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Exponential Acceleration To Improve Simpson's 3/8-rule

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Exponential acceleration to improve Simpson's 3/8-rule

<p>Authors Names a. Salah Mahdi Aflook b. Adil AL_Rammahi</p> <p>Article History Received on: 8/11/2020 Revised on: 7 /12/2020 Accepted on: 17/12/2020</p> <p>Keywords: Simpson's 3/8 Rule Newton Cotes AL-Tememe accelerations</p> <p>DOI: https://doi.org/10.29350/jops.2021.26.1.1240</p>	<p>ABSTRACT</p> <p>The main objective of this research is to find new accelerations of Newton Coates methods and in particular Simpson 3/8 and use it to find numerical results for continuous integration sooner and accuracy where the error rate is lower than its predecessors of roads and reach accurate results in easier ways and less fragmentation, as new exponential laws were derived for the Simpson 3/8 method and through these advanced accelerations we have obtained high accuracy in improving numerical integration results.</p>
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1. Introduction

Numerical analysis is one of the branches of mathematics characterized by finding approximate methods of great importance in applied sciences. These methods are based on their efficiency, accuracy, and use. Numerical integration is one of the topics of numerical analysis that involves finding approximate values of the definite integral. Numerical integration is also used to solve many physical problems such as finding the velocity, acceleration, volumes and areas. Numerical integration is needed in spite of the appearances of the error rate. There are many

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studies and developed issues of numerical integration. In 1967, L. Fox studied the way of Romberg which is the class of individual integrals which takes into account the conditions of the correction obtained and uses them together with the extrapolation of three quadratic formulas [2]. In 1970, the same scientist clarified some concepts with a study on the definite integral of a singular integral. He used the power of h by using the newton-cot's rules [1]. In 1972 Shanks JA, using Romberg's tables of single integration, demonstrated accelerated methods and he proved Fox's work with quadratic correction equations based on the foundations of the laws of error and produced the same result in shorter and easier ways [6]. In 2019, a scholars Ali Hassan and Asmahan Abed presented the new acceleration methods for methods of accelerating the base of the trapezoid and the midpoint to improve the values of integrals numerically, reduce the error rate, and reach better results and they called it Al-Tememe acceleration methods of the first type [3]. In 2019, Ali Hassan and Shatha H. Theyab introduced a new accelerated method based on Simpson 1/3 rule, which was helpful in reducing error rate and they called it Al-Tememe Acceleration Methods of the second type [4]. In this paper, new accelerated laws which were derived by Fox's laws and al-Tememe acceleration rules by using the exponential accelerations for Simpson's 3/8 rule. Our improved method aims to achieve correct results and reduce the error rate for nearest and lowest number of sections. In order to test the efficiency of our method, there was a comparison with the known Newton Coats methods Simpson's 3/8, Trapezoidal and Simpson's 1/3 rules.

2. The basic Concepts

There are digital ways to calculate individual integration processes in their integration breaks.

Newton's formulas are called coats.

This study will provide an upgraded method called "Expedition" for "Simpsons" 3/8 to find approximate values to integrate a continuous integrand by using

Relative acceleration methods and algorithms, which are the development of a series of Tmeme accelerations. in this part we introduce the Simpson 3/8 rule as follows

Let's assume that integration I is

$$I = \int_a^b f(x) dx \quad (1)$$

Where $f(x)$ is a continuous integrand located above the x -axis in the interval $[a, b]$, the approximate value of the integral is as follows

$$I = \int_a^b f(x)dx = G(h) + E_{G(h)} + R_G \quad (2)$$

Where $G(h)$ is (Lagrangian – Approximation) and G is the type of rule. $E_{G(h)}$ is the correction terms (error) that can be added to $G(h)$, and R_G is the remainder. The value of the Simpson 3/8 rule we will referred to by $S^*(h)$ is:

$$S^*(h) = \frac{3h}{8} [f(a) + 3f(a+h) + 3f(a+2h) + 2f(a+3h) + 3f(a+4h) + \dots + 2f(n-3h) + 3f(n-2h) + 3f(n-h) + f(b)]$$

Where

$h = \frac{b-a}{n}$. The general form for main error $E_{G(h)}$ is as follows

$$E_{G(h)} = \frac{3}{80} h^5 f^{iv}(\bar{x}) \text{ so we can write. } E_{G(h)} = \frac{3(b-a)}{80n} h^4 f^{iv}(\bar{x}); \quad \bar{x} \in (a, b), \quad [5]$$

Notice that the error form in Simpson's Rule 3/8 is very close to the error formula in Simpson's Rule 1/3, and according to Fox, they are both of the power of four

Fox [1] When the integrand is continuous and its derivatives continuous at each point of integrand during the interval $[a, b]$, the error formula in Simpson 3/8 rule can be written as follows:

$$E = I - S^*(h) \cong (A_1 h^4 + A_2 h^6 + A_3 h^8 + \dots) \quad (3)$$

Such that A_1, A_2, A_3, \dots they are constants whose value does not depend on the values of the exponents of h .

3. Exponential acceleration functions for the Simpson 3/8-rule

In this part we will introduce a new method to accelerate the values of Simpson 3/8 rule.

From (3) can be written

$$E = h^4(A_1 + A_2 h^2 + A_3 h^3 + \dots) \cong h^4 e^{h^2} \quad (4)$$

Depending on the

$$(e^{h^2} = (1 + \frac{1}{2}h^2 + \frac{1}{4}h^4 + \dots)) \quad \text{Maclurian series [7]} \quad (5)$$

Now we can write the error as follows:

$$E = I - S^*(h) \cong h^4 e^{h^2}$$

If we assume that we calculate two values numerically based on the Simpson 3/8 rule as follows:

$S^*(h_1)$ when $h = h_1$, $S^*(h_2)$, when $h = h_2$, so.

$$I - S^*(h_1) \cong h_1^4 e^{h_1^2} \quad (6)$$

$$I - S^*(h_2) \cong h_2^4 e^{h_2^2} \quad (7)$$

From the equations (6) and (7), we get:

$$A_{\beta_1}^{S^*} \cong \frac{(h_1^4 e^{h_1^2}) S^*(h_2) - (h_2^4 e^{h_2^2}) S^*(h_1)}{h_1^4 e^{h_1^2} - h_2^4 e^{h_2^2}} \quad ; h_1 \neq h_2 \quad (8)$$

We call the formula (8) the law of exponential acceleration, and we denote it by the symbol $A_{\beta_1}^{S^*}$. In the same way we can find the second exponential law.

where

$$E = (A_1 h^4 + A_2 h^6 + A_3 h^8 + \dots) = h^4 (A_1 + A_2 h^2 + A_3 h^4 + \dots) \cong h^4 e^{-h^2} \quad (9)$$

Depending on the

$$(e^{-h^2} = 1 - \frac{1}{2} h^2 + \frac{1}{4} h^4 - \dots) \quad \text{Maclurian series [7]}$$

Similarly, as in the first law of exponential acceleration above, we can get the following

$$I - S^*(h_1) \cong h_1^4 e^{-h_1^2} \quad (10)$$

$$I - S^*(h_2) \cong h_2^4 e^{-h_2^2} \quad (11)$$

From the equations (10) and (11), we get:

$$A_{\beta_2}^{S^*} \cong \frac{(h_1^4 e^{-h_1^2}) S^*(h_2) - (h_2^4 e^{-h_2^2}) S^*(h_1)}{h_1^4 e^{-h_1^2} - h_2^4 e^{-h_2^2}} \quad ; h_1 \neq h_2 \quad (12)$$

We call the formula (12) the second exponential acceleration law, and we denote it by $A_{\beta_2}^{S^*}$.

4.Example

We take some integrations that have continuous integrands and find their numerical values by using the Simpson 3/8 method and apply the new acceleration and after that we will compare the values of new accelerations with the Simpson 3/8 method to see how the results improve for this method. In the first two examples, the new accelerations will compared with the Simpson

3/8 method, and we will compare them with the rest of the methods as in the third example to prove their superiority in the speed of improving results and reducing the error rate.

1. $\int_1^2 \sqrt{x} dx$ Its analytical value 1.21895141649746, is rounded to 14 decimals.
2. $\int_1^2 \frac{1}{x+1} dx$ Its analytical value 0.40546510810816, is rounded to 14 decimals.
3. $\int_0^1 x^5 dx$ Its analytical value 0.16666666666667, is rounded to 14 decimals.

Table. (1) Integration Calculation $\int_1^2 \sqrt{x} dx = 1.21895141649746$, Numerically using The Sipson 3/8 rule with The exponential acceleration.

N	Simpson3/8	Error	$A^{S^*}_{\beta_1}$	Error	$A^{S^*}_{\beta_2}$	Error
3	1.21891231546478					
6	1.21894861362646	2.803E-06	1.21895082821659	5.883E-07	1.21895125912804	1.574E-07
9	1.21895084479094	5.717E-07	1.21895138355937	3.294E-08	1.21895140468520	1.181E-08
12	1.21895123341102	1.831E-07	1.21895141187271	4.625E-09	1.21895141471526	1.782E-09
15	1.21895134107229	7.543E-08	1.21895141544878	1.049E-09	1.21895141608134	4.161E-10
18	1.21895138000767	3.649E-08	1.21895141617891	3.186E-10	1.21895141636917	1.283E-10
21	1.21895139676306	1.973E-08	1.21895141637971	1.178E-10	1.21895141644964	4.782E-11
24	1.21895140491490	1.158E-08	1.21895141644732	5.014E-11	1.21895141647698	2.048E-11
27	1.21895140926024	7.237E-09	1.21895141647373	2.373E-11	1.21895141648774	9.720E-12
30	1.21895141174616	4.751E-09	1.21895141648526	1.220E-11	1.21895141649244	5.020E-12

Table. (2) Integration Calculation $\int_1^2 \frac{1}{x+1} dx = 0.40546510810816$, Numerically using The Simpson 3/8 rule with

N	Simpson3/8	Error	$A^{S^*}_{\beta_1}$	Error	$A^{S^*}_{\beta_2}$	Error
3	0.40550595238095					
6	0.40546790831717	2.800E-06	0.40546558720766	4.791E-07	0.40546513556984	2.746E-08
9	0.40546567213615	5.640E-07	0.40546513215636	2.405E-08	0.40546511098303	2.875E-09
12	0.40546528783542	1.797E-07	0.40546511135724	3.249E-09	0.40546510854629	4.381E-10
15	0.40546518196887	7.386E-08	0.40546510883224	7.241E-10	0.40546510821023	1.021E-10
18	0.40546514379235	3.568E-08	0.40546510832609	2.179E-10	0.40546510813955	3.139E-11
21	0.40546512739075	1.928E-08	0.40546510818830	8.014E-11	0.40546510811984	1.168E-11
24	0.40546511941934	1.131E-08	0.40546510814216	3.400E-11	0.40546510811316	4.998E-12
27	0.40546511517314	7.065E-09	0.40546510812422	1.606E-11	0.40546510811053	2.372E-12
30	0.40546511274512	4.637E-09	0.40546510811641	8.248E-12	0.40546510810939	1.227E-12

The exponential acceleration.

Table. (3) Integration Calculation $\int_0^1 x^5 dx = 0.16666666666667$, Numerically using The Trapezoidal , Simpson 1/3 and Simpson 3/8 rules with The exponential acceleration.

N	Trapezoidal	Simpson 1/3	Simpson3/8	$A^{S^*}_{\beta_1}$	$A^{S^*}_{\beta_2}$
3	0.21193415637860	0.17592592592593		
6	0.17817644032922	0.16692386831276	0.16724537037037	0.16671576027904	0.16661270958573
9	0.17179799827262	0.16678097850937	0.16666883990399	0.16666444279892
12	0.16955616640947	0.16668274176955	0.16670283564815	0.16666695096704	0.16666637939376
15	0.16851687242798	0.16668148148148	0.16666672921208	0.16666660374681
18	0.16795188106488	0.16666984199563	0.16667381115684	0.16666668536995	0.16666664789052
21	0.16761106054919	0.16667052308452	0.16666667351874	0.16666665979580
24	0.16738979512281	0.16666767136060	0.16666892722801	0.16666666956693	0.16666666376049
27	0.16723806907383	0.16666807792398	0.16666666803426	0.16666666529691
30	0.16712952674897	0.16666707818930	0.16666759259259	0.16666666736796	0.16666666596450

5. THE RESULTS

1. We notice from the first table that we got nine correct decimal places in acceleration $A^{S^*}_{\beta_2}$ when $n=15$ while we got eight correct decimal places only in Simpson's 3/8 rule at $n=30$ which means that the new acceleration is twice more accurate than Simpson's method.

2. We notice from the second table that the acceleration $A^{S^*}_{\beta_2}$ is faster in reaching accurate results than the acceleration $A^{S^*}_{\beta_1}$ and both got nine decimal places when $n=15$ while we got only seven correct decimal places in Simpson 3/8 rule at $n=30$. It assures us that the new laws of exponential acceleration give more accuracy than Simpson 3/8 rule.

3. We notice from the third table that the new accelerations got best results by compared them with the rest methods, where they got eight decimal places when $n=30$, while the Trapezoidal got two decimal places and Simpson's 1/3 & Simpson's 3/8 got five decimal places.

6. conclusion

From the above tables, the new exponential acceleration methods have roughly the same efficiency with little difference according to the given function and period.

The results of the new accelerations value reach in accuracy approximately twice the Simpson's 3/8 rule in each n .

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