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Semi-Analytical Method for Solving Coupled Nonlinear Partial Differential Equations Using Hybrid Iterative Method

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Abstract

This article aims to propose a new efficient hybrid method for solving different types of nonlinear differential equations. The method combines mixed Shehu and Sumudu integral transforms with the variational iteration method. The proposed method is termed the multiple transform iteration method to solve different nonlinear partial differential equations, by engaging a new time and frequency domain. The outcomes that arise from this method show the efficiency, accuracy, and simplicity of applying the approach.

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1. Introduction (use 10 point Century font)

Nonlinear differential equations (NLDEs) are mathematical models used to describe significant phenomena observed in many science and engineering fields such as bioengineering, fluid dynamics, quantum mechanics, plasma physics, and signal processing. Thus, many researchers focused on finding a suitable efficient method for solving such equations, numerically or analytically. The Adomain decomposition method (ADM) [1], Homotopy perturbation method (HPM) [2], Variational iteration method (VIM) [3], Homotopy analysis method (HAM) [4], and Differential Transform Method (DTM) are some of such methods. Current studies combine the previous iterative methods with some integral transforms [5-21], to solve nonlinear partial differential equations (NLPDE) and integral equations [22,23]. In this paper, we suggest multiple integral transforms, the Shehu-Sumudu transform (SST) mixed with the VIM to solve the coupled nonlinear Burger's equations, in the form [24].

$$\begin{cases} g_t - \alpha_1 g_{ȿȿ} + \beta_{1,1} g_{ȿ} + \beta_{1,2}(g y)_{ȿ} \\ \quad = 0, t \in (0, Ţ], ȿ \in [ȿ_1, ȿ_2] \\ y_t - \alpha_2 y_{ȿȿ} + \beta_{2,2} y_{ȿ} + \beta_{2,1}(g y)_{ȿ} \\ \quad = 0, t \in (0, Ţ], ȿ \in [ȿ_1, ȿ_2] \end{cases} \quad \dots (1)$$

with the initial conditions

$$\begin{cases} g(ȿ, 0) = \omega_1(ȿ), & ȿ \in [ȿ_1, ȿ_2] \\ y(ȿ, 0) = \omega_2(ȿ), & ȿ \in [ȿ_1, ȿ_2] \end{cases}$$

where α_1, α_2 are nonzero positive viscosity parameters and $\beta_{1,1}, \beta_{1,2}, \beta_{2,2}, \beta_{2,1}$ are constants depending on the increase of the velocity. Besides that, we solve the coupled nonlinear Schrödinger-Kortweg-de Veries (SKdV) equation in the form [25].

$$\begin{cases} i\, g_t + g_{ȿȿ} + |g|^2 g + \gamma\, g ʄ = 0, \\ ʄ_t + ʄ_{ȿȿȿ} + ʄ\, ʄ_{ȿ} + \frac{1}{2} \gamma\, (|g|^2)_{ȿ} = 0, \end{cases} \quad \dots (2)$$

where $g = g(ȿ, t) \in ₵$, $ʄ = ʄ(ȿ, t) \in \mathbb{R}$, and $\gamma \in \mathbb{R}$ is the real coupling coefficient, the System in Eq. (2) arrived at the appearance of interactions between the short wave, defined by $g(ȿ, t)$, and the long dispersive wave, defined by $ʄ(ȿ, t)$. The primary goal of this research is to develop an efficient method termed (MTIM) for solving nonlinear PDEs without difficulty and obtain highly accurate solutions with the least amount of calculations.

2. Basic Definitions and Theorems

In this Section, we introduce the basic concepts that our new approach based on:

Definition 1 [26]: The Sumudu Transform is:

$$\mathbf{D}_t[\mathbf{z}(t)] = \mathbf{Z}(\text{v}) = \frac{1}{\text{v}}\int_0^{\infty} \exp\left(\frac{-t}{\text{v}}\right) \mathbf{z}(t)\, dt\,, t \geq 0$$

Definition 2 [27]: The Shehu Transform is:

$$\mathbf{U}_{ş}\,[\mathbf{w}(ş)] = \mathbf{W}(ą, ƀ) = \int_0^{\infty} \exp\left(-\frac{ą}{ƀ}\,ş\right) \mathbf{w}(ş) dş, ş \geq 0$$

Definition 3: Given a piecewise continuous function g of two independent variables $ş$ and t, the multiple Shehu-Sumudu transform (MSST) is defined by:

$$\mathbf{U}_{ş}\,\mathbf{D}_t[g(ş,t)] = \mathbf{G}((ą,ƀ),\text{v})$$
$$= \frac{1}{\text{v}} \iint_0^{\infty} \exp\left(-\left(\frac{ą}{ƀ}\,ş + \frac{t}{\text{v}}\right)\right) g(ş,t) dt dş\,; \quad ş, t \geq 0$$
$$= \lim_{\rho,\sigma\to\infty} \frac{1}{\text{v}} \int_0^{\rho}\int_0^{\sigma} \exp\left(-\left(\frac{ą}{ƀ}\,ş + \frac{t}{\text{v}}\right)\right) g(ş,t) dt dş.$$

The inverse of MSST is given by:

$$\mathbf{U}_{ş}\,\mathbf{D}_t^{-1}[\mathbf{G}((ą,ƀ),\text{v})] =$$
$$\frac{1}{2\pi i}\int_{\rho-i\infty}^{\rho+i\infty} \frac{1}{ƀ} \exp\left(\frac{ą}{ƀ}\,ş\right) dą . \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} \exp\left(\frac{t}{\text{v}}\right) \mathbf{G}((ą,ƀ),\text{v})\, d\text{v}$$
$$= g(ş,t)$$

where the function $g(ş,t)$ is of exponential order of k, c > 0 on $0 \leq ş, t < \infty$, and belongs to the set

$$Ꝑ = \Big\{ f(ş,t) \,|\, \exists\, Ϻ > 0, \text{s.t} \,|f(ş,t)| \leq Ꝁ * \exp(kş + ct)\,,$$
$$\forall ş, t \geq 0 \text{ where } Ϻ \text{ is constant and } \lim_{ş,t\to\infty} \exp\left(-\left(\frac{ą}{ƀ}\,ş + \frac{t}{\text{v}}\right)\right) |f(ş,t)| = 0 \Big\}$$

Theorem 1 (Existence of the MSST)

Let $g(ş,t)$ be a piecewise continuous function in $0 \leq ş \leq \rho$ and $0 \leq t \leq \sigma$, and of exponential order $qş + pt$, then the MSST of $g(ş,t)$ exists for all ρ and σ as long as $Re[\rho] > q$ and $Re[\sigma] > \frac{1}{p}$.

Proof: Applying definition 3 for positive ρ and σ,

$$\mathbf{U}_{ş}\,\mathbf{D}_t[g(ş,t)] = \frac{1}{\text{v}} \iint_0^{\infty} \exp\left(-\left(\frac{ą}{ƀ}\,ş + \frac{t}{\text{v}}\right)\right) g(ş,t) dt dş$$
$$= \frac{1}{\text{v}} \left\{ \underbrace{\int_0^{\rho}\int_0^{\sigma} \exp\left(-\left(\frac{ą}{ƀ}\,ş + \frac{t}{\text{v}}\right)\right) g(ş,t) dt dş}_{\textit{Exists since the fun. is piecewise cont.}} + \underbrace{\int_{\rho}^{\infty}\int_{\sigma}^{\infty} \exp\left(-\left(\frac{ą}{ƀ}\,ş + \frac{t}{\text{v}}\right)\right) g(ş,t) dt dş}_{\textit{Solve it by "integration by parts"}} \right\}$$

For the second integral part, we have:

$$\left|\int_{\rho}^{\infty}\int_{\sigma}^{\infty} \exp\left(-\left(\frac{ą}{ƀ}\,ş + \frac{t}{\text{v}}\right)\right) g(ş,t) dt dş\right| \leq$$
$$\int_{\rho}^{\infty}\int_{\sigma}^{\infty} \exp\left(-\left(\frac{ą}{ƀ}\,ş + \frac{t}{\text{v}}\right)\right) |g(ş,t)|\, dt dş$$
$$\leq ₦ \int_{\rho}^{\infty}\int_{\sigma}^{\infty} \exp\left(-\left(\frac{ą}{ƀ}\,ş + \frac{t}{\text{v}}\right)\right) \exp(qş + pt)\, dt dş$$
$$= ₦ \lim_{ş,t\to\infty} \left[\exp\left(\left(q - \frac{ą}{ƀ}\right)ş + \left(p - \frac{1}{\text{v}}\right) t\right)\right]_{\rho,\sigma}^{\infty}$$
$$= \frac{₦}{\left(q - \frac{ą}{ƀ}\right)\left(p - \frac{1}{\text{v}}\right)} \blacksquare$$

Theorem 2 (Linearity of the MSST)

If $g(ş,t)$, and $h(ş,t)$ are two functions in the set Ꝑ, $\vartheta\, g(ş,t) + \mu\, h(ş,t) \in$ Ꝑ, where ϑ and μ are nonzero constants, then:

$$\mathbf{U}_{ş}\,\mathbf{D}_t[\vartheta\, g(ş,t) + \mu\, h(ş,t)]$$
$$= \vartheta\, \mathbf{U}_{ş}\,\mathrm{D}_t[g(ş,t)] + \mu\, \mathbf{U}_{ş}\,\mathrm{D}_t[h(ş,t)]$$

Proof: Applying definition 3, we get

$$\mathbf{U}_{ş}\,\mathbf{D}_t[\vartheta\, g(ş,t) + \mu\, h(ş,t)] = \frac{1}{\text{v}} \iint_0^{\infty} \exp\left(-\left(\frac{ą}{ƀ}\,ş + \frac{t}{\text{v}}\right)\right) \{\vartheta\, g(ş,t) + \mu\, h(ş,t)\}\, dt dş$$
$$= \vartheta . \frac{1}{\text{v}} \iint_0^{\infty} \exp\left(-\left(\frac{ą}{ƀ}\,ş + \frac{t}{\text{v}}\right)\right) g(ş,t) dt dş +$$
$$\mu . \frac{1}{\text{v}} \iint_0^{\infty} \exp\left(-\left(\frac{ą}{ƀ}\,ş + \frac{t}{\text{v}}\right)\right) h(ş,t) dt dş$$
$$= \vartheta\, \mathbf{G}((ą,ƀ),\text{v}) + \mu\, \mathbf{H}((ą,ƀ),\text{v})$$
$$= \vartheta\, \mathbf{U}_{ş}\,\mathbf{D}_t[g(ş,t)] + \mu\, \mathbf{U}_{ş}\,\mathbf{D}_t[h(ş,t)] \blacksquare$$

Theorem 3 (MSST for the Basic Derivatives)

Let $\mathbf{U}_{ş}\,\mathbf{D}_t[g(ş,t)] = \mathbf{G}((ą,ƀ),\text{v})$, then:

I. $\mathbf{U}_{ş}\,\mathbf{D}_t\left[\frac{\partial}{\partial t}\, g(ş,t)\right] = \frac{1}{\text{v}}\, \mathbf{G}((ą,ƀ),\text{v}) - \frac{1}{\text{v}}\, \mathbf{U}_{ş}\,[g(ş,0)]$

II. $\mathbf{U}_{ş}\,\mathbf{D}_t\left[\frac{\partial^2}{\partial t^2}\, g(ş,t)\right] = \frac{1}{\text{v}^2}\, \mathbf{G}((ą,ƀ),\text{v}) - \frac{1}{\text{v}}\mathbf{U}_{ş}\,[g(ş,0)] - \mathbf{U}_{ş}\,[g_t(ş,0)]$

III. $\mathbf{U}_{ş}\,\mathbf{D}_t\left[\frac{\partial}{\partial ş}\, g(ş,t)\right] = \frac{ƀ}{ą}\, \mathbf{G}((ą,ƀ),\text{v}) - \frac{ƀ}{ą}\, \mathbf{D}_t[g(0,t)]$

IV. $\mathbf{U}_{ş}\,\mathbf{D}_t\left[\frac{\partial^2}{\partial ş^2}\, g(ş,t)\right] = \frac{ƀ^2}{ą^2}\mathbf{G}((ą,ƀ),\text{v}) - \frac{ƀ}{ą}\mathbf{D}_t[g(0,t)] - \mathbf{D}_t[g_{ş}(0,t)]$

Proof: Applying definition 3, we get:

i. $$\mathbf{U}_{ş}\,\mathrm{D}_t\left[\frac{\partial}{\partial t}\, g(ş,t)\right] = \frac{1}{\text{v}} \iint_0^{\infty} \exp\left(-\left(\frac{ą}{ƀ}\,ş + \frac{t}{\text{v}}\right)\right) \left\{\frac{\partial}{\partial t}\, g(ş,t)\right\} dt dş$$
$$= \int_0^{\infty} \exp\left(-\frac{ą}{ƀ}\,ş\right) dş . \underbrace{\frac{1}{\text{v}}\int_0^{\infty} \exp\left(-\frac{t}{\text{v}}\right) \left\{\frac{\partial}{\partial t}\, g(ş,t)\right\} dt}_{\textit{solve it by "integration by parts"}}$$

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$$= \frac{1}{\text{v}} \iint_0^\infty \exp\left(-\left(\frac{\text{ą}}{\text{ƀ}}\,\text{ş} + \frac{t}{\text{v}}\right)\right) g(\text{ş}, t)\,dt\,d\text{ş} - \frac{1}{\text{v}} \int_0^\infty \exp\left(-\frac{\text{ą}}{\text{ƀ}}\,\text{ş}\right) d\text{ş}$$

$$= \frac{1}{\text{v}} \iint_0^\infty \exp\left(-\left(\frac{\text{ą}}{\text{ƀ}}\,\text{ş} + \frac{t}{\text{v}}\right)\right) g(\text{ş}, t)\,dt\,d\text{ş} - \frac{1}{\text{v}} \int_0^\infty \exp\left(-\frac{\text{ą}}{\text{ƀ}}\,\text{ş}\right) d\text{ş}$$

$$= \frac{1}{\text{v}}\, \mathbf{G}\big((\text{ą}, \text{ƀ}), \text{v}\big) - \frac{1}{\text{v}}\, \mathbf{U}_{\text{ş}}\, [g(\text{ş}, 0)] \ \blacksquare$$

Similarly, we can prove part (ii).

iii. $$\mathbf{U}_{\text{ş}}\, \mathrm{D}_{t}\left[\frac{\partial}{\partial \text{ş}}\, g(\text{ş}, t)\right] = \frac{1}{\text{v}} \iint_0^\infty \exp\left(-\left(\frac{\text{ą}}{\text{ƀ}}\,\text{ş} + \frac{t}{\text{v}}\right)\right) \left\{\frac{\partial}{\partial \text{ş}}\, g(\text{ş}, t)\right\} dt\,d\text{ş}$$

$$= \underbrace{\int_0^\infty \exp\left(-\frac{\text{ą}}{\text{ƀ}}\,\text{ş}\right)\left\{\frac{\partial}{\partial \text{ş}}\, g(\text{ş}, t)\right\} ds}_{\textit{solve it using "integration by parts"}} \cdot \int_0^\infty \exp\left(-\frac{t}{\text{v}}\right) dt$$

$$= \frac{\text{ƀ}}{\text{ą}} \iint_0^\infty \exp\left(-\left(\frac{\text{ą}}{\text{ƀ}}\,\text{ş} + \frac{t}{\text{v}}\right)\right) g(\text{ş}, t)\,dt\,d\text{ş} - \frac{\text{ƀ}}{\text{ą}} \int_0^\infty \exp\left(-\frac{t}{\text{v}}\right) dt$$

$$= \frac{\text{ƀ}}{\text{ą}}\, \mathbf{G}\big((\text{ą}, \text{ƀ}), \text{v}\big) - \frac{\text{ƀ}}{\text{ą}}\, \mathbf{D}_{t}[g(0, t)] \ \blacksquare$$

Similarly, we can prove part (iv).

Table 1: MSST of some functions, where $\mathbf{U}_{\text{ş}}\, \mathrm{D}_{t}[g(\text{ş}, t)] = \mathbf{G}\big((\text{ą}, \text{ƀ}), \text{v}\big)$

$g(\text{ş}, t)$	$\mathbf{G}\big((\text{ą}, \text{ƀ}), \text{v}\big)$
1	$\frac{\text{ƀ}}{\text{ą}}$
ş	$\frac{\text{ƀ}^2}{\text{ą}^2}$
$\exp(k\text{ş} + lt)$	$\frac{\text{ƀ}}{(k\text{ƀ} - \text{ą})(l\text{v} - 1)}$
$\exp\big((k\text{ş} + lt)\mathrm{i}\big)$	$\frac{\text{ƀ}(kl\text{ƀ} - \text{ą}) + \text{ą}(k\text{ƀ} + \text{ą}k)\mathrm{i}}{((k\text{ƀ})^2 + \text{ą}^2)(l^2 + 1)}$
$\cos\,(k\text{ş} + lt)$	$\frac{\text{ƀ}(kl\text{ƀ} - \text{ą})}{((k\text{ƀ})^2 + \text{ą}^2)}$
$\sin\,(k\text{ş} + lt)$	$\frac{\text{ą}(k\text{ƀ} + \text{ą}k)\mathrm{i}}{(l^2 + 1)}$
$\text{ş}^{\mathrm{j}}\, t^{\mathrm{m}};$ $\mathrm{j}, \mathrm{m} = 0,1,2, \ldots$	$\mathrm{j}!\ \mathrm{m}!\ \left(\frac{\text{ƀ}}{\text{ą}}\right)^{\mathrm{j}+1} (\text{v})^{\mathrm{m}+1}$

3. Structure of the MTIM

Consider the following NLPDE:

$$\mathcal{L}g(\text{ş}, t) + \mathcal{N}g(\text{ş}, t) = \mathit{q}(\text{ş}, t) \qquad \ldots(3)$$

with the initial condition:

$$g(\text{ş}, 0) = \varphi(\text{ş}) \qquad \ldots(4)$$

where $g(\text{ş}, t)$ is a piecewise continuous function, $\mathcal{L} = \frac{\partial}{\partial t}$ is a linear operator, $\mathcal{N}$ is a nonlinear operator, and $\mathit{q}(\text{ş}, t)$ is the inhomogeneous or homogeneous term.

Applying the MSST on both sides of Eq. ‹1›, yields

$$\mathbf{U}_{\text{ş}}\, \mathbf{D}_{t}[\,\mathcal{L}g(\text{ş}, t) + \mathcal{N}g(\text{ş}, t)] = \mathbf{U}_{\text{ş}}\, \mathbf{D}_{t}[\mathit{q}(\text{ş}, t)] \qquad \ldots(5)$$

Using some derivatives and the linearity property of DSST for Eq. (5) leads to

$$\mathbf{U}_{\text{ş}}\, \mathbf{D}_{t}[\,\mathcal{L}g(\text{ş}, t)] + \mathbf{U}_{\text{ş}}\, \mathrm{D}_{t}[\mathcal{N}g(\text{ş}, t)] = \mathbf{U}_{\text{ş}}\, \mathrm{D}_{t}[\mathit{q}(\text{ş}, t)], \qquad \ldots(6)$$

$$\mathbf{U}_{\text{ş}}\, [g(\text{ş}, 0)] = \mathbf{U}_{\text{ş}}\, [\varphi(\text{ş})] = \mathbf{G}\big((\text{ą}, \text{ƀ}), 0\big) \qquad \ldots(7)$$

where $\mathrm{U}_{\text{ş}}$ is a single Shehu transform.

Substitute the transformed initial condition, Eq. (7), in Eq. (6) utilizing some properties of MSST in the previous theorems, which leads to

$$\frac{1}{\text{v}}\, \mathbf{G}\big((\text{ą}, \text{ƀ}), \text{v}\big) - \frac{1}{\text{v}}\, \mathbf{U}_{\text{ş}}\, [g(\text{ş}, 0)] + \mathbf{U}_{\text{ş}}\, \mathrm{D}_{t}[\mathcal{N}g(\text{ş}, t)] = \mathbf{U}_{\text{ş}}\, \mathrm{D}_{t}[\mathit{q}(\text{ş}, t)] \qquad \ldots(8)$$

$$\mathbf{G}\big((\text{ą}, \text{ƀ}), \text{v}\big) - \mathbf{U}_{\text{ş}}\, [g(\text{ş}, 0)] + \text{v}\, \mathbf{U}_{\text{ş}}\, \mathrm{D}_{t}[\mathcal{N}g(\text{ş}, t)] = \text{v}\, \mathbf{U}_{\text{ş}}\, \mathbf{D}_{t}[\mathit{q}(\text{ş}, t)] \qquad \ldots(9)$$

Take the inverse of MSST on both sides of Eq. (9), which gives

$$g(\text{ş}, t) + \mathbf{U}_{\text{ş}}\, \mathbf{D}_{t}^{-1}\left[\text{v}\, \mathbf{U}_{\text{ş}}\, \mathbf{D}_{t}[\mathcal{N}g(\text{ş}, t)]\right] - \square(\text{ş}, t) = 0 \qquad \ldots(10)$$

where

$$\square(\text{ş}, t) = \mathbf{U}_{\text{ş}}\, \mathbf{D}_{t}^{-1}\left[\mathbf{U}_{\text{ş}}\, [g(\text{ş}, 0)] + \text{v}\, \mathbf{U}_{\text{ş}}\, \mathrm{D}_{t}[\mathit{q}(\text{ş}, t)]\right]$$

put $\frac{\partial}{\partial t}$ on both sides of Eq. (10),

$$\frac{\partial}{\partial t} g(\text{ş}, t) + \frac{\partial}{\partial t} \mathbf{U}_{\text{ş}}\, \mathbf{D}_{t}^{-1}\left[\text{v}\, \mathbf{U}_{\text{ş}}\, \mathbf{D}_{t}[\mathcal{N}g(\text{ş}, t)]\right] - \frac{\partial}{\partial t} \square(\text{ş}, t) = 0 \qquad \ldots(11)$$

then, use VIM to solve Eq. (11), where the correction functional is

$$g_{\eta+1}(\text{ş}, t) = g_{\eta}(\text{ş}, t) + \int_0^t \lambda(\xi) \left\{\frac{\partial}{\partial \xi}\, g_{\eta}(\text{ş}, \xi) + \frac{\partial}{\partial \xi}\, \mathbf{U}_{\text{ş}}\, \mathbf{D}_{t}^{-1}\left[\text{v}\, \mathbf{U}_{\text{ş}}\, \mathrm{D}_{t}[\mathcal{N}\tilde{g}_{\eta}(\text{ş}, \xi)]\right]\right\} d\xi, \qquad \ldots(12)$$

λ is the Lagrange multiplier, which can be identified from variational theory [28], and $\tilde{g}_{\eta}$ is the restricted

variation, i.e. $\delta g_k = 0$, with stationary conditions $\lambda'(\xi)|_{\xi=t} = 0$, and $1 + \lambda(\xi)|_{\xi=t} = 0$.
Put $\lambda(\xi) = -1$ in Eq. (12), we get

$$g_{\eta+1}(s,t) = g_\eta(s,t) - \int_0^t \left\{\frac{\partial}{\partial\xi} g_\eta(s,\xi) + \frac{\partial}{\partial\xi} U_s D_t^{-1}\left[v\, U_s D_t[\mathcal{N} g_\eta(s,\xi)]\right]\right\} d\xi, \quad \eta \geq 0 \quad ...(13)$$

Hence, the solution is

$$g(s,t) = \lim_{\eta\to\infty} g_{\eta+1}(s,t).$$

4. Applications of the MTIM

In this section, we construct some examples to examine the applicability of the (MTIM):

<u>Example 1[29,30]</u>: Consider the coupled Burgers' equations:

$$\begin{cases} g_t - g_{ss} - 2gg_s + (gy)_s = 0, t > 0, s \in [-\pi,\pi] \\ y_t - y_{ss} - 2yy_s + (gy)_s = 0, t > 0, s \in [-\pi,\pi] \end{cases} ..(14)$$

with the initial conditions

$$\begin{cases} g(s,0) = \sin(s), \; s \in [-\pi,\pi] \\ y(s,0) = \sin(s), \; s \in [-\pi,\pi] \end{cases} \quad ... (15)$$

Solution: Apply the MSST to Eq. (14,15), we obtain,

$$U_s D_t[g_t] - U_s D_t\left[g_{ss} + 2gg_s - (gy)_s\right] = 0 \quad ...(16)$$

$$U_s D_t[y_t] - U_s D_t\left[y_{ss} + 2yy_s - (gy)_s\right] = 0 \quad ...(17)$$

$$\frac{1}{v} G((a,b),v) - \frac{1}{v} U_s[g(s,0)] - U_s D_t[g_{ss} + 2gg_s - (gy)_s] = 0 \quad ...(18)$$

$$\frac{1}{v} \mathcal{Y}((a,b),v) - \frac{1}{v} U_s[y(s,0)] - U_s D_t[y_{ss} + 2yy_s - (gy)_s] = 0 \quad ...(19)$$

where $U_s[g(s,0)] = U_s[\sin(s)]$, and $U_s[y(s,0)] = U_s[\sin(s)]$, are single Shehu Transforms
substitutes in Eq.'s (18,19)

$$G((a,b),v) - U_s[\sin(s)] - v\, U_s D_t[g_{ss} + 2gg_s - (gy)_s] = 0 \quad ... (20)$$

$$\mathcal{Y}((a,b),v) - U_s[\sin(s)] - v\, U_s D_t[y_{ss} + 2yy_s - (gy)_s] = 0 \quad ...(21)$$

Take the MSST inverse for Eq.'s (20,21)

$$g(s,t) - g(s,0) - U_s D_t^{-1}\left[v\, U_s D_t[g_{ss} + 2gg_s - (gy)_s]\right] = 0 \quad ...(22)$$

$$y(s,t) - y(s,0) - U_s D_t^{-1}\left[v\, U_s D_t[y_{ss} + 2yy_s - (gy)_s]\right] = 0 \quad ...(23)$$

put $\frac{\partial}{\partial t}$ on each side of Eq.'s (22,23)

$$\frac{\partial}{\partial t} g(s,t) - \frac{\partial}{\partial t} U_s D_t^{-1}\left[v\, U_s D_t[g_{ss} + 2gg_s - (gy)_s]\right] = 0 \quad ...(24)$$

$$\frac{\partial}{\partial t} y(s,t) - \frac{\partial}{\partial t} U_s D_t^{-1}\left[v\, U_s D_t[y_{ss} + 2yy_s - (gy)_s]\right] = 0 \quad ...(25)$$

Solve Eq.'s (24,25) using the VIM, where the correct functionals are

$$g_{\eta+1}(s,t) = g_\eta(s,t) + \int_0^t \lambda_1(\xi) \left\{\frac{\partial}{\partial\xi} g_\eta(s,\xi) - \frac{\partial}{\partial\xi} U_s D_t^{-1}\left[v\, U_s D_t\left[\frac{\partial^2}{\partial s^2} g_\eta(s,\xi) + 2\, g_\eta(s,\xi).\frac{\partial}{\partial s} g_\eta(s,\xi) - \frac{\partial}{\partial s}\left(g_\eta(s,\xi).y_\eta(s,\xi)\right)\right]\right]\right\} d\xi,$$
...(26)

$$y_{\eta+1}(s,t) = y_\eta(s,t) + \int_0^t \lambda_2(\xi) \left\{\frac{\partial}{\partial\xi} y_\eta(s,\xi) - \frac{\partial}{\partial\xi} U_s D_t^{-1}\left[v\, U_s D_t\left[\frac{\partial^2}{\partial s^2} y_\eta(s,\xi) + 2\, y_\eta(s,\xi).\frac{\partial}{\partial s} y_\eta(s,\xi) - \frac{\partial}{\partial s}\left(g_\eta(s,\xi).y_\eta(s,\xi)\right)\right]\right]\right\} d\xi,$$
...(27)

in Eq.'s (26,27), Put $\lambda_1(\xi) = \lambda_2(\xi) = -1$, where the stationary conditions are
$\lambda'_1(\xi)|_{\xi=t} = 0$, $1 + \lambda_1(\xi)|_{\xi=t}\, 0$ and
$\lambda'_2(\xi)|_{\xi=t} = 0$, $1 + \lambda_2(\xi)|_{\xi=t} = 0$,

These yields

$$g_{\eta+1} = g_\eta - \int_0^t \left\{\frac{\partial}{\partial\xi} g_\eta - \frac{\partial}{\partial\xi} U_s D_t^{-1}\left[v\, U_s D_t\left[(g_\eta)_{ss} - 2\, g_\eta(g_\eta)_s - (g_\eta\, y_\eta)_s\right]\right]\right\} d\xi, \quad \eta \geq 0$$

$$y_{\eta+1} = y_\eta - \int_0^t \left\{\frac{\partial}{\partial\xi} y_\eta - \frac{\partial}{\partial\xi} U_s D_t^{-1}\left[v\, U_s D_t\left[(y_\eta)_{ss} - 2\, y_\eta(y_\eta)_s - (g_\eta\, y_\eta)_s\right]\right]\right\} d\xi, \quad \eta \geq 0$$

$$g_0 = g(s,0) = \sin(s),$$
$$y_0 = y(s,0) = \sin(s),$$

$$g_1 = g_0 - \int_0^t \left\{\frac{\partial}{\partial\xi} g_\eta - \frac{\partial}{\partial\xi} U_s D_t^{-1}\left[v\, U_s D_t[(g_0)_{ss} - 2\, g_0(g_0)_s - (g_0\, y_0)_s]\right]\right\} d\xi,$$

$$= \sin(\text{ş}) - \sin(\text{ş})\, t,$$

$$y_1 = y_0 - \int_0^t \left\{\frac{\partial}{\partial \xi} y_0 - \frac{\partial}{\partial \xi} U_{\text{ş}} D_t^{-1}\left[v\, U_{\text{ş}} D_t[(y_0)_{\text{şş}} - 2\, y_0 (y_0)_{\text{ş}} - (g_0\, y_0)_{\text{ş}}]\right]\right\} d\xi,$$

$$= \sin(\text{ş}) - \sin(\text{ş})\, t,$$

$$g_2 = g_1 - \int_0^t \left\{\frac{\partial}{\partial \xi} g_1 - \frac{\partial}{\partial \xi} U_{\text{ş}} D_t^{-1}\left[v\, U_{\text{ş}} D_t[(g_1)_{\text{şş}} - 2\, g_1 (g_1)_{\text{ş}} - (g_1\, y_1)_{\text{ş}}]\right]\right\} d\xi,$$

$$= \sin(\text{ş}) - \sin(\text{ş})\, t + \sin(\text{ş}) \frac{t^2}{2!},$$

$$y_2 = y_1 - \int_0^t \left\{\frac{\partial}{\partial \xi} y_1 - \frac{\partial}{\partial \xi} U_{\text{ş}} D_t^{-1}\left[v\, U_{\text{ş}} D_t[(y_1)_{\text{şş}} - 2\, y_1 (y_1)_{\text{ş}} - (g_1\, y_1)_{\text{ş}}]\right]\right\} d\xi,$$

$$= \sin(\text{ş}) - \sin(\text{ş})\, t + \sin(\text{ş}) \frac{t^2}{2!},$$

$$g_3 = g_2 - \int_0^t \left\{\frac{\partial}{\partial \xi} g_2 - \frac{\partial}{\partial \xi} U_{\text{ş}} D_t^{-1}\left[v\, U_{\text{ş}} D_t[(g_2)_{\text{şş}} - 2\, g_2 (g_2)_{\text{ş}} - (g_2\, y_2)_{\text{ş}}]\right]\right\} d\xi,$$

$$= \sin(\text{ş}) - \sin(\text{ş})\, t + \sin(\text{ş}) \frac{t^2}{2!} - \sin(\text{ş}) \frac{t^3}{3!},$$

$$y_3 = y_2 - \int_0^t \left\{\frac{\partial}{\partial \xi} y_2 - \frac{\partial}{\partial \xi} U_{\text{ş}} D_t^{-1}\left[v\, U_{\text{ş}} D_t[(y_2)_{\text{şş}} - 2\, y_2 (y_2)_{\text{ş}} - (g_2\, y_2)_{\text{ş}}]\right]\right\} d\xi,$$

$$= \sin(\text{ş}) - \sin(\text{ş})\, t + \sin(\text{ş}) \frac{t^2}{2!} - \sin(\text{ş}) \frac{t^3}{3!},$$

$$\vdots$$

and so on, providing the approximate solutions

$$g_{\eta+1}(\text{ş}, t) = \sin(\text{ş}) \left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \cdots\right),$$

$$y_{\eta+1}(\text{ş}, t) = \sin(\text{ş}) \left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \cdots\right)$$

The two functions $g(\text{ş}, t)$ and $y(\text{ş}, t)$ in the closed-form solution is given as

$$\begin{cases} g(\text{ş}, t) = \sin(\text{ş})\, e^{-t} \\ y(\text{ş}, t) = \sin(\text{ş})\, e^{-t} \end{cases}$$

<u>Example 2[31]</u>: Consider the coupled Schrödinger-KdV equation:

$$\begin{cases} g_t - y_{\text{şş}} - y f = 0, \\ y_t + g_{\text{şş}} + g f = 0, \\ f_t + 6 f f_{\text{ş}} + f_{\text{şşş}} - 2 g g_{\text{ş}} - 2 y y_{\text{ş}} = 0 \end{cases} \quad \dots (28)$$

with the initial conditions

$$\begin{cases} g(\text{ş}, 0) = \cos(\text{ş}), \\ y(\text{ş}, 0) = \sin(\text{ş}), \\ f(\text{ş}, 0) = \frac{3}{4} \end{cases} \quad \dots (29)$$

Solution: Apply the MSST to Eq.'s (28,29), we obtain,

$$U_{\text{ş}} D_t[g_t] - U_{\text{ş}} D_t\left[y_{\text{şş}} + y f\right] = 0, \quad \dots (30)$$

$$U_{\text{ş}} D_t[y_t] + U_{\text{ş}} D_t\left[g_{\text{şş}} + g f\right] = 0, \quad \dots (31)$$

$$U_{\text{ş}} D_t[f_t] + U_{\text{ş}} D_t\left[6 f f_{\text{ş}} + f_{\text{şşş}} - 2 g g_{\text{ş}} - 2 y y_{\text{ş}}\right] = 0 \quad (32)$$

$$\frac{1}{v} G((a, b), v) - \frac{1}{v} U_{\text{ş}}[g(\text{ş}, 0)] - U_{\text{ş}} D_t[y_{\text{şş}} + y f] = 0 \quad (33)$$

$$\frac{1}{v} Y((a, b), v) - \frac{1}{v} U_{\text{ş}}[y(\text{ş}, 0)] + U_{\text{ş}} D_t[g_{\text{şş}} + g f] = 0 \quad (34)$$

$$\frac{1}{v} F((a, b), v) - \frac{1}{v} U_{\text{ş}}[f(\text{ş}, 0)] + U_{\text{ş}} D_t[6 f f_{\text{ş}} + f_{\text{şşş}} - 2 g g_{\text{ş}} - 2 y y_{\text{ş}}] = 0 \quad \dots (35)$$

where $U_{\text{ş}}[g(\text{ş}, 0)] = U_{\text{ş}}[\cos(\text{ş})]$,
$U_{\text{ş}}[y(\text{ş}, 0)] = U_{\text{ş}}[\sin(\text{ş})]$, and
$U_{\text{ş}}[f(\text{ş}, 0)] = U_{\text{ş}}\left[\frac{3}{4}\right]$, are single Shehu Transforms,

substitute in Eq.'s (33-35)

$$G((a, b), v) - U_{\text{ş}}[\cos(\text{ş})] - v\, U_{\text{ş}} D_t[y_{\text{şş}} + y f] = 0, \quad \dots (36)$$

$$Y((a, b), v) - U_{\text{ş}}[\sin(\text{ş})] + v\, U_{\text{ş}} D_t[g_{\text{şş}} + g f] = 0, \quad \dots (37)$$

$$F((a, b), v) - U_{\text{ş}}[f(\text{ş}, 0)] + v\, U_{\text{ş}} D_t[6 f f_{\text{ş}} + f_{\text{şşş}} - 2 g g_{\text{ş}} - 2 y y_{\text{ş}}] = 0 \quad \dots (38)$$

Take the MSST inverse for Eq.'s (36-38).

$$g(\text{ş}, t) - g(\text{ş}, 0) - U_{\text{ş}} D_t^{-1}\left[v\, U_{\text{ş}} D_t[y_{\text{şş}} + y f]\right] = 0, \quad \dots (39)$$

$$y(\text{ş}, t) - y(\text{ş}, 0) + U_{\text{ş}} D_t^{-1}\left[v\, U_{\text{ş}} D_t[g_{\text{şş}} + g f]\right] = 0, \quad \dots (40)$$

$$f(\text{ş}, t) - f(\text{ş}, 0) + U_{\text{ş}} D_t^{-1}\left[v\, U_{\text{ş}} D_t[6 f f_{\text{ş}} + f_{\text{şşş}} - 2 g g_{\text{ş}} - 2 y y_{\text{ş}}]\right] = 0 \quad \dots (41)$$

put $\frac{\partial}{\partial t}$ on each side of Eqs. (39-41),

$$\frac{\partial}{\partial t} g(\text{ş}, t) - \frac{\partial}{\partial t} U_{\text{ş}} D_t^{-1}\left[v\, U_{\text{ş}} D_t[y_{\text{şş}} + y f]\right] = 0 \quad \dots (42)$$

$$\frac{\partial}{\partial t} y(\text{ş}, t) + \frac{\partial}{\partial t} U_{\text{ş}} D_t^{-1}\left[v\, U_{\text{ş}} D_t[g_{\text{şş}} + g f]\right] = 0 \quad \dots (43)$$

$$\frac{\partial}{\partial t} f(\text{ş}, t) + \frac{\partial}{\partial t} U_{\text{ş}} D_t^{-1}\left[v\, U_{\text{ş}} D_t[6 f f_{\text{ş}} + f_{\text{şşş}} - 2 g g_{\text{ş}} - 2 y y_{\text{ş}}]\right] = 0 \quad \dots (44)$$

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Solve Eq.'s (42-44) using the VIM, where the correct functionals are

Solve Eq.'s (42-44) using the VIM, where the correct functionals are

$$\boldsymbol{g}_{ŋ+1}(ş,\boldsymbol{t}) = \boldsymbol{g}_{ŋ}(ş,\boldsymbol{t}) + \int_0^t \boldsymbol{\lambda}_1(\xi)\left\{\frac{\partial}{\partial\xi}\boldsymbol{g}_{ŋ}(ş,\xi) - \frac{\partial}{\partial\xi}\mathbf{U}_ş \mathrm{D}_t^{-1}\left[\mathrm{v}\,\mathbf{U}_ş \mathrm{D}_t\left[\frac{\partial^2}{\partial ş^2}\boldsymbol{y}_{ŋ}(ş,\xi) + \left(\boldsymbol{y}_{ŋ}(ş,\xi).ƒ_{ŋ}(ş,\xi)\right)\right]\right]\right\} d\xi, \quad \ldots(45)$$

$$\boldsymbol{y}_{ŋ+1}(ş,\boldsymbol{t}) = \boldsymbol{y}_{ŋ}(ş,\boldsymbol{t}) + \int_0^t \boldsymbol{\lambda}_2(\xi)\left\{\frac{\partial}{\partial\xi}\boldsymbol{y}_{ŋ}(ş,\xi) - \frac{\partial}{\partial\xi}\mathbf{U}_ş \mathrm{D}_t^{-1}\left[\mathrm{v}\,\mathbf{U}_ş \mathrm{D}_t\left[\frac{\partial^2}{\partial ş^2}\boldsymbol{g}_{ŋ}(ş,\xi) + \left(\boldsymbol{g}_{ŋ}(ş,\xi).ƒ_{ŋ}(ş,\xi)\right)\right]\right]\right\} d\xi, \quad \ldots(46)$$

$$ƒ_{ŋ+1}(ş,\boldsymbol{t}) = ƒ_{ŋ}(ş,\boldsymbol{t}) + \int_0^t \boldsymbol{\lambda}_3(\xi)\left\{\frac{\partial}{\partial\xi}ƒ_{ŋ}(ş,\xi) + \frac{\partial}{\partial\xi}\mathbf{U}_ş \mathrm{D}_t^{-1}\left[\mathrm{v}\,\mathbf{U}_ş \mathrm{D}_t\left[6\,ƒ_{ŋ}(s,\xi).\frac{\partial}{\partial ş}ƒ_{ŋ}(ş,\xi) + \frac{\partial^3}{\partial ş^3}ƒ_{ŋ}(ş,\xi) - 2\,\boldsymbol{g}_{ŋ}(ş,\xi).\frac{\partial}{\partial ş}\boldsymbol{g}_{ŋ}(ş,\xi) - 2\,ƒ_{ŋ}(ş,\xi).\frac{\partial}{\partial ş}ƒ_{ŋ}(ş,\xi)\right]\right]\right\} d\xi, \quad \ldots(47)$$

In Eqs. ‹43-45›, put $\boldsymbol{\lambda}_1(\xi) = \boldsymbol{\lambda}_2(\xi) = \boldsymbol{\lambda}_3(\xi) = -1$, where the stationary conditions are

$\boldsymbol{\lambda}'_1(\xi)|_{\xi=t} = 0,\ 1 + \boldsymbol{\lambda}_1(\xi)|_{\xi=t} = 0,$
$\boldsymbol{\lambda}'_2(\xi)|_{\xi=t} = 0,\ 1 + \boldsymbol{\lambda}_2(\xi)|_{\xi=t} = 0$, and
$\boldsymbol{\lambda}'_3(\xi)|_{\xi=t} = 0,\ 1 + \boldsymbol{\lambda}_3(\xi)|_{\xi=t} = 0$

$$\boldsymbol{g}_{ŋ+1} = \boldsymbol{g}_{ŋ} - \int_0^t\left\{\frac{\partial}{\partial\xi}\boldsymbol{g}_{ŋ} - \frac{\partial}{\partial\xi}\mathbf{U}_ş \mathrm{D}_t^{-1}\left[\mathrm{v}\,\mathbf{U}_ş \mathrm{D}_t\left[\frac{\partial^2}{\partial ş^2}\boldsymbol{y}_{ŋ} + (\boldsymbol{y}_{ŋ}.ƒ_{ŋ})\right]\right]\right\} d\xi, \quad ŋ \geq 0$$

$$\boldsymbol{y}_{ŋ+1} = \boldsymbol{y}_{ŋ} - \int_0^t\left\{\frac{\partial}{\partial\xi}\boldsymbol{y}_{ŋ} - \frac{\partial}{\partial\xi}\mathbf{U}_ş \mathrm{D}_t^{-1}\left[\mathrm{v}\,\mathbf{U}_ş \mathrm{D}_t\left[\frac{\partial^2}{\partial ş^2}\boldsymbol{g}_{ŋ} + (\boldsymbol{g}_{ŋ}.ƒ_{ŋ})\right]\right]\right\} d\xi, \quad ŋ \geq 0$$

$$ƒ_{ŋ+1} = ƒ_{ŋ} - \int_0^t\left\{\frac{\partial}{\partial\xi}ƒ_{ŋ} + \frac{\partial}{\partial\xi}\mathbf{U}_ş \mathrm{D}_t^{-1}\left[\mathrm{v}\,\mathbf{U}_ş \mathrm{D}_t\left[6\,ƒ_{ŋ}.\frac{\partial}{\partial ş}ƒ_{ŋ} + \frac{\partial^3}{\partial ş^3}ƒ_{ŋ} - 2\boldsymbol{g}_{ŋ}.\frac{\partial}{\partial ş}\boldsymbol{g}_{ŋ} - 2ƒ_{ŋ}.\frac{\partial}{\partial ş}ƒ_{ŋ}\right]\right]\right\} d\xi, \quad ŋ \geq 0$$

$\boldsymbol{g}_0 = \boldsymbol{g}(ş,0) = \cos(ş),$
$\boldsymbol{y}_0 = \boldsymbol{y}(ş,0) = \sin(ş),$
$ƒ_0 = ƒ(ş,0) = \frac{3}{4}$

$$\boldsymbol{g}_1 = \boldsymbol{g}_0 - \int_0^t\left\{\frac{\partial}{\partial\xi}\boldsymbol{g}_0 - \frac{\partial}{\partial\xi}\mathbf{U}_ş \mathrm{D}_t^{-1}\left[\mathrm{v}\,\mathbf{U}_ş \mathrm{D}_t\left[\frac{\partial^2}{\partial ş^2}\boldsymbol{y}_0 + (\boldsymbol{y}_0.ƒ_0)\right]\right]\right\} d\xi,$$
$$= \cos(ş) - \frac{1}{4}\,t\sin(ş)$$

$$\boldsymbol{y}_1 = \boldsymbol{y}_0 - \int_0^t\left\{\frac{\partial}{\partial\xi}\boldsymbol{y}_0 - \frac{\partial}{\partial\xi}\mathbf{U}_ş \mathrm{D}_t^{-1}\left[\mathrm{v}\,\mathbf{U}_ş \mathrm{D}_t\left[\frac{\partial^2}{\partial ş^2}\boldsymbol{g}_0 + (\boldsymbol{g}_0.ƒ_0)\right]\right]\right\} d\xi,$$
$$= \sin(ş) + \frac{1}{4}\,t\cos(ş)$$

$$ƒ_1 = ƒ_0 - \int_0^t\left\{\frac{\partial}{\partial\xi}ƒ_0 + \frac{\partial}{\partial\xi}\mathbf{U}_ş \mathrm{D}_t^{-1}\left[\mathrm{v}\,\mathbf{U}_ş \mathrm{D}_t\left[6\,ƒ_0.\frac{\partial}{\partial ş}ƒ_0 + \frac{\partial^3}{\partial ş^3}ƒ_0 - 2\boldsymbol{g}_0.\frac{\partial}{\partial ş}\boldsymbol{g}_0 - 2ƒ_0.\frac{\partial}{\partial ş}ƒ_0\right]\right]\right\} d\xi,$$
$$= \frac{3}{4}$$

$$\boldsymbol{g}_2 = \boldsymbol{g}_1 - \int_0^t\left\{\frac{\partial}{\partial\xi}\boldsymbol{g}_1 - \frac{\partial}{\partial\xi}\mathbf{U}_ş \mathrm{D}_t^{-1}\left[\mathrm{v}\,\mathbf{U}_ş \mathrm{D}_t\left[\frac{\partial^2}{\partial ş^2}\boldsymbol{y}_1 + (\boldsymbol{y}_1.ƒ_1)\right]\right]\right\} d\xi,$$
$$= \cos(ş) - \frac{1}{4}\,t\sin(ş) + \frac{1}{32}\,t^2\cos(ş)$$

$$\boldsymbol{y}_2 = \boldsymbol{y}_1 - \int_0^t\left\{\frac{\partial}{\partial\xi}\boldsymbol{y}_1 - \frac{\partial}{\partial\xi}\mathbf{U}_ş \mathrm{D}_t^{-1}\left[\mathrm{v}\,\mathbf{U}_ş \mathrm{D}_t\left[\frac{\partial^2}{\partial ş^2}\boldsymbol{g}_1 + (\boldsymbol{g}_1.ƒ_1)\right]\right]\right\} d\xi,$$
$$= \sin(ş) + \frac{1}{4}\,t\cos(ş) - \frac{1}{32}\,t^2\sin(ş)$$

$$ƒ_2 = ƒ_1 - \int_0^t\left\{\frac{\partial}{\partial\xi}ƒ_1 + \frac{\partial}{\partial\xi}\mathbf{U}_ş \mathrm{D}_t^{-1}\left[\mathrm{v}\,\mathbf{U}_ş \mathrm{D}_t\left[6ƒ_1.\frac{\partial}{\partial ş}ƒ_1 + \frac{\partial^3}{\partial ş^3}ƒ_1 - 2\boldsymbol{g}_1.\frac{\partial}{\partial ş}\boldsymbol{g}_1 - 2ƒ_1.\frac{\partial}{\partial ş}ƒ_1\right]\right]\right\} d\xi,$$
$$= \frac{3}{4}$$

$$\vdots$$

and so on, providing the approximate solutions

$\boldsymbol{g}_{ŋ+1} = \cos(ş) - \frac{1}{4}\,t\sin(ş) + \frac{1}{32}\,t^2\cos(ş) - \cdots,$
$\boldsymbol{y}_{ŋ+1} = \sin(ş) + \frac{1}{4}\,t\cos(ş) - \frac{1}{32}\,t^2\sin(ş) + \cdots,$
$ƒ_{ŋ+1} = \frac{3}{4}$

where the exact solutions of Eqs. ‹26› is given by

$$\begin{cases} \boldsymbol{g}(ş,\boldsymbol{t}) = \cos\left(ş + \frac{t}{4}\right), \\ \boldsymbol{y}(ş,\boldsymbol{t}) = \sin\left(ş + \frac{t}{4}\right), \\ ƒ(ş,\boldsymbol{t}) = \frac{3}{4} \end{cases}$$

5. Conclusion

This paper introduces a new efficient Hybrid Iterative Method to solve the coupled nonlinear Burgers' equations and the coupled nonlinear Schrödinger-Kortweg-de Veries (SKdV) equations. The recommended method is easy to use and practical in solving different types of coupled nonlinear partial differential equations (NLPDEs). The obtained results provided as a series converge rapidly to the exact solution. The main goal of this new strategy is to use a mix of the Shehu-Sumudu Integral Transforms with a successful Variational Iteration Method (VIM) to avoid the difficulties in solving the NLPDEs in comparison with traditional semi-analytical methods and to minimize the steps of the calculations with high accurate results.

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Conflicts of Interest

The authors declare, there is no conflict of interest.

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