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# Using Fibonacci Sequence in Nature

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## Abstract

In this work, primarily focuses of the Fibonacci sequence (FS) by compute each number (N) is the total of the two preceding numbers. The quantities (N) that are associated with the Fibonacci sequence (FS) are referred to as Fibonacci numbers (FN), which are typically written as  $F_n$ . The order (S) commonly starts from 0 to 1, and presented formula for Fibonacci sequence (FS), understand Fibonacci numbers (FN) through solved various examples. Moreover introduced connection between the Fibonacci sequence (FS) and Golden Ratio (GR), relation between Fibonacci sequence (FS) and Geometric Sequence (GS) and so comparison between Lucas Sequence And (FS). Also give applied Fibonacci sequence (FS) in nature.

**Keywords:** Fibonacci sequence, Fibonacci number, Golden Ratio, Lucas Sequence

## 1. Introduction

One of the most common scientific experiments that involves the (FS) is discussed below is his experiment with rabbits [1]. Mathematics is a specialized field of science that concerns (N), shapes, numbers, and relations, (S), series. In Ref. [2] researcher is attributed with the initial description of the (S) of (N), which occurred during the (5th B.C. and 2nd or 3rd century A.D.). Since Fibonacci documented the series for the Western world, it has been recognized as significant. In The Da Vinci Code, the (FS) is integral to a significant revelation. Another take [3]. The (FS) is said for Leonardo Pisano (fame Fibonacci), an Italian mathematician. The (S) presented by Fibonacci to answer the following question: "How many duo of rabbits will be produced in a year, beginning with a single duo, if in every month each duo bears a new duo which becomes productive from the 2nd month on?" The outcome is expressed as 1, 1, 2, 3, 5, 8, 13, 21, 34,.. [4]. The Fibonacci poem's verse employs the same syllable progression (N) on each line, following the pattern of Fibonacci. (FN) Also, they can be employed to define a circle and have a beneficial effect on biologists and physicists because

they are common in the natural world and are observed frequently. In Refs. [5–14] introduce patterns of branching in trees and leaves, as well as the distribution of seeds in a raspberry's berry are all indicative of the (FS). In this survey, presented role (FS) with many branches and applied its for simple examples.

## 2. Methodology of (FS)

In this part, presented a (FS) is the one in which every (N) offered is a sum of the previous 2 (N), where (FS) can be computed in mathematic. As each (N) in (S) is presented a idiom, that is rewrite by the idiom  $F_n$ . The n be inverted the (N) situation in the (S), initiate as zero. For instance, the (6th, 7th) term is mention to as  $F_5$ ,  $F_6$  respectively. The numbering of (FS) proposed as:

- $F_0 = 0$  (implies exclusive the 1st integer)
- $F_1 = 1$  (implies exclusive the 2nd integer)
- $F_n = F_{n-1} + F_{n-2}$  (implies to all other integers)

The above equation show that each (N) of the (S) is realized by making the preceding (N). For test, to realize the 5th (N) is ( $F_4$ ), by the same way realize ( $F_2$ ,  $F_3$ ,...,  $F_n$ ) like Employing Table 1 to locate the

Table 1. Sequence numbers preceding the target term value.

Term position	Fn value	Fibonacci number
1st	F0	0
2nd	F1	1
3rd	F2	1
4th	F3	2
5th	F4	3
6th	F5	5
7th	F6	8
8th	F7	13
9th	F8	21
10th	F9	34
11th	F10	55
12th	F11	89
13th	F12	144
14th	F13	233

sequence numbers preceding the target expression value. For test, Now computation locate the (FN) for the expression in the 10th situation (F9):

$$F_9 = F_{(9-1)} + F_{(9-2)} = F_8 + F_7 = 21 + 13 = 34$$

The defy with a recursive formula is that it constantly relies on knowing the previous (FN) in order to compute a specific (N) in the (S). For test, we need calculate (98th and 99th) terms before the value of the 100th term. Other equations are possible, but Binet's formula is the most popular, and it provides a closed-form solution to the (FS) problem. Also many programs of applications for recursive formula code such as Java, Python or PHP. It is that soft test to find is  $21 + 34 = 55$  from (s) above show that as

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811,... as shown in Table 2.

From (S) above can be represented the 'Rule' of (FS).

So 6th expression is submitted ( $x_6 = 8$ ). Lastly we get the following:

$$x_n = x_{(n-1)} + x_{(n-2)}$$

Where:

$x_n$  is idiom (N) "n"

$x_{(n-1)}$  is the previous idiom (n-1)

$x_{(n-2)}$  is the idiom before that (n-2)

Table 2. Rule of (FS).

n=	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	...
xn=	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	...

### 2.1. Connection between (FS) and (GR)

The survey fundamentally focuses on the utilize of the (GR) and the (FS). The connection between them find in nature.

With the assist of the (FS) scientists, many mysteries associated with nature have been resolved. In order to sequence the numbers, they are organized in a specific manner and follow a common pattern. Four primary types of (S) exist (Arithmetic Sequence (AS), Geometric Sequence (GS) and Harmonic Sequence (HS)). (FN) can be explained as a circle with the (N)'s representing the widths of the (S)'s. The squares seem to fit together quite well, creating a circle. In this test,  $(5 + 8 = 13)$ ,  $(8 + 13 = 21)$ , etc. The (FS) is often associated with the (GR), a proportion (roughly 1:1.6) that is common in the natural world and is employed in multiple disciplines of human activity. The (FS) and (GR) are employed to direct the design of architecture as shown in Fig. 1.

### 2.2. Relation between (FS) and (GS)

In this section, show that (FS) is not (GS) since  $(\frac{F_1}{F_0}, \frac{F_2}{F_1}, \frac{F_3}{F_2})$  are (not defined,  $\frac{1}{1}, \frac{2}{1}$ ) have different values. Instead of being dissuaded, we should calculate several ratios of successive numbers. (FN) are expressed in Table 3 as:

As  $n$  turn out larger, the (FS) increasingly like a (GS) with a collective share of around 1.6. Since the upcoming integer values of the (FS) seem to have a significant impact, let's not concern ourselves with the primary values of the (S). Instead, we aim for a soft outcome (GS)  $x_n = ar^n$  that implies the Fibonacci recurrence  $x_{n+2} = x_{n+1} + x_n$ . That is, we want to consider values of  $r$  for which  $ar^{n+2} = ar^{n+1} + ar^n$ ,

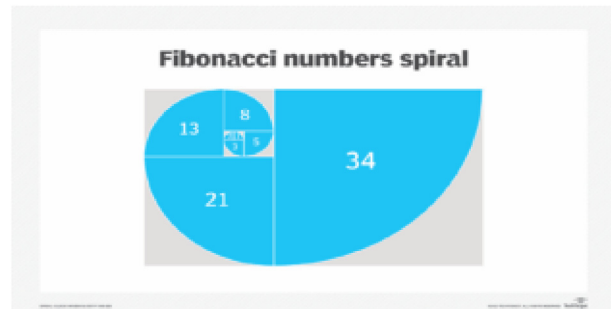


Fig. 1. Fibonacci numbers spiral.

Table 3. Ratios of successive (FN).

n	0	1	2	3	4	5	6	7	8
$F_n$	0	1	1	2	3	5	8	13	21
$\frac{F_{n+1}}{F_n}$	-	1.0	2.0	1.5	1.667	1.600	1.625	1.615	1.615

a constant  $r \neq 0$ , which  $r^{n+2} = r^{n+1} + r^n$ ,  $r^2 = r + 1$  is the quadratic formula, we obtain there are two roots  $r$  of the quadratic equation:  $p = \frac{1+\sqrt{5}}{2}$  and  $q = \frac{1-\sqrt{5}}{2}$ .  $p = 1.618$ , We believe we've identified the magnitude we sought in the numerical table above. For any two arbitrary numbers  $a$  and  $b$ , the geometric sequences  $ap^n$  and  $bq^n$  satisfy the Fibonacci law. Additionally, since  $ap^{n+2} = ap^{n+1} + ap^n$  and  $bq^{n+2} = bq^{n+1} + bq^n$ , we can add these two equations to see that

$$(ap^{n+2} + bq^{n+2}) = (ap^{n+1} + bq^{n+1}) + (ap^n + bq^n).$$

That is, for any values of  $a$  and  $b$ ,  $x_n = ap^n + bq^n$  solves the Fibonacci recurrence.

After that get the several result as:

**Theorem (1).** The Fibonacci recursion formula  $x_{n+2} = x_{n+1} + x_n$ , is solved by  $x_n = ap^n + bq^n$  where  $p = \frac{1+\sqrt{5}}{2}$ ,  $q = \frac{1-\sqrt{5}}{2}$ , and  $a$  and  $b$  are arbitrary constants. The constant  $p = \frac{1+\sqrt{5}}{2} = 1.6180\dots$  is the celebrated (GR). It was recognized (but not named) in ancient Greek mathematics, because it resolved the problem: determine the point  $C$  on a line segment  $\overline{AB}$  so that  $AB/AC = AC/BC$ . The second solution  $q = \frac{1-\sqrt{5}}{2} = -0.618\dots$ , of  $r^2 - r - 1 = 0$  can be write as quadratic polynomial.

$$r^2 - r - 1 = (r - p)(r - q) = r^2 - (p + q)r + pq$$

Then the coefficients presented as  $p + q = 1$  and  $pq = -1$ . It's show the relationships without requiring the explicit formulas for  $p$  and  $q$  that involve  $\sqrt{5}$ .

### 2.3. Comparison between Lucas Sequence And (FS)

When take is  $a = 1, b = 1$  for  $x_n = ap^n + bq^n$  in the theorem above get:

$$L_n = p^n + q^n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n. \text{ For instance, } L_0 = p^0 + q^0 = 1 + 1 = 2 \text{ and } L_1 = p + q = 1, \text{ and}$$

we found that no  $\sqrt{5}$  expression appears. Moreover, because  $L_{n+2} = L_{n+1} + L_n$  and  $L_0 = 2, L_1 = 1$ , the expression of the sequence are  $\forall Z^+$ . We have rediscovered the (FS): 2, 1, 3, 4, 7, 11, 18, 29, ... dedicated to Edouard Lucas (1842–1909). Despite this, the (S) is less commonly recognized. The (FS), the Lucas sequence is also characterized by several

properties that benefit. To get the (FN), we must assess the potential for choosing the constants  $a$  and  $b$  to produce the desired result  $F_n = ap^n + bq^n$ . Because  $F_0 = 0$  and  $ap^0 + bq^0 = a + b$ , if  $tb = -a$ . Also, since  $F_1 = 1$  and

$ap^1 + bq^1 = a(p - q) = a\left(\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2}\right) = a\sqrt{5}$ , we must take  $a = \frac{1}{\sqrt{5}}$ . Thus, the  $n$ th (FN) is given by the explicit formula

$$F_n = \frac{p^n - q^n}{\sqrt{5}} = \frac{1}{\sqrt{5}} \left[ \left(\frac{1 + \sqrt{5}}{2}\right)^n - \left(\frac{1 - \sqrt{5}}{2}\right)^n \right]$$

This formula is referred to as Binet's formula for the (FN), after Jacques Binet (1786–1865) who found it in 1843. However, the formula's discovery was first made in 1718 by Abraham DeMouivre (1667–1754). Since Binet's formula implies that  $F_0 = 0$ , we can now understand why Jason began with 0 in his list of (FN). The formula also demonstrates why the (FS) despite not being geometric, is increasingly similar to one. Since  $q = \frac{1-\sqrt{5}}{2} \approx -0.618$  we see that  $|q^n|$  is increasingly small as  $n$  becomes large. Then, we have the approximate equality  $F_n \approx \frac{1}{\sqrt{5}}p^n$ . This explains why the (FS) is nearly a (GS), as we noticed in the table of values compute earlier. If we let  $\{x\}$  denote the "round to the integer nearest" to  $x$  function, it is easy to check that  $F_n = \left\{ \frac{p^n}{\sqrt{5}} \right\}$  for all  $n \geq 0$ . Similarly, the Lucas numbers can be written as  $L_n = \{p^n\}$  when  $n \geq 1$ .

### 3. Applications

In the part, implied formula  $x_n = x_{n-1} + x_{n-2}$  for some examples of Fibonacci Numbers as:

**Example (1).** calculate the addition of the 1st-10th (FN).

**Sol.** The list of (FN) is introduced like: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34.

$$\sum = 0 + 1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 + 34 = 88.$$

**Example (2).** Write the value of the (12th, 13th) (FN). The 9th and 10th terms in the sequence are 21 and 34.

**Sol.** we require the (12th, 13th) (FN) as: Firstly present 11th term from (9th, 10th) term as  $21 + 34 = 55$ , 12th term as  $34 + 55 = 89$ , 13th term as  $55 + 89 = 144$ .

**Example (3).** Let that  $F_{14} = 377$ , compute the next (FN).

Sol. from  $F_{n+1} = F_n \times (GR)$ . Here,  $(GR) = 1.61803398875$ . Thus,  $F_{15} = F_{14} \times 1.615 = 377 \times 1.61803398875 \approx 610$ .

### 3.1. Applied (FN) in nature

#### 3.1.1. Shells

Shells are probably the most celebrated test of the (S) because the lines are very distinct and apparent, and because the math is small and personal as shown in Fig. 2.

#### 3.1.2. Trees

Initially, tree's appear as if they are part of a larger structure that analyses the growth of the part from the tree and the other? No, because you're typical and have a more favorable test result (FS).

#### 3.1.3. Flower pistils

The portion of the flower that is in the middle of the petals (the pistil) has a greater degree of similarity to the (FS) than other parts of nature. The pattern that results from the repeated composition of the sequence is aesthetically pleasing and intricate as shown in Fig. 3.

#### 3.1.4. Flower petals

All varieties of flowers follow this pattern, but roses are the most preferred type of flower for use as



Fig. 2. (FS) of shells.



Fig. 3. (FS) of flower pistils.

a test of (FS). I enjoy it because the petals aren't uniformly distributed, and the espiral is more apparent and lucid as shown in Fig. 4.

#### 3.1.5. Leaves

The veins on the leaves are progressing toward a more and more outward-oriented pattern as the (FS) increases as shown in Fig. 5.

#### 3.1.6. Storms

Specifically, hurricanes and tornadoes are frequented by many systems of storm. I think this isn't very attractive, but it's more intriguing as shown in Fig. 6.

#### 3.1.7. Human body

The human body has several ways of expressing the (FS) proportions, including your face, your ear, and your hands, as well as other ways. You're now mathematically brilliant as shown in Fig. 7.



Fig. 4. Fibonacci sequence of flower petals.



Fig. 5. (FS) of leaves.

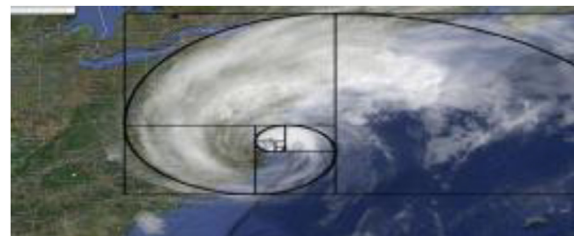


Fig. 6. (FC) of storms.

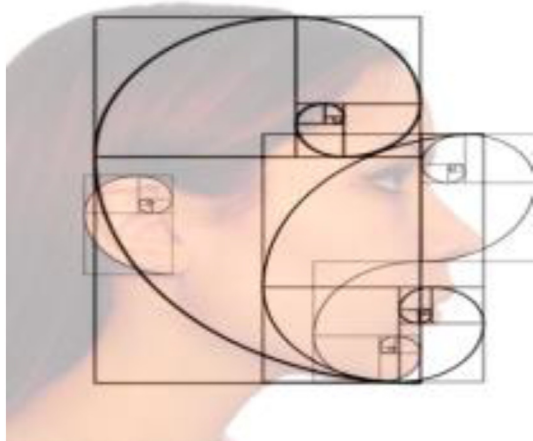


Fig. 7. (FC) of human body.

#### 4. Conclusion

In this paper, understanding the (FN) are Nature's numbering system, (FS), the (1st, 2nd) idiom is (0, 1) respectively and The (FN) have numerous practical uses in computer science, music, financial transactions, and other fields. (FN) are present in nature in various patterns and forms. As a result, the number of (FS) will change depending on their position in the series. To calculate the general formula:

$x_n = x_{n-1} + x_{n-2}$ . Also study the difference (Fs) and Lucas through The creation of both Fibonacci and Lucas numbers is identical (the first number is derived from the previous two, every number is added to them). The sole discrepancy is that (FN) begins from 0 to 1, whereas Lucas' numbers begin from 2 to 1, and show that connection between the (FS) and (GR), relation between (FS) and (GS).

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