A Comparison between Sinusoidal Oscilation and Constant Temperature Boundary Conditions in Annulus Filled with Porous Media Saturated with Nanofluid

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ABSTRACT

A numerical study has been carried out to investigate the heat transfer by natural convection of nanofluid taking copper as nano particles and the water as based fluid in a three dimensional annulus enclosure filled with porous media between two horizontal concentric cylinders with 12 annular fins of 3mm length and 2.4 mm thickness attached to the inner cylinder under steady state condition and different wall temperature boundary conditions. The governing equations which used are continuity, momentum and energy equations under an assumptions used Darcy law and Boussinesq's approximation which are transformed to dimensionless equations. The finite difference approach is used to obtain all the computational results using the MATLAB- 7. The parameters affected on the system are modified Rayleigh number ($10 \leq Ra^* \leq 500$), radius ratio Rr (0.293, 0.365 and 0.435) and the volume fraction $\left(0 \leq \varphi \leq 0.3\right)$. The results show that, increasing of fin length decreases the heat transfer rate and for $Ra^* < 100$, decreasing Rr cause to decrease Nu and adding Cu nanoparticles with $\varphi = 0.35$ cause 27.9% enhancement in heat transfer. A correlation for Nu in terms of Ra, H_f and φ, has been developed for inner hot cylinder.

Keywords: Three Dimensional Annulus, Laminar Flow, Natural Convection, Nanofluid, Porous Media.

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مقارنة بين درجة الحرارة الثابتة والمتغيرة جيبيا لظروف تخوم محتوى حلقي مملوء بوسط مسامي مشبع بمائع النانو

الخلاصة:

أجريت في هذا البحث دراسة عددية لإنتقال الطاقة الحرارية بالحمل الحر لمائع نانو باستخدام جُسيمات نانو النحاس Cu في الماء كمائع أساس في فجوة حلقية ثلاثية الأبعاد مملؤة بوسط مسامي بين أسطوانتين أفقيتين متحدتي المركز بوجود 12 زعنفة بطول 3mm وسمك 2.4 mm متصلة بالإسطوانة الداخلية تحت شروط حالة الإستقرار. المعادلات الحاكمة هي معادلة الاستمرارية والزخم والطاقة. كُتبت المعادلات تحت فرضيات قانون دارسي وتقريب بوسنسك وتحويلها إلى معادلة بدلالة الجهد المتجه والتي بدورها حُلت عدديا باستخدام طريقة الفروق المحددة ثم

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استخدام برنامج MATLAB- 7 لإيجاد النتائج الحسابية. العناصر الإستدلالية المؤثرة هي عدد رايلي المعدل (10 ≥ Ra^* ≥ 500) وزمن لابعدي ثابت 0.2 =′τ والسعة اللابعدية(0.8 ≥ a ≥ 0) وكسر الحجوم (0.3 ≥ φ ≥ 0) .

بينت النتائج تعزيز في معدل أنتقال الطاقة الحرارية للظروف الحديّة لدرجة الحرارة المتغيرة جيبيا وتعزيز إضافي عند إضافة جسيمات النانو. تزداد نسبة عدد نسلت (NUR) بزيادة Ra^* حتى تصل قيمة ثابتة عند Ra^*>100. مثلت النتائج بمعادلة تجريبية للأسطوانة الداخلية الساخنة تربط عدد نسلت بعدد رالي المعدّل كسر الحجوم والسعة اللابعدية.

INTRODUCTION

Coupled heat and mass transfer driven by buoyancy due to temperature and concentration variations in fluid-saturated porous media has been of growing interest during the last several decades because of its great practical applications in modern industry, such as the design of building components for energy consideration, control of pollutant spread in groundwater, compact heat exchangers, solar power collectors and food industries [1]. For the first time **Lage and Bejan, 1993 [2]** investigated the buoyancy driven flows in a square enclosure with periodic heat flux and examined the effects of oscillation frequency of heat generation on natural convection. Other researchers carried out similar studies by considering a clear base fluid within the enclosure as in [3] and [4]. **Choi, 1995[5]** showed that introducing nanofluids containing nanoparticles with substantially higher thermal conductivities improves the heat transfer performance. The results of this study have also been confirmed by other researchers [6–8]. **Mina et al., 2011[9]** Studied numerically laminar conjugate heat transfer by natural convection and conduction in a vertical annulus formed between an inner heat generating solid circular cylinder and an outer isothermal cylindrical boundary. It is assumed that the two sealed ends of the tube to be adiabatic. The governing equations have been solved using the finite volume approach, using SIMPLE algorithm on the collocated arrangement.

Ghasemi and Aminossadati, 2010 [10] Examined the periodic natural convection in an enclosure filled with nanofluids. Whilst a heat source with oscillating heat flux is located on the left wall of the enclosure, the right wall is maintained at a relatively low temperature and the other walls are thermally insulated. The utilization of nanoparticles, in particular Cu, enhances the heat transfer especially at low Rayleigh numbers. In addition, the oscillation period of heat generation affects the maximum operational temperature of the heat source.

Mostafa and Seyed, 2012 [11] investigated numerically Natural convection fluid flow and heat transfer inside C-shaped enclosures filled with Cu-Water nanofluid using finite volume method and SIMPLER algorithm. It was found from the obtained results that the mean Nusselt number increased with increase in Rayleigh number and volume fraction of Cu nanoparticles regardless aspect ratio of the enclosure. Moreover the obtained results showed that the rate of heat transfer increased with decreasing the aspect ratio of the cavity. Also it was found that the rate of heat transfer increased with increase in nanoparticles volume fraction.

Kuang and Angela, 2010[12] Analyzed the heat transfer and fluid flow of natural convection in a cavity filled with Al_2O_3/ water nanofluid that operates under differentially heated walls. The heat transfer rates are examined for parameters of

non-uniform nanoparticle size, mean nanoparticle diameter, nanoparticle volume fraction, Prandtl number, and Grashof number.

Heat transfer and fluid flow due to buoyancy forces in a partially heated enclosure using nanofluids is carried out by **Hakan, Oztop and Abu-Nada, 2008 [13]** using different types of nanoparticles. The flush mounted heater is located to the left vertical wall with a finite length. The temperature of the right vertical wall is lower than that of heater while other walls are insulated. The finite volume technique is used to solve the governing equations.

Mohammadreza et al., 2011[14] Investigated the heat transfer performance in an enclosure including nanofluids with a localized heat source. The hydrodynamics and thermal fields are coupled together using the Boussinesq's approximation.

In the present study, the heat transfer by natural convection of nanofluid taking copper as nano particles and the water as based fluid in a three dimensional annulus enclosure filled with porous media between two horizontal concentric cylinders under steady state condition and for thermal boundary condition of two cases constant or sinusoidal temperature oscillation at the wall of the inner cylinder and for modified Rayleigh number ($10 \leq Ra^* \leq 500$), radius ratio Rr (0.293, 0.365 and 0.435) and the volume fraction $\left(0 \leq \varphi \leq 0.3\right)$.

Mathematical Model

The effective thermal conductivity of the nano-fluid is approximated by Maxwell-Garnetts model:

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\varphi\left(k_f - k_s\right)}{k_s + 2k_f + \varphi\left(k_f - k_s\right)} \qquad \ldots(1)$$

The use of this equation is restricted to spherical nano-particles where it does not account for other shapes of nano-particles. This model is found to be appropriate for studying heat transfer enhancement using nanofluid **[9, 15]**.

The thermo-physical properties of pure water and nanoparticles are given in **Table (1) [10].**

Table (1) Thermo-physical properties of pure fluid and nanoparticles

physical properties	Pure water	Cu
$P(kg/m^3)$	997.1	8933
Cp(J/kg K)	4179	385
K(W/m K)	0.613	400
B(1/K)	21×10^{-5}	1.67×10^{-5}

The viscosity of the nanofluid can be approximated as viscosity of a base fluid μ_f containing dilute suspension of fine spherical particles and is given by **[16]**:

$$\mu_{nf} = \frac{\mu_f}{\left(1-\varphi\right)^{2.5}} \qquad \qquad \dots(2)$$

The governing equations used are continuity, momentum and energy equations which are transformed to dimensionless equations and the vector potential equation was obtained in the dimensionless form as [17]and [18]:

$$\frac{\partial U_r}{\partial R} + \frac{U_r}{R} + \frac{1}{R}\frac{\partial U_\phi}{\partial \phi} + \frac{\partial U_z}{\partial Z} = 0 \qquad \qquad \dots (3)$$

$$Ra^* \Pr \ast C_1 \ast \left(\sin\phi \ \frac{\partial \theta}{\partial Z} \right) = \begin{array}{c} -\dfrac{\partial^2 \psi_r}{\partial R^2} - \dfrac{1}{R^2}\dfrac{\partial (R\psi_r)}{\partial R} - \dfrac{2}{R}\dfrac{\partial \psi_r}{\partial R} - \dfrac{1}{R^2}\dfrac{\partial^2 \psi_r}{\partial \phi^2} \\[4mm] -\dfrac{\partial^2 \psi_r}{\partial Z^2} - \dfrac{2}{R}\dfrac{\partial \psi_z}{\partial Z} \end{array}$$

$$\dots (4)$$

$$Ra^* \Pr \ C_1 \ast \left(\cos\phi \ \frac{\partial \theta}{\partial Z} \right) = \begin{array}{c} -\dfrac{\partial^2 \psi_\phi}{\partial Z^2} - \dfrac{\partial^2 \psi_\phi}{\partial R^2} - \dfrac{1}{R^2}\dfrac{\partial^2 \psi_\phi}{\partial \phi^2} \\[4mm] -\dfrac{2}{R^2}\dfrac{\partial \psi_r}{\partial \phi} + \dfrac{\psi_\phi}{R^2} - \dfrac{1}{R}\dfrac{\partial \psi_\phi}{\partial R} \end{array}$$

$$\dots (5)$$

$$-\frac{Ra^* \Pr \ C_1}{\alpha_{nf}} \left(\frac{1}{R}\cos\phi\frac{\partial \theta}{\partial \phi} + \sin\phi\frac{\partial \theta}{\partial R} \right) = \begin{array}{c} -\dfrac{\partial^2 \psi_z}{\partial R^2} - \dfrac{1}{R}\dfrac{\partial \psi_z}{\partial R} \\[4mm] -\dfrac{1}{R^2}\dfrac{\partial^2 \psi_z}{\partial \phi^2} - \dfrac{\partial^2 \psi_z}{\partial Z^2} \end{array}$$

$$\dots(6)$$

Where

$$C_1 = \frac{\alpha_f}{\alpha_{nf}} \left[\left(1-\varphi\right) + \varphi\frac{(\rho\beta)_s}{(\rho\beta)_f} \right] \left(1-\varphi\right)^{2.5} \qquad \qquad \dots(7)$$

And the energy equation will be:

$$\left(\frac{1}{R}\frac{\partial \psi_z}{\partial \phi}-\frac{\partial \psi_\phi}{\partial Z}\right)\frac{\partial \theta}{\partial R}+\frac{1}{R}\left(\frac{\partial \psi_r}{\partial Z}-\frac{\partial \psi_z}{\partial R}\right)\frac{\partial \theta}{\partial \phi}$$

$$+\left(\frac{\psi_\phi}{R}+\frac{\partial \psi_\phi}{\partial R}-\frac{1}{R}\frac{\partial \psi_r}{\partial \phi}\right)\frac{\partial \theta}{\partial Z} \qquad \ldots(8)$$

$$=\frac{\alpha_{nf}}{r_2}[\frac{\partial^2 \theta}{\partial R^2}+\frac{1}{R}\frac{\partial \theta}{\partial R}+\frac{1}{R^2}\frac{\partial^2 \theta}{\partial \phi^2}+\frac{\partial^2 \theta}{\partial Z^2}]$$

And fin equation will be [12]:

$$\frac{\partial \theta}{\partial R}+\frac{\theta}{R}+\frac{1}{R}\frac{\partial \theta}{\partial \phi}+\frac{\partial \theta}{\partial Z}=0 \qquad \ldots(9)$$

For the vector potential field, the boundary conditions are given in figure (1).

a

b
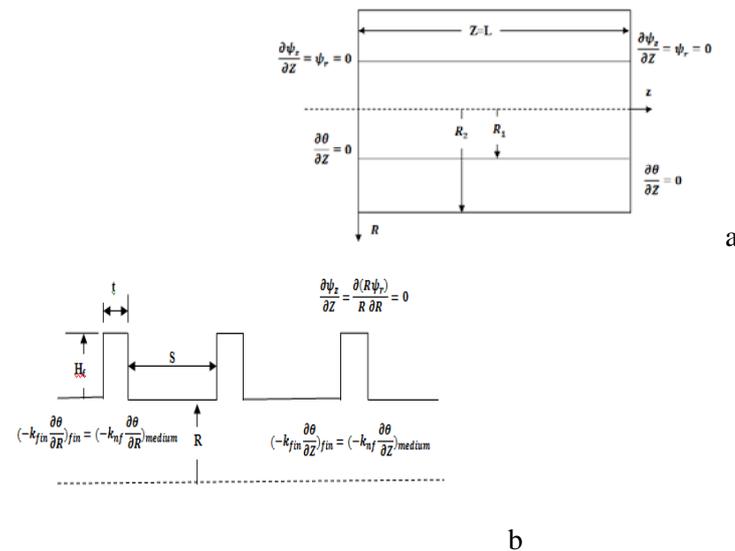
**Figure (1) a. Annulus boundary conditions**

b. Fins boundary conditions

Computational Technique

The equations were transformed into the finite difference approximation, where the upwind differential method in the left hand side of the energy equation and the centered – space differential method for the other terms were used, and solved by using (SOR) method [10].

A computer program was built using MATLAB to meet the requirements of the problem. The value of the vector potential ψ calculated at each node, in which the value of vector potential is unknown, the other node will appear in the right hand side of each equation.
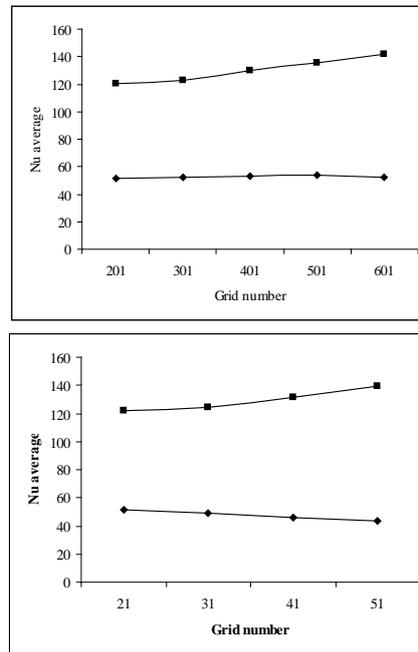
a
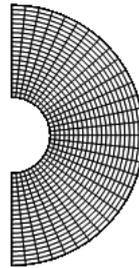
b

Figure (2) a. Grid Size Study for point at the inner hot cylinder,　　b. grid generation in the R-∅ section

Figure (2) illustrates the influence of number of grid points for a test case of Ra^{*} =500, a=0.8, η=0.01and φ =0.3. The figure shows the average Nusselt number for point located on the inner hot cylinder. The number of grid points used was 21 grid points in the R – direction, 31 in the ϕ – direction and 301 in the Z – direction.

Dimensionless Hydraulic Boundary Conditions:

$$\frac{1}{R}\frac{\partial}{\partial R}(R\psi) = 0 \qquad at \ R = R_{1},1$$

$$\frac{\partial \psi}{\partial Z} = 0 \qquad at \ Z = 0, \ L$$

And for the fin, the boundary conditions are given as:

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R \ \psi_r\right) = \frac{\partial \ \psi_Z}{\partial \ Z} = 0$$

On the fin faces which were located on the following planes:
At $R = R_1$ for (fin base)
At $r = r_1 + H_f$ for (fin tip)

Dimensionless Thermal Boundary Conditions:

$$\theta = 0 \qquad at \ R = R_2 = 1$$

Case 1: constant wall temperature

$$\theta = 1 \qquad at \ R = R_1 = r_{in} / r_{out}$$

Case 2:

$$\theta = 1 + a \ \sin \left(2\pi \ \tau' / \eta'\right) \qquad at \ R = R_1 = r_{in} / r_{out}$$

$$\frac{\partial \theta}{\partial Z} = 0 \qquad at \ Z = 0, \ L$$

At $R = H_1$

$$-k_{fin}\frac{\partial \theta}{\partial R}\bigg|_{fin} = -k_{nf}\frac{\partial \theta}{\partial R}\bigg|_{nf}$$

At S_1 for any R and S_2 for any R

$$-k_{fin}\frac{\partial \theta}{\partial Z}\bigg|_{fin} = -k_{nf}\frac{\partial \theta}{\partial Z}\bigg|_{nf}$$

Calculation of The Local and Average Nusselt Number

The local Nusselt number Nu_1 and Nu_2 on the inner and the outer cylinders are written in the form**:**

$$Nu_1 = -\left(1 - R_1\right) \frac{k_{nf}}{k_f} \left(\frac{\partial \theta}{\partial R}\right)_{R=R_1} \qquad \qquad …(11)$$

$$Nu_2 = -\left(1 - R_1\right) \frac{k_{nf}}{k_f} \left(\frac{\partial \theta}{\partial R}\right)_{R=1_1} \qquad \qquad …(12)$$

The average Nusselt number Nu_{in} and Nu_{out} on the inner and the outer cylinders are defined as:

$$Nu_{in} = -(1-R_1)\frac{1}{\pi L}\int_0^L \int_0^\pi \left(\frac{\partial \theta}{\partial R}\right)_{R=R} d\phi\ dZ$$

$$\dots (13)$$

$$Nu_{out} = -(1-R_1)\frac{1}{\pi L}\int_0^L \int_0^\pi \left(\frac{\partial \theta}{\partial R}\right)_{R=1} d\phi\ dZ$$

$$\dots(14)$$

The effect of the nanofluids on heat transfer rate introduced as a variable called Nusselt number ratio (NUR) with its definition given as:

$$NUR = \frac{Nu_{out}|_{nanofluid}}{Nu_{out}|_{pure\ fluid}} \qquad \dots(15)$$

A correlation for Nu in terms of Ra, φ and dimensionless amplitude (a) has been developed for inner hot cylinder as follow:
$Nu = 13.07\ (Ra^*)^{0.4304}\ \varphi^{0.533}\ a^{0.4003}$

RESULTS AND DISCUSSION
Temperature, streamlines and velocity Fields
The dimensionless temperature distribution and the axial velocity within the enclosure is presented in a contour map form. For isotherms, one section was selected in the (Z-R) plane along the length of the annulus, and the other in the (R-ϕ) plane, in a manner allowed studying the temperature distribution and streamlines within each plane. For comparison purposes, parameters are selected of constant wall temperature, $Ra^*=500$ and $\varphi=0$ in Fig.(3)and $\varphi=0.3$ in Fig. (4). Fig.(3) shows the temperature distribution for pure fluid where the isotherms shift towards the outer (cold) cylinder and a thicker cold layer in the lower region of the annulus wall exist while a high temperature exist in the upper half of the annulus where the waviness in temperature distribution is due to the existence of the fins**.** Fig. (4) illustrates the enhancement in heat transfer where the temperature decrease clearly in the regions between fins. For the streamlines it can be seen from figures that one vortices occur which indicates that the heated flow moves up from the inner heated cylinder of the enclosure and impinges to the cold outer cylinder. The difference between nanofluid and pure fluid cases are the higher intensity of streamlines in nanofluid. The velocity field shows a clear decrease in the axial velocity due to the enhancement in heat transfer caused the fluid to be cooled and as a result the velocity decreased.

For sinusoidal oscillation in the inner cylinder wall temperature, a selected case is taken for dimensionless amplitude a= 0.8, $\eta=0.01$, $Ra^*=500$, $\varphi=0$ in Fig.(5)and $\varphi=0.3$ in Fig. (6). Fig.(5) shows the temperature distribution for pure fluid and that the increase in amplitude cause to enhance heat transfer so the values of isotherms decrease and a decrease in the axial velocity and

streamlines intensity observed. In Fig. (6) as φ increase to 0.3 an extra enhancement in heat transfer obtained and the enclosure cooled with more decrease in the velocity and streamlines intensity.

Figures (7) and (8) illustrate the variation of Local Nusselt number along the length of the inner cylinder. It is clear from these figures, as mentioned previously that heat transfer enhances by sinusoidal oscillation boundary condition and adding nano particles to the pure fluid causes extra enhancement in heat transfer.

The variation of the average Nu with the modified Rayleigh number for various volume fractions (φ) and with various wall temperatures boundary condition is illustrated in Fig. (9). This figure shows that for $Ra^*=10$, as the volume fraction increase from zero to 0.3 the enhancement percent in heat transfer is 1.327% for constant wall temperature and 1.42% for sinusoidal oscillation boundary condition for amplitude equal 0.8.

Figure (10) shows the variation of the average Nu with the modified Rayleigh number for various dimensionless amplitudes in pure fluid and in nanofluid with volume fraction φ = 0.3. For constant wall temperature with $Ra^*=10$, as the dimensionless amplitude increase from zero to 0.8 the enhancement percent in heat transfer is 0.78 % where in nanofluid with φ=0.3 the enhancement percent is 0.857 %.

The variation of the average Nusselt number with the dimensionless period ή for dimensionless amplitude a=0.8 and $Ra^*=300$ is shown in Fig. (11); where it is clear that Nu decrease with the increase of the dimensionless period ή.

Fig. (12) shows that the Nusselt number ratio (NUR) increase with the modified Rayleigh number increase until Ra^* equal about 100 and then the values (for various volume fractions (φ) and a=0.8) reach a steady state. In Fig. (13) the variation of the Nusselt number ratio with the dimensionless amplitude (a) for various volume fractions (φ) and $Ra^*=500$ is illustrated and it is clear that NUR is nearly constant with the dimensionless amplitude and increase when adding more copper particles.

CONCLUSION

From the present work results and for the annulus that described previously, the following conclusions can be obtained:

1- The increase in the dimensionless amplitude cause to enhance heat transfer which causes the values of the isotherms to decrease and as consequent a decrease in the axial velocity values and the streamlines intensity observed.

2- Adding copper particles to the pure fluid cause an extra enhancement in heat transfer and the enclosure cooled with more decrease in the velocity and streamlines intensity.

3- For $Ra^*=10$, as the volume fraction increase from zero to 0.3 the enhancement percent in heat transfer is 1.327% for constant wall temperature and 1.42% for sinusoidal oscillation boundary condition.

4- For constant wall temperature with $Ra^*=10$, as the dimensionless amplitude increase from zero to 0.8 the enhancement percent in heat transfer is 0.78 % where in nanofluid the enhancement percent is 0.857 %.

5- As Ra^* increase, the Nusselt number ratio (NUR) increase until Ra^* equal about 100 where it reaches a constant value.

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Nomenclature
Latin Symbols

Symbol	Description	Units
a	Dimensionless amplitude(a= amplitude/ temperature difference of cylinders wall)	__
C_p	Specific heat at constant pressure	kJ/kg°C
Da	Darcy number $Da=K/r^2$	
g	Acceleration due to gravity	m/s^2
H_f	Fin length	m
H_1	Dimensionless fin length (H_f/r_{out})	__
$k_{fin.}$	Fin thermal conductivity	W/m K
k_f	Thermal conductivity of the fluid	W/m K

k_s	Thermal conductivity of the solid	W/m K
$k_{nf.}$	Thermal conductivity of nanofluid	W/m K
K	Permeability	m²
L	Dimensionless length of the cylinder	___
r_{in}	Radius of the inner cylinder	m
r_{out}	Radius of the outer cylinder	m
R	Dimensionless radial coordinate	
R_1	Dimensionless radius for inner cylinder	m
R_2	Dimensionless radius for outer cylinder	m
Ra^*	Modified Rayleigh number $Ra^*=Ra\,Da$	__
Ra	Rayleigh number	
s	Fin spacing (fin pitch)	m
S_1	Dimensionless fin spacing $S_1 = \dfrac{s}{2\,r_{out}}$	__
S_2	Dimensionless fin spacing $S_2 = \dfrac{\frac{s}{2}+t}{r_{out}}$	__
U_r	Dimensionless velocity component in R -direction	__
$U_\emptyset$	Dimensionless	

	velocity component in Ø-direction	—
Uz	Dimensionless velocity component in Z - direction	—
Z	Dimensionless axial coordinate	—

Greek Letters

Symbol	Description	Units
α_f	Thermal diffusivity of fluid	m^2/s
α_{nf}	thermal diffusivity of nanofluid	m^2/s
β	Volumetric thermal expansion coefficient	$1/K$
θ	Dimensionless temperature	—
μ_f	Dynamic viscosity of fluid	$N.s/m^2$
μ_{nf}	Dynamic viscosity of nanofluid	$N.s/m^2$
ψ_r	Streamline in radial direction	—
$\psi_{\emptyset}$	Streamline in angular direction	—
ψ_z	Streamline in axial direction	—
$\acute{\tau}$	Dimensionless time $\acute{\tau}=$time xα_f/R^2	—
$\acute{\eta}$	Dimensionless period $\acute{\eta}=$period xα_f/R^2	_

Subscripts

Symbol	Description	Units
$()_f$	Fluid	--
$()_s$	Solid	--
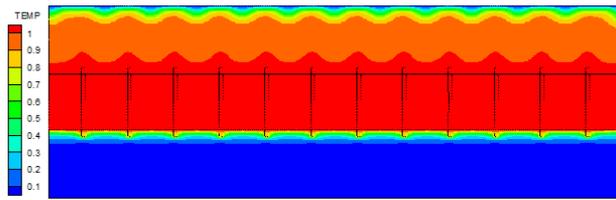
a. Isotherms along annulus length   b. isotherms (left), streamlines (right)
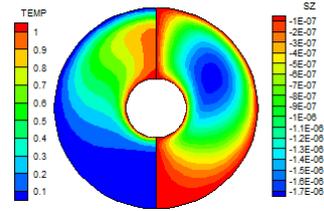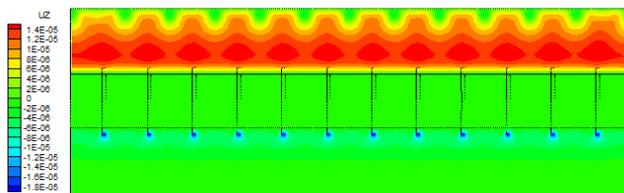
c. Axial velocity along annulus length

Figure. (3) Ra=500, a=0 and φ=0.
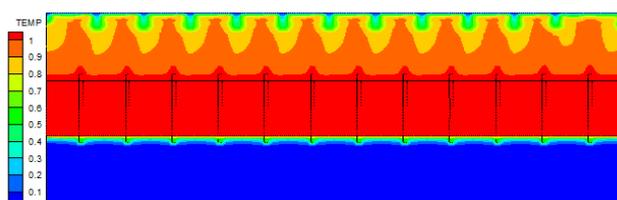
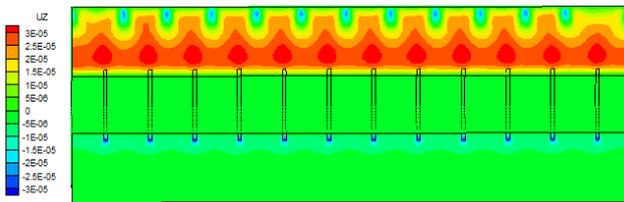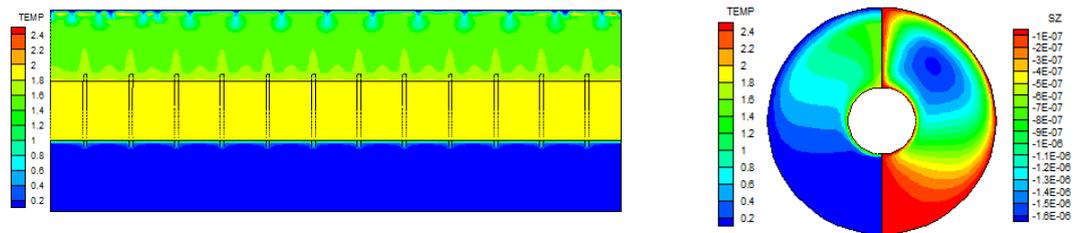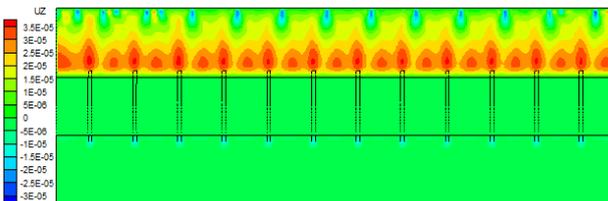a. Isotherms along annulus length   b. isotherms (left), streamlines (right)
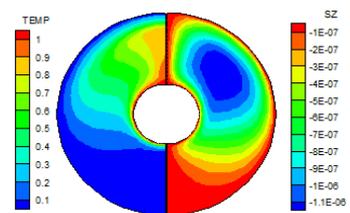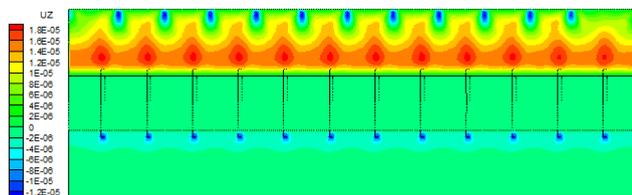
c. Axial velocity along annulus length

Figure. (4) Ra=500, a=0 and φ=0. 3.
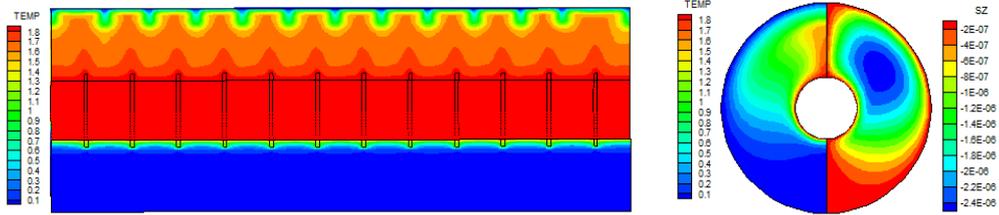
a. Isotherms along annulus length   b. isotherms (left), streamlines (right)

c. Axial velocity along annulus length

Figure. (5) Ra=500, a=0.8, η=0.01 and φ=0

a. Isotherms along annulus length   b. isotherms (left), streamlines (right)

c. Axial velocity along annulus length

Figure. (6) Ra=500, a=0.8, η=0.01 and φ=0. 3

(a) φ = 0   (b) φ = 0.3
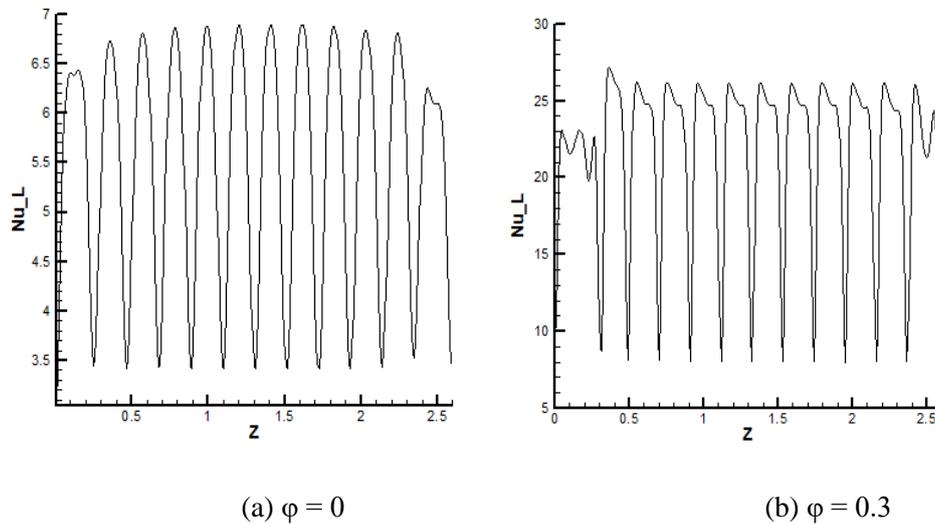
Figure (7) The variation of Local Nusselt number along the length of the inner cylinder for Ra*=500 and for constant wall temperature boundary condition (a=0) .

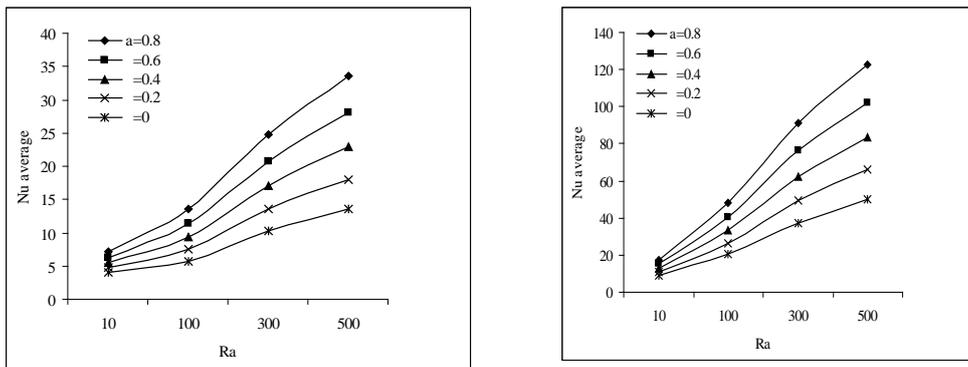

(a) φ = 0   (b) φ = 0.3
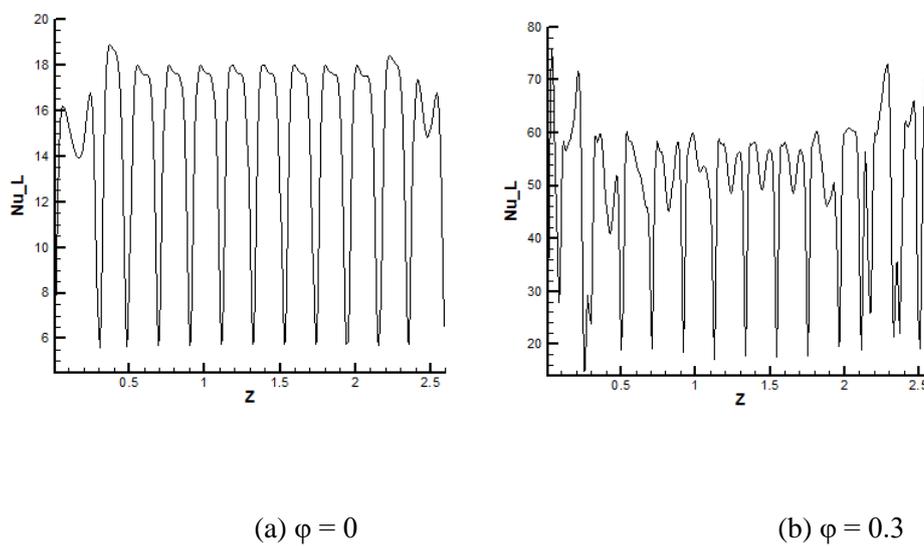
Figure. (8) The variation of Local Nusselt number along the length of the inner cylinder for Ra*=500 and sinusoidal oscillation temperature boundary condition (a=0.8) .
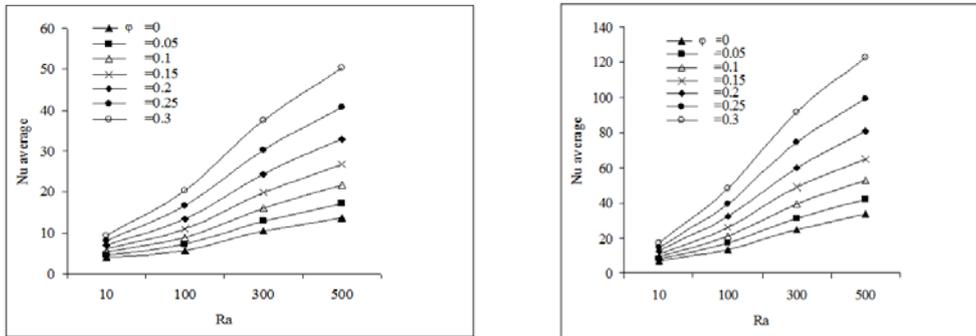
(a) Dimensionless amplitude (a) =0   (b) Dimensionless amplitude (a) =0.8
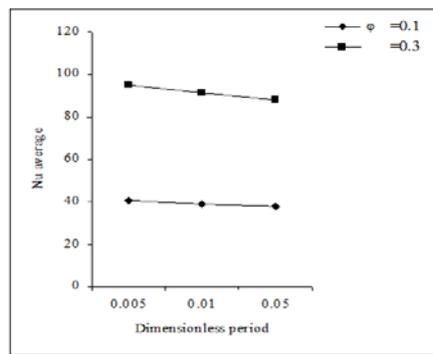
Figure. (9) The variation of the average Nu with the modified Rayleigh number for various volume fractions (φ)

(a) φ =0   (b) φ =0.3

Figure. (10) The variation of the average Nu with the modified Rayleigh number for various dimensionless amplitudes (a)

Figure. (11) The variation of the average Nusselt number with the dimensionless period $\acute{\eta}$ for dimensionless amplitude a=0.8 and Ra*=300.
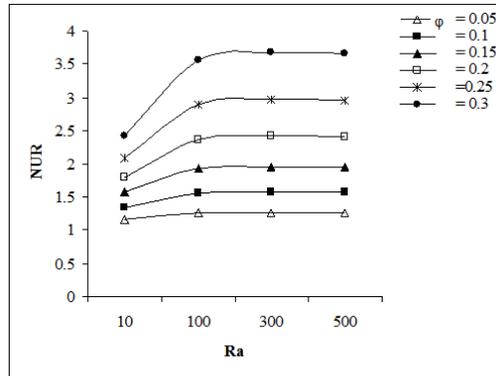
**Figure. (12) The variation of the Nusselt number ratio with the modified
Rayleigh number for various volume fractions (φ) and a=0.8**

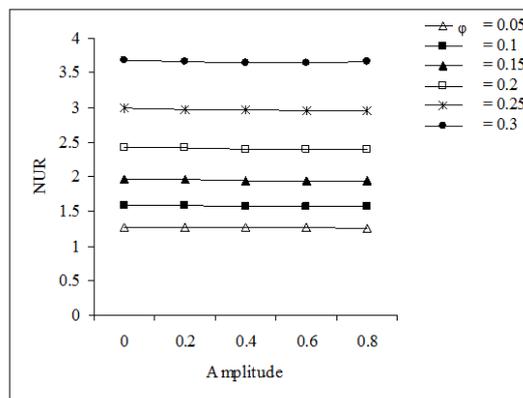**Figure. (13) The variation of the Nusselt number ratio with the dimensionless
amplitude (a) for various volume fractions (φ) and Ra*=500**