Numerical Simulations of Charged Analogues of Isentropic Superdense Star Model

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Abstract

Numerical simulations of charged analogues of isentropic superdense star model by using a particular eclectic field, which involves a parameter *K* have been obtained and the general solution for star is derived for all negative values *K*. Runge-Kutta method has been used to represent superdense star models with ultrahigh surface density of the order $2\times10^{14}\ gm\ cm^{-3}$. The physical properties have been studied and analyzed numerically subject to the energy conditions throughout the star and found to be physically acceptable.

Keywords: Einstein- Maxwell equations, Numerical simulations, Charged stars.

I Introduction

In general relativity and allied theories, the distribution of the mass, momentum, and stress due to matter and to any non-gravitational fields is described by the energy-momentum tensor (or *matter tensor*) T^{ab}. However, the Einstein field equation is not very choosy about what kinds of states of matter or non-gravitational fields are admissible in a space-time model. For the last four decades research workers have been busy in deriving the solutions for charged fluid spheres to provide source of Reissener (1916) and Nordstrom(1918) solutions. Such fluid models are not likely to undergo gravitational collapse to reduce into a point singularity, in presence of charges. The gravitational attraction may be nullified by electrostatic repulsion and pressure gradient. Several workers have studied the number of charged fluids in different contexts such as Bonner (1960), Tikekar (1990), Felice et al (1995, 1999), Gupta et al (1986), Ray et al (2003). Moodely et al (2003) have found a class of accelerating, expanding and shearing solutions which is characterized geometrically by conformal killing vector while Thirukkanesh and Maharaj ([2], [3]) found a new class of exact solutions in closed form and have mention that a physical analysis indicated that the model may be used to describe a charge sphere. Gupta, et al (2005, 2010), have charged the Vaidya-Tikekar type solutions, then followed have charged Buchdahl's fluid spheres, while, in (2010), have found a class of analogues of Durgopal and Floria superdense star and the member of this class are seen to satisfy the various physical conditions.

In the present article, we have charged the generalized superdense star which obtained by Gupta-Jasim (2000, 2003), using Runge-Kutta. The result so obtained have been compared with Gupta- Mukesh (2005) and found having a good approximation. The intensive studies and the limitation of the parameter λ for different models of charged as well as for null charged have been produced, which ensure the physically acceptable to their aspects.

II Field Equations

The space time of charged fluid sphere model is appropriately described by the metric [2]

$$ds^2 = -e^{\lambda(r)}dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) + e^{\nu(r)}dt^2, \tag{1}$$

The metric potential functions with some modification [6]:

$$\lambda(r) = \ln\left(\frac{1 - K\dfrac{r^2}{R^2}}{1 - \dfrac{r^2}{R^2}}\right)$$

(1a)

$$\nu(r) = \ln\left(\left(1 - K\frac{r^2}{R^2}\right)(1-K)\right)^3$$

(1b)

satisfying the Einstein –Maxwell equations[6]

$$R^i_j - \frac{1}{2}R\delta^i_j = -8\pi\frac{G}{C^4}T^i_j \tag{2}$$

where C=1, G=1 and $T^i_j = M^i_j + E^i_j$ (3)

In the interior, M^i_j can be described by an isentropic pressure P and the mass density ρ, to take the form [6]:

$$M^i_j = (P+\rho)u_i u^i - P\delta^i_j \tag{4}$$

Where $u^i = \left(0,0,0,e^{-\nu/2}\right)$ (5)

while, E^i_j, the electromagnetic contribution to the stress energy tensor can be written as:

$$E^i_j = -\frac{1}{4\pi}\left(F^{im}F_{jm} - \frac{1}{4}\delta^i_j F_{mn}F^{mn}\right) \quad (6)$$

F_{ik}, being the skew symmetric electromagnetic field tensor satisfying the Maxwell equations

$$F_{ik,j} + F_{kj,i} + F_{ji,k} = 0 \quad (7)$$

$$\frac{\partial}{\partial x^k}\left(\sqrt{-g}F^{ik}\right) = -4\pi\sqrt{-g}\, j^i \quad (8)$$

where $j^i = \sigma v^i$ represents the four-current vector of charged fluid with σ as the charged density.

In view of (1) and (2) with (3), the field equation can be furnished as [5]:

$$8\pi\rho = \frac{\lambda'}{r}e^{-\lambda} + \frac{\left(1-e^{-\lambda}\right)}{r^2} - 8\pi\left(E^i_j\right)^2 \quad (9)$$

$$8\pi P = \frac{\nu'}{r}e^{-\lambda} - \frac{\left(1-e^{-\lambda}\right)}{r^2} + 8\pi\left(E^i_j\right)^2 \quad (10)$$

$$8\pi P = \left[\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r}\right]e^{-\lambda} - 8\pi\left(E^i_j\right)^2 \quad (11)$$

where, prime denotes the differentiation with respect to *r* and

$$E^i_j = 4\pi\int_0^r \sigma r^2 e^{\lambda/2}dr = r^2\sqrt{-F_{14}F^{14}} = r^2F^{41}e^{(\lambda+\nu)/2} \quad (12)$$

which represents the total charge contained within the sphere of radius *r* in view of (1).

The consistency of the field equations (9)-(11) using (1a) & (1b) and the electric field intensity (Gupta, et al, 2005)

$$\frac{q^2}{r^4} = \frac{K^2r^2\gamma^2}{2R^2\left(K+R^2\right)} \quad (13)$$

yield the hypergeometric equation (Gupta et al 2005)

$$(1-X^2)\frac{d^2y}{dX^2} + X\frac{dy}{dX} + \left(1-K+K\gamma^2\right)y = 0 \quad (14)$$

where $X=\sqrt{\frac{K}{K-1}}\sqrt{1+\frac{Cr^2}{K}}$, $K<0$ or $K>1$ and $y=\sqrt{e^{\nu}}$

So, (14) can be solved exactly for two cases:

Case I: Absence of charged by putting $\gamma=0$ (Gupta-Jasim, 2000 and 2003) [4, 5]

Case II: for charged case, the case have been discussed by Gupta-Mukesh (2005, 2010)[7, 8] which they have produced and discuss some physical properties of such kind problem while in our search in the next section, we have built an algorithm for discussing charged and null charged problem with the point of view of its physical properties for different value of *K* using 4th order Runge-Kutta method.

III Algorithm

To approximate the solution of initial value problem (14) with its physical properties at (N+1) equality spaced number (i.e. $x=\frac{r}{a}=0(0.1)1$)

Input lambda (l), a, $H=10^{-5}$, M_S , z=18.66106* 10^{-28} , $d=2*10^{14}$, K

I For j=1 to 101
 Read c(j)
 h=0.01
 c(j)=1-(j-1)*h
 if c(j)<0.000001 then c(j)=0
 next j
 $m=SQR(2-K)$

II $R=d/l$
 $zd=SQR(d*z)$
 Compute m(r),

III for i=1 to n
 Set k_1, k_2, $k3$, k_4
 Compute $\rho(n)$ and p(n), q(n)

IV Compute some other physical quantity like weak energy condition, strong energy condition, adiabatic index, speed of the sound $\frac{dp}{d\rho}$ etc.

V Output

Stop

End

IV Sensitive analysis to solution

In general relativity an energy condition is one of various alternative conditions which can be applied to the matter content of the theory, when it is either not possible or desirable to specify this content explicitly. The hope is then that any reasonable matter theory will satisfy this condition or at least will preserve the condition if it is satisfied by the starting conditions. So, the physical validity of the charged fluid spheres depends on the following reality as well as energy conditions inside and on the sphere at $r = a$ such that [6]

(i) The matter density ρ and the fluid pressure P should be positive through out the star.

(ii) The gradients $\frac{d\rho}{dr}, \frac{dP}{dr}$ should be negative with increasing radius.

(iii) The interior metric should match continuously with the Schwarzschild exterior solution

$$ds^2 = -(1-\frac{2M}{r})^{-1}dr^2 - r^2(d\theta^2 + \sin^2(\theta)d\varphi^2) + c^2(1-\frac{2M}{r})dt^2$$

(iv) In addition to that the weak energy (WE) and strong energy (SE) conditions should be positive.

(v)The adiabatic velocity of sound $\left(\frac{dP}{d\rho}\right)^{1/2}$ should be less than the velocity of light.

(vi) The electric field intensity E should be real $\left(E^2 \geq 0\right)$ and at the center, $r = 0,\ \left((E_0)^2 = 0\right)$

Besides to the above the charged fluid spheres is expected to join smoothly with Nordstrom metric, which requires the continuity of e^{λ} , e^{v} and q across the pressure free interface $r = a$ [9,10].

$$\left(\frac{1-\frac{r^2}{R^2}}{1-K\frac{r^2}{R^2}}\right) = 1 - \frac{2m(a)}{a} + \frac{q(a)^2}{a^2}$$

(15)

$$y^2 = \left(1 - \frac{2m(a)}{a} + \frac{q(a)^2}{a^2} \right) \qquad (16)$$

$$E_a = \frac{q}{a^2} \qquad (17)$$

$$p_{(r=a)} = 0 \qquad (18)$$

The conditions (15) and (17) are automatically satisfied which can be manipulated in the forthcoming tables and graphs. Various physical quantities have been calculated numerically for $0 \le \left(x = \frac{r}{a} \right) \le 1$ and the data so obtained is analyzed subject to the reality conditions. The data for various cases reveals the following information:

- The value of pressure in charged fluid spheres are less than the pressure value in null charged case.
- The charge at the center ($r = 0$) of the fluid sphere are zero (i.e. $\left((E_0)^2 = 0 \right)$) and increasing gradually until the surface ($r=a$) where the charge takes the maximum.
- The comparison of the values of pressure, and effect of charge on the behavior of the pressure with fixing density, we reviled that when the fluid sphere provided with charge, this leads to reduction in pressure.
- The comparison of the radii with maximum mass may be shown in the figure(1) below:

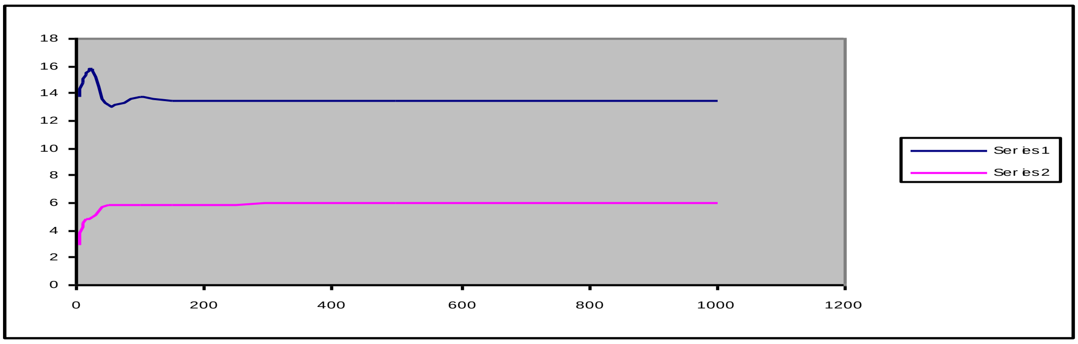

Fig. (1), shows the comparison of the radii with maximum mass for charged and null charged fluid sphere

The energy conditions represent such criteria. Roughly speaking, they crudely describe properties common to all (or almost all) states of matter and all non-gravitational fields which are well-established in physics, while being sufficiently strong to rule out many unphysical "solutions" of the Einstein field equation. (It does not hold for matter described by a super-field, i.e., the Dirac field!)

Mathematically speaking, the most apparent distinguishing feature of the energy conditions is that they are essentially restrictions on the eigenvalues and eigenvectors of the matter tensor.

V Conclusions

Numerical simulations have been carried out, e.g. using smooth particle hydrodynamics with soft and stiff equations of state. Those results suggest that, depending on the mass ratio, at least part of the neutron star is tidally disrupted before the merger.

Recently these systems have received more attention and there are some results in Newtonian gravity with relativistic modifications to the gravitational potential. There are also results using full General Relativity, to create initial data for a mixed, orbiting system. So the following may be concluded:

1. The constraint equations together with the Euler equations for the neutron star matter in a co-rotating frame, in which the system is assumed to be stationary, have been solved.
2. A dynamical evolution will be required at least from this point on, the black hole is only included as background metric around the neutron star and is not included in the computational domain.
3. An extensive study have been carried and compared with different values of -*K* and $\lambda = \frac{\rho a}{\rho 0}$ for charged fluid spheres and analyzed to get more interest limitation of –*K* with concerned to its Lambda's as follows:
   - For –*K=1.5*, $\lambda = \frac{\rho a}{\rho 0}$ will be $0.17 \le \lambda < 1$
   - For –*K=3*, $\lambda = \frac{\rho a}{\rho 0}$ will be $0.11 \le \lambda < 1$
   - And so on for other values of –K

Also we may mention that all the solutions obtained by others in the prevailing circumstances and conditions, can be seen as special cases of the present work.

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