$.
2. Let $A_1, A_2, \dots, A_n \in \mathcal{F}$,and put $A_k = \emptyset$ for all k$> n$

Since $\mathcal{F}$ is fuzzy α-σ-field , then $\cup_{i=1}^{\infty} A_i \in \mathcal{F}$, but $A_i = \emptyset \ \forall i > n$ then $\cup_{i=1}^{\infty} A_i = \cup_{i=1}^{n} A_i$,hence $\cup_{i=1}^{n} A_i \in \mathcal{F}$

Hence $\mathcal{F}$ is a fuzzy α-field

■ in general the convers of proposition (3.16) is not true and example (3.10) indicate that ,$\mathcal{F}$ is fuzzy α-field but not fuzzy α-σ-field ,because $\emptyset \notin \mathcal{F}$.

Theorem (3.17): Every fuzzy σ-field is fuzzy α-σ-field .

Proof : Let $\mathcal{F}$ be a fuzzy σ-field (by definition of $\mathcal{F}$) we get

$\Omega \in \mathcal{F}$,and $\emptyset = \Omega^c \in \mathcal{F}$,hence

1. $\emptyset, \Omega \in \mathcal{F}$
2. Let $A_1, A_2, \dots \in \mathcal{F}$, then $\cup_{i=1}^{\infty} A_i \in \mathcal{F}$

Hence $\mathcal{F}$ is fuzzy α-σ-field

■ in general the converse of theorem (3.17) is not true and example (3.15) indicate that $\mathcal{F}$ is fuzzy α-σ-field ,but not fuzzy σ-field , because A$\in \mathcal{F}$,but $A^c \notin \mathcal{F}$

Theorem (3.18): Let $\{\mathcal{F}_i\}_{i\in I}$ *be* family of fuzzy α-σ-field on a set Ω ,then$\cap_{i\in I} \mathcal{F}_i$ is a fuzzy α-σ-field on Ω.

Proof :1. Since $\mathcal{F}_i$ *is a fuzzy* α-σ-field on Ω ,for all i$\in I$,then $\emptyset,\Omega \in \mathcal{F}_i$

For all i$\in I$,hence$\emptyset, \Omega \in \cap_{i\in I} \mathcal{F}_i$

2.Let $A_1, A_2 \dots, \in \cap_{i\in I} \mathcal{F}_i$,then $A_1, A_2, \dots \in \mathcal{F}_i$ for all i$\in I$
Since $\mathcal{F}_i$is fuzzy α-σ-field for all i$\in I$,then$\cup_{i=1}^{\infty} A_i \in \mathcal{F}_i$ for all i$\in I$ hence $\cup_{i=1}^{\infty} A_i \in \cap_{i\in I} \mathcal{F}_i$

Therefore $\cap_{i\in I} \mathcal{F}_i$ *is* a fuzzy α-σ-field

■ the next example indicate that the union of two fuzzy α-σ-field on Ω is not necessary a fuzzy α-σ-field on Ω

Example (3.19): Let Ω=[0,1] and let A and B are two fuzzy sets on Ω define as follows

$$A(x)=\begin{cases} 0 & 0 \le x \le \frac{1}{2} \\ 1 & \frac{1}{2} < x \le 1 \end{cases} \quad , \quad B(x)=\begin{cases} x & 0 \le x \le \frac{1}{2} \\ 1 & \frac{1}{2} < x \le 1 \end{cases}$$

And let $\mathcal{F}_1 = \{\emptyset , A, \Omega\}$ *and* $\mathcal{F}_2 = \{\emptyset, B , \Omega\}$, then $\mathcal{F}_1$ *and* $\mathcal{F}_2$ are fuzzy α-σ-field on a set Ω ,But $\mathcal{F}_1 \cup \mathcal{F}_2 = \{\emptyset, A, B, \Omega\}$ is not fuzzy α-σ-field on Ω

Since $(A\cup B)(x) = \max\{A(x), B(x)\}$

If $x=\frac{1}{2} \rightarrow (A\cup B)(\frac{1}{2}) = \max\{A\left(\frac{1}{2}\right), B\left(\frac{1}{2}\right)\} = \max\{0, \frac{1}{2}\}$

$$=\frac{1}{2} \notin \mathcal{F}_1 \cup \mathcal{F}_2.$$

Definition (3.20) : Let $\mathcal{F}$ be a nonempty family of fuzzy set on a set Ω ,then$\mathcal{F}$ is called fuzzy β-field if the following condition satisfied :

1. $\emptyset \in \mathcal{F}$.
2. If $A_1, A_2, \dots, A_n \in \mathcal{F}, then\ \cap_{i=1}^{n} A_i \in \mathcal{F}$.

Example (3.21): Let Ω=[0,1] and A be fuzzy set on Ω ,define as follows

$$A(x)=\begin{cases} 0 & 0 \le x \le \frac{1}{2} \\ 1 & \frac{1}{2} < x \le 1 \end{cases}$$ and $\mathcal{F} = \{\emptyset, A\}$ then $\mathcal{F}$ is fuzzy β-field

Theorem (3.22): Every fuzzy field is fuzzy β-field

Proof: Let $\mathcal{F}$ be fuzzy field (by definition of fuzzy field) we get

1. $U \in \mathcal{F}$, $and\ \emptyset = U^c \in \mathcal{F}\ therefore\ \emptyset \in \mathcal{F}$.
2. Let $A_1, A_2, \dots, A_n \in \mathcal{F}$,then $\cup_{i=1}^{\infty} A_i \in \mathcal{F}$, by demorgan laws we have $\cap_{i=1}^{\infty} A_i = (\cup_{i=1}^{\infty} {A_i}^c)^c$,but $A_1, A_2, \dots, A_n \in \mathcal{F}$

Then $A_1^c, A_2^c, \dots, A_n^c \in \mathcal{F}$,and$\cup_{i=1}^{\infty} A_i^c \in \mathcal{F}$,hence $(\cup_{i=1}^{\infty} A_i^c)^c \in \mathcal{F}$ therefore $\cap_{i=1}^{\infty} A_i \in \mathcal{F}$

Hence $\mathcal{F}$ is a fuzzy β-field on a set Ω

■ in general the converse of theorem (3.22) is not true and example (3.21) indicate that ,$\mathcal{F}$ is a fuzzy β-field but not fuzzy field ,because A∈ $\mathcal{F}$ but $A^c \notin \mathcal{F}$

Remark (3.23): Every fuzzy σ-field is fuzzy β- field , but the converse is not true

Definition (3.24): Let $\mathcal{F}$ be anon empty family of fuzzy sets on a set Ω , then $\mathcal{F}$ is called fuzzy $\beta - \sigma -$field if the following conditions satisfied :

1. $\emptyset , \Omega \in \mathcal{F}$

2. if$A_1 , A_2 , \dots \dots \in \mathcal{F}$ then $\bigcap_{i=1}^{\infty} A_i \in \mathcal{F}$

Definition (3.25): Let Ω be anon empty set and $\mathcal{F}$ is fuzzy $\beta - \sigma -$ field on Ω then the pair (Ω,$\mathcal{F}$)is called $\beta - fuzzy$ measurable space over $\mathcal{F}$ and the member of $\mathcal{F}$ are called β- fuzzy measurable set .

Example (3.26): Let Ω=[0,1] and , $$A(x)=\begin{cases} 0 & 0 \le x \le \frac{1}{2} \\ 1 & \frac{1}{2} < x \le 1 \end{cases}$$

and$\mathcal{F} = \{\emptyset , A , \Omega\}$ then $\mathcal{F}$ is a fuzzy $\beta - \sigma -$field

proposition (3.27): Let $\{\mathcal{F}_i\}_{i \in I}$be family of fuzzy $\beta - \sigma -$field on Ω ,then$\bigcap_{i \in I} \mathcal{F}_i$ is a fuzzy $\beta - \sigma - field$

Proof:1.since$\mathcal{F}_i$ is fuzzy $\beta - \sigma -$field $\forall i \in I$, then $\emptyset , \Omega \in \mathcal{F}_i\ \forall i \in I$ hence $\emptyset , \Omega \in \bigcap_{i \in I} \mathcal{F}_i$

2.letA_1 , A_2 , … … $\in \bigcap_{i\in I} \mathcal{F}_i$ then A_1 , A_2 , … . . $\in \mathcal{F}_i$ for all i$\in I$ since $\mathcal{F}_i$ is a fuzzy $\beta - \sigma -$field $\forall i \in I$ then $\bigcap_{k=1}^{\infty} A_k \in \mathcal{F}_i$ $\forall i \in I$ which is implies that $\bigcap_{k=1}^{\infty} A_k \in \bigcap_{i\in \mathcal{F}} \mathcal{F}_i$

hence$\bigcap_{i\in I} \mathcal{F}_i$ is fuzzy $\beta - \sigma -$field

Proposition (3.28): Every fuzzy σ-field is a fuzzy β-σ-field .

Proof : Direct

Remark :in general the convers of theorem (3.28) is not true and example (3.26) indicate that , $\mathcal{F}$ is fuzzy β-σ-field , but not fuzzy σ-field because A$\in \mathcal{F}$, but $A^c \notin \mathcal{F}$.

Theorem (3.29): Every fuzzy β-σ-field is fuzzy β-field

Proof : let$\mathcal{F}$ be fuzzy β-σ-field , then

1. $\emptyset$, $\Omega \in \mathcal{F}$
2. Let $A_1, A_2, \dots, A_n \in \mathcal{F}$,and put $A_k = \Omega$ for all k$> n$

Then $\cap_{i=1}^{n} A_i = \cap_{i=1}^{\infty} A_i$, since $\mathcal{F}$ be fuzzy β-σ-field ,

then $\cap_{i=1}^{\infty} A_n \in \mathcal{F}$, hence $\cap_{i=1}^{n} A_i \in \mathcal{F}$

therefore$\mathcal{F}$ is fuzzy β-field on Ω .

■in general the convers of theorem (3.29) is not true and example (3.21) indicate that $\mathcal{F}$ is fuzzy β-field ,but not fuzzy β-σ-field ,because $\Omega \notin \mathcal{F}$.

References

[1] Ibrahim, S. A,; Hassan, H. E., Generalization of σ-field and New Collections of Sets Noted by δ-field, AIP Conference proceedings , 2019, 2096, 020019, 020019_1-020019_6, doi.org/10.1063/1.5097816.

[2]Karrar, S. H. ; ;On Fuzzy Measure With Respect To Fuzzy Sets ,2017.

[3] Klement, E. P., "Fuzzy u-algebras and fuzzy measurable functions", Fuzzy Sets and Systems4 (1980) 83-93

[4] Sugeno, .M "Theory of fuzzy Integrals and Its Applications", p h .D. Dissertation , Tokyo Institute of Technology, 1975

[5] Zadeh, L. A., "Fuzzy sets, Information and Control", 8 (1965) 338-353.

[6] Zadeh, L., A., "Fuzzy Sets", Information Sets, Edited by Yagar R. R., Ovchinnikos S., Tong R. M. and Ngnyenw T., John Wiley and Sons, Inc., 1987.

[7] Zhong, Q., "Riesz's theorem and Lebesgue's theorem on the fuzzy measure space", busefal 29, (1987), 33-41.

[8] Zimmerman, H. J., "fuzzy set theory and Its Application", Kluwer Academic Publishers, 2001.