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Anti – fuzzy AT – ideals of AT – algebras

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Abstract.

In this paper , we introduce the notion of anti – fuzzy AT – ideals in AT – algebra, several appropriate examples are provided and theirsome properties are investigated. The image and the inverse imageof anti – fuzzy AT – ideals in AT – algebra are defined and how theimage and the inverse image of anti – fuzzy AT – ideals in AT – algebra become anti – fuzzy AT – ideals are studied. Moreover, the Cartesian product of anti – fuzzy AT – ideals are given .

Keywords : AT – ideal, anti – fuzzy AT – ideals , image and pre-image of anti – fuzzy AT – ideals .

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1 Introduction

BCK − algebras form an important class of logical algebras introducedby K. Iseki [4] and was extensively investigated by several researchers. Theclass of all BCK − algebras is quasi variety. J. Meng and Y. B. Jun posed an interesting problem (solved in[7]) whether the class of all BCK − algebras is a variety. In connection with this problem , Komori introduced in [6]a notion of BCC − algebras. W.A. Dudek (cf.[2],[5]) redefined the notion of BCC − algebras by using a dual form of the ordinary definition inthe sense of Y. Komori and studied ideals and congruences of BCC-algebras. In ([10],[11]), C. Prabpayak and U. Leerawat introduced a new algebraic structure, which is called KU − algebra. They gave the concept of homomorphisms of KU − algebras and investigated some related properties. L.A. Zadeh [13] introduced the notion of fuzzy sets. At present this concept has been applied to many mathematical branches, such as group, functional analysis , probability theory , topology, and soon. In 1991 , O.G. Xi [12] applied this concept to BCK − algebras, and heintroduced the notion of fuzzy sub-algebras (ideals) of the BCK − algebras with respect to minimum, and since then Jun et al studiedfuzzy ideals (cf.[1],[5],[12]), and moreover several fuzzystructures in BCC-algebras are considered (cf.[2],[6]). S. Mostafa , M. Abd-Elnaby, F. Abdel-Halim and A.T. Hameed (in [7]) introduced the notion of fuzzy KUS − ideals of KUS − algebras and they investigated severalbasic properties which are related to fuzzy KUS − ideals. they describedhow to deal with the homomorphism image and inverse image of fuzzy KUS − ideals. And in [8], the anti − fuzzy KUS − ideals of KUS − algebras is introduced. Several theorems are stated and proved. In [3], Areej Tawfeeq Hameed introduced and studied new algebraic structure, called AT − algebra and investigate some of its properties. She introduced the notion of fuzzy AT − ideal of AT − algebra, several theorems, properties are stated and proved.

In this paper, we introduce the notion of anti − fuzzy AT − ideals of AT − algebras and then we study the homomorphism image and inverse image of anti − fuzzy AT − ideals.We also prove that the Cartesian product of anti − fuzzy AT − ideals are anti − fuzzy AT − ideals .

2. Preliminaries

In this section we give some basic definitions and preliminaries lemmas of AT − ideals and fuzzy AT − ideals of AT − algebra.

Definition 2.1[3]. An **AT-algebra** is a nonempty set X with a constant (0) and a binary operation ($*$) satisfying the following axioms: for all x, y, z∈X,

(i) (x*y)*((y*z)*(x*z))=0,

(ii) 0* x =x,

(iii) x* 0 =0.

In X we can define a binary relation (≤) by :x ≤ y if and only if , y * x = 0 .

Remark 2.2[3]. (X ;*, 0) is an AT − algebra if and only if, it satisfies that: for allx, y, z∈X,

(i') : (y *z) *(x *z) ≤ x *y ,

(ii') : x ≤ y if and only if, y * x = 0.

Example 2.3 [3]. Let X = {0, 1, 2, 3, 4} in which (*) is defined by the following table:

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	3	3
3	0	0	2	0	2
4	0	0	0	0	0

It is easy to show that (X ;*, 0) is an AT − algebra.

Example 2.4[3]. Let X = {0, 1, 2, 3,4} be a set with the following table:

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	3	3
3	0	0	2	0	2
4	0	0	0	0	0

Then (X ;*, 0)is an AT − algebra.

Proposition 2.5 [3]. In any AT − algebra (X ;*, 0), the following properties holds: for all x, y, z ∈X;

a) z * z = 0,
b) z*(x *z) = 0 ,
c) y * ((y* z) * z) = 0,
d) x * y = 0 implies that x * 0 = y * 0 ,
e) 0*x=0*y implies that x=y .

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Proposition 2.6[3]. In any AT – algebra (X ;*, 0), the following properties holds: for all x, y, z $\in$X;

a) $x \le y$ implies that $y * z \le x * z$,

b) $x \le y$ implies that $z * x \le z * y$

c) $z * x \le z * y$ implies that $x \le y$(left cancellation law).

Proposition 2.7[3]. In any AT – algebra (X ;*, 0), the following properties holds: for all x, y,z $\in$X;

a) $x = 0 * (0 * x)$,

b) $x * y \le z$ imply $z * y \le x$.

Definition 2.8[3]. A nonempty subset S of an AT – algebra X is called **an AT – subalgebra of AT – algebra X** if $x*y \in S$, whenever x, y$\in$S.

Definition 2.9[3]. A nonempty subset I of an AT – algebra X is called **an AT-ideal of AT-algebra X** if it satisfies the following conditions: for all x, y, z $\in$ X .

AT_1) $0 \in I$;

AT_2) $x * (y * z) \in I$ and $y \in I$ imply $x * z \in I$.

Proposition 2.10[3]. Every AT – ideal of AT – algebra X is an AT – subalgebra.

Definition 2.11[3]. Let X be an AT – algebra.

A fuzzy set μ in X is called a fuzzy AT – subalgebra of X if it satisfies the following conditions: for all x, y $\in$ X,

$\mu(x * y) \ge \min\{\mu(x), \mu(y)\}$.

Definition 2.12[3]. Let X be an AT – algebra.

A fuzzy set μ in X is called a fuzzy AT – ideal of X if it satisfies the following conditions: for all x, y and z $\in$ X,

(AT_1) $\mu(0) \ge \mu(x)$.

(AT_2) $\mu(x * z) \ge \min\{\mu(x*(y*z)), \mu(y)\}$.

Proposition 2.13[3]. Every fuzzy AT – ideal of AT – algebra X is fuzzy AT – subalgebra.

3. Anti-fuzzy AT-ideals of AT-algebras

In this section, we will introduce a new notion called an anti – fuzzy AT – ideal of AT – algebra and study several basic properties of it.

Definition 3.1[13]. Let X be a nonempty set, a fuzzy set μ in X is a function $\mu : X \to [0, 1]$.

Definition 3.2. Let X be an AT – algebra. A fuzzy set μ in X is called an anti- fuzzy AT – ideal of X if it satisfies the following conditions: for all x, y and z $\in$ X,

(AAT_1) $\mu(0) \le \mu(x)$.

(AAT_2) $\mu(x * z) \le \max\{\mu(x*(y*z)), \mu(y)\}$.

Example 3.3. Let X = {0, 1, 2, 3} be a set with the following table:

*	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	0	0	3
3	0	0	0	0

Then (X ;*, 0) is an AT – algebra. It is easy to show that $I_1 = \{0, 1\}$and $I_2 = \{0, 3\}$ are AT-ideals of X .

Define a fuzzy set $\mu : X \to [0, 1]$ by$\mu(0) = t_1, \mu(1) = \mu(2) = \mu(3) = t_2$,where t_1, $t_2 \in [0, 1]$ with $t_1 < t_2$.

Routine calculation gives that μ is an anti- fuzzy AT – ideal of AT – algebras X.

Lemma 3.4. Let μ be an anti- fuzzy AT – ideal of AT – algebra X and if $x \le y$, then $\mu(y) \le \mu(x)$, for all x, y$\in$ X.

Proof: Assume that $x \le y$, then $y * x = 0$, and $\mu(0 * y) = \mu(y) \le \max\{\mu(0 * (x*y)), \mu(x)\} = \max\{\mu(0), \mu(x)\} = \mu(x)$.

Hence $\mu(y) \le \mu(x)$. ⌂

Proposition 3.5. Let μ be an anti- fuzzy AT – ideal of AT – algebra X. If the inequality $y*x\leq z$ hold in X, then $\mu(x) \leq \max\{\mu(y),\mu(z)\}$.

Proof: Assume that the inequality $y*x\leq z$ hold in X, by lemma(3.4),

$\mu(z) \leq \mu(y*x)$ --- (1).

By(AAT_2),$\mu(z*x) \leq \max\{\mu(z*(y*x)), \mu(y)\}$.Put z=0,then

$\mu(0*x) = \mu(x) \leq \max\{\mu(0*(y*x)), \mu(y)\} = \max\{\mu(y*x),\mu(y)\}$--- (2) .

From (1) and (2), we get $\mu(x) \leq \max\{\mu(y), \mu(z)\}$, for all x, y, z $\in$X. ⌂

Theorem 3.6. Let μ be an anti-fuzzy set in X then μ is an anti – fuzzy AT – ideal of X if and only if, it satisfies:

For all $\alpha\in[0, 1]$, $U(\mu,\alpha) \neq\emptyset$ implies $U(\mu,\alpha)$ is an AT – ideal of X----(A),

where $U(\mu,\alpha) = \{x\in X|\mu(x) \leq\alpha\}$.

Proof: Assume that μ is an anti – fuzzy AT – ideal of X, let $\alpha\in[0,1]$ be such that $U(\mu,\alpha) \neq\emptyset$, and let x, y $\in$ X be such that $x \in U(\mu,\alpha)$, then $\mu(x) \leq\alpha$and so by (AAT_1),$\mu(0) \leq\mu(x)\leq\alpha$. Thus $0 \in U(\mu,\alpha)$.

Now let $(z*(y*x)),y\in U(\mu,\alpha)$. It follows from ($AAT_2$) that

$\mu(z*x) \leq \max\{\mu(z*(y*x)),\mu(y)\}= \alpha$, so that $(z*x)\in U(\mu,\alpha)$. Hence $U(\mu,\alpha)$ is an AT – ideal of X.

Conversely, suppose that μ satisfies (A), assume that (AAT_1) is false, then there exist $x \in X$ such that $\mu(0) >\mu(x)$.If we take $t = \frac{1}{2}(\mu(x)+\mu(0))$, then $\mu(0) > t$ and $0 \leq\mu(x) < t \leq 1$, thus$x \in U(\mu,t)$ and $U(\mu,t) \neq\emptyset$.As $U(\mu,t)$ is an AT – ideal of X, we have $0 \in U(\mu,t)$, and so $\mu(0) \leq t$.This is a contradiction.

Hence $\mu(0) \leq\mu(x)$ for all $x \in X$. Now, assume (AAT_2) is not true thenthere exist x, y, z $\in$Xsuch that

$\mu(z*x) > \max\{\mu(z*(y*x)),\mu(y)\}$,

taking $\beta_0 = \frac{1}{2}[\mu(z*x) + \max\{\mu(z*(y*x)),\mu(y)\}]$,

we have $\beta_0\in[0, 1]$ and

$\max\{\mu(z*(y*x)),\mu(y)\}, <\beta_0<\mu(z*x)$, it follows that

$\max\{\mu(z*(y*x)),\mu(y)\}, \in U(\mu,\beta_0)$and $z*y\notin U(\mu,\beta_0)$, this is a contradiction and therefore μ is an anti – fuzzy AT – ideal of X. ⌂

4. Characterization of anti-fuzzy AT-ideals by their level AT-ideals

Theorem 4.1. A fuzzy subset μ of an AT – algebra X is an anti – fuzzy AT – ideal of X if and only if, for every $t\in[0,1]$, μ_t is an AT – ideal of X, where

$\mu_t = \{x\in X|\mu(x)\leq t\}$.

Proof: Assume that μ is an anti – fuzzy AT – ideal of X, by (AAT_1), we have

$\mu(0) \leq\mu(x)$ for all $x \in X$, therefore $\mu(0) \leq\mu(x) \leq t$, for $x \in \mu_t$ and so $0 \in \mu_t$.

Let $(z*(y*x)) \in \mu_t$ and $(y)\in \mu_t$, then $\mu(z*(y*x))\leq t$ and $\mu(y) \leq t$,since μ is an anti – fuzzy AT – ideal it follows that $\mu(z*x) \leq\{\mu(z*(y*x)),\mu(y)\}\leq t$ and that

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$(z*x) \in \mu_t$ .Hence μ_t is an AT − ideal of X.

Conversely, we only need to show that (AAT_1) and (AAT_2) are true. If (AAT_1) is false, then there exist $x \in X$such that $\mu(0) > \mu(x)$. If we take $t = \frac{1}{2}(\mu(x) + \mu(0))$, then $\mu(0) > t$ and $0 \leq \mu(x) < t \leq 1$ thus $x \in \mu_t$ and $\mu_t \neq \emptyset$. As μ_t is an AT − ideal of X, we have $0 \in \mu_t$ and so $\mu(0) \leq t$. This is a contradiction.

Now,assume (AAT_2) is not true, then there exist x, y and $z \in X$such that,

$\mu(z*x) > \max\{\mu(z*(y*x)),\mu(y)\}$.

Putting $t = \frac{1}{2}[\mu(z*x) + \max\{\mu(z*(y*x)), \mu(y)\}]$,

then $\mu(z*x) > t$ and

$0 \leq \max\{\mu(z*(y*x)),\mu(y)\}, < t \leq 1$, hence $\mu(z*(y*x)) < t$ and $\mu(y) < t$, which imply that $(z*y) \in \mu_t$ and$(y*x) \in \mu_t$, since μ_t is an anti − fuzzy AT − ideal, it follows that $(z*x) \in \mu_t$ and that $\mu(z*x) \leq t$, this is also a contradiction. Hence μ is an anti − fuzzy AT − ideal of X . ⌂

Corollary 4.2. If a fuzzy subset μ of AT − algebra X is an anti − fuzzy AT − ideal, then for every $t \in Im(\mu)$, μ_t is an AT − ideal of X.

Definition 4.3. Letμ be an anti − fuzzy AT − ideal of AT − algebra X, then the AT − ideal μ_t , $t \in [0,1]$ are called level AT − ideals of μ.

Corollary 4.4. Let I be an AT − ideal of an AT − algebra X, then for any fixed number t in an open interval (0,1), there existan anti − fuzzy AT − ideal μ of X such that $\mu_t = I$.

Proof: Define $\mu : X \to [0:1]$ by$\mu(x) =$

$$\begin{cases} 0, & \text{if } x \in I; \\ t, & \text{if } x \notin I. \end{cases}$$

Where t is a fixed number in (0,1). Clearly, $\mu(0) \leq \mu(x)$ and we have one two level sets $\mu_0 = I$, $\mu_t = X$, which are AT − ideals of X, then from Theorem (4.1)μ is an anti − fuzzy AT − ideal of X. ⌂

5. Image and Pre-image of anti-fuzzy AT-ideals

Definition 5.1. $f : (X;*,0) \to (Y;*`,0`)$be a mapping from a nonempty set X to a nonemptyset Y. If β is a fuzzy subset of X, then the fuzzy subset μ of Y defined by: $f(\mu)(y) = \beta(y) =$

$$\begin{cases} \inf_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) = \{x \in X, f(x) = y\} \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

is said to be the image of μ under f .

Similarly if μ is a fuzzy subset of Y, then the fuzzy subset $\mu = (\beta o f)$ in X (i.e., the fuzzy subset defined by $\mu(x) = \beta(f(x))$, for all $x \in X$) is called the

pre-image of β under f .

Theorem 5.2. An into homomorphic pre-image of anti − fuzzy AT − ideal is also an anti − fuzzy AT − ideal.

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Proof: Let $f:(X;*,0) \to (Y;*`,0`)$be an onto homomorphism of AT – algebras, β is an anti – fuzzy AT – ideal of Y and μ the pre-image of β under f , then

$\beta(f(x)) = \mu(x)$, for all $x \in X$. Let $x \in X$, then $\mu(0) = \beta(f(0)) < \beta(f(x)) = \mu(x)$.

Now let $x, y, z \in X$, then $\mu(z*x) = \beta(f(z*x)) = \beta(f(z)*' f(x))$

$\leq \max\{\beta(f(z)*' f(y)), \beta(f(y)*' f(x))\}$

$= \max\{\beta(f(z*y)), \beta(f(y*x))\}$

$= \max\{\mu(z*y), \mu(y*x)\}$,and the proof is completed. ⌂

Definition 5.3. An anti fuzzy subset μ of X has inf property if for any subset T of X, there exist $t_0 \in T$ such that $\mu(t) = \inf_{t\in T} \mu(t)$.

Theorem 5.4. Let $f:(X;*,0) \to (Y;*`,0`)$be a homomorphism between AT – algebras X and Y respectively . For every anti – fuzzy AT – ideal μ in X, $f(\mu)$ is an anti – fuzzy AT – ideal of Y.

Proof: By definition $\beta(y') = f(\mu)(y') = \inf_{x\in f^{-1}(y')} \mu(x)$, for all $y' \in Y$ and $\varnothing = 0$.

We have to prove that $\beta(z'*x') \leq \max\{\beta(z'*(y'*x')), \beta(y')\}$, for all x', y', z'∈ Y.

Let $f : X \to Y$ be an onto homomorphism of AT – algebras, μ is an anti – fuzzy AT – ideal of X with inf property and β the image of μ under f , since μ is anti – fuzzy AT – ideal of X, we have $\mu(0) \leq \mu(x)$ for all $x \in X$.

Note that $0 \in f^{-1}(0')$, where 0, 0'are the zero of X and Y, respectively.

Thus $\beta(0') = \inf_{t\in f^{-1}(x')} \mu(t) = \beta(x')$, for all $x \in X$, which implies that

$\beta(0') \leq \inf_{t\in f^{-1}(x')} \mu(t) = \beta(x')$, for any $x' \in Y$.For any x', y', z'∈ Y, let $x_0 \in f^{-1}(x')$, $y_0 \in f^{-1}(y')$, $z_0 \in f^{-1}(z')$ be such that

$\mu(z_0*(y_0*x_0)) = \inf_{t\in f^{-1}(z'*(y'*x'))} \mu(t)$, $\mu(y_0) = \inf_{t\in f^{-1}(y')} \mu(t)$ and

$\mu(z_0*x_0) = \inf_{t\in f^{-1}(z'*x')} \mu(t)$. Then

$\beta(z'*x') = \inf_{t\in f^{-1}(z'*x')} \mu(t) = \mu(z_0*x_0)$

$\leq \max\{\mu(z_0*(y_0*x_0)), \mu(y_0)\}$

$= \max[\inf_{t\in f^{-1}(z'*(y'*x'))} \mu(t), \inf_{t\in f^{-1}(y')} \mu(t)]$

$= \max\{\beta(z'*(y'*x')), \beta(y')\}$.

Hence β is an anti – fuzzy AT – ideal of Y. ⌂

6. Cartesian product of anti-fuzzy AT-ideals

Definition 6.1 ([1],[9]). A fuzzy relation R on any set S is a fuzzy subset

$R: S\times S \to [0,1]$.

Definition 6.2 ([1]). If R is a fuzzy relation on sets S and β is a fuzzy subset of S , then Ris a fuzzy relation on β if $R(x,y) \geq \max\{\beta(x), \beta(y)\}$, for all $x, y \in S$.

Definition 6.3([1]). Let μ and β be fuzzy subsets of a set S . The Cartesian product of μ and β is defined by$(\mu\times\beta)(x,y) = \max\{\mu(x), \beta(y)\}$, , for all $x, y \in S$.

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Lemma 6.4([1]). Let S be a set and μ and β be fuzzy subsets of S . Then,

(1) $\mu\times\beta$ is a fuzzy relation on S ,

(2) $(\mu\times\beta)_t = \mu_t \times\beta_t$, for all t ∈[0,1].

Definition 6.5([1]). Let S be a set and β be fuzzy subset of S . The strongest fuzzy relation on S , that is, a fuzzy relation on β is R_β given by

R_β (x,y) = max {β(x) , β(y)},for all x, y∈S .

Lemma 6.6([1]). For a given fuzzy subset β of a set S, let R_β be the strongest fuzzy relation on S. Then for t ∈[0,1], we have$(R_\beta)_t = \beta_t\times\beta_t$.

Proposition 6.7. For a given fuzzy subset β of an AT – algebra X , let R_β be the strongest fuzzy relation on X . If β is an anti – fuzzy AT – ideal of $X\times X$, then R_β (x,x) ≥ R_β (0,0) , for all x∈X .

Proof: Since R_β is a strongest fuzzy relation of $X\times X$, it follows from that,

R_β (x,x) = max{β(x), β(x)}≥ max{β(0), β(0)}= R_β (0,0),which implies that

R_β (x,x) ≥ R_β (0,0).⌂

Proposition 6.8. For a given fuzzy subset β of an AT – algebra X , let R_β be the strongest fuzzy relation on X . If R_β is an anti – fuzzy AT – ideal of $X\times X$, then

β(0) ≤ β(x) , for all x∈X .

Proof: Since R_β is an anti – fuzzy AT – ideal of $X\times X$, it follows from (AAT_1),

R_β (x,x) ≥ R_β (0,0),where (0,0) is the zero element of $X\times X$. But this means that , max{β(x), β(x)} ≥ max{β(0), β(0)}which implies that β(0) ≤ β(x) . ⌂

Remark 6.9([9]). Let X and Y be AT – algebra*s*, we define (*) on $X\times Y$ by: for all(x,y),(u,v) ∈ $X\times Y$, (x,y) ∗ (u,v) = (x ∗ u,y ∗ v). Then clearly ($X\times Y$; ∗ .(0,0)) is anAT-algebra.

Theorem 6.10. Let μ and β be an anti – fuzzy AT – ideal of AT – algebra X . Then $\mu\times\beta$ is an anti – fuzzy AT – ideal of $X\times X$.

Proof: Note first that for every (x,y)∈ $X\times X$,

$(\mu\times\beta)$(0,0) = max {μ(0) , β(0)} ≤ max {μ(x) , β(y)} = $(\mu\times\beta)$(x,y) .

Now let (x_1,x_2) , (y_1,y_2) , (z_1,z_2) ∈ $X\times X$. Then

$(\mu\times\beta)(x_1 * z_1, x_2 * z_2)$ = max { $\mu(x_1 * z_1)$,$\beta(x_2 * z_2)$}

≤ max { max{$\mu(x_1 * (y_1 * z_1))$,$\mu(y_1)$}, max{$\beta(x_2 * (y_2 * z_2))$,$\beta(y_2)$}}

= max { max{$\mu((x_1 * (y_1 * z_1)))$, $\beta(x_2 * (y_2 * z_2))$}, max{ $\mu(y_1)$,$\beta(y_2)$}}

= max { $(\mu\times\beta)((x_1 * (y_1 * z_1)) ,(x_2 * (y_2 * z_2)))$, $(\mu\times\beta)(y_1,y_2)$}

Hence $(\mu\times\beta)$ is an anti – fuzzy AT – ideal of $X\times X$.⌂

Theorem 6.11. Let μ and β be anti-fuzzy subsets of AT – algebra X such that $\mu\times\beta$ is an anti – fuzzy AT – ideal of $X\times X$. Then for all x∈X,

(i) either μ (0) ≤ μ (x) or β (0) ≤ β (x) .

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(ii) $\mu(0) \leq \mu(x)$ for all $x \in X$, then either $\beta(0) \leq \beta(x)$ or $\beta(0) \leq \mu(x)$.

(iii) If $\beta(0) \leq \beta(x)$ for all $x \in X$, then either $\mu(0) \leq \mu(x)$ or $\mu(0) \leq \beta(x)$.

(iv) Either μ or β is an anti – fuzzy AT – ideal of X .

Proof.

(i) Suppose that $\mu(0)>\mu(x)$ and $\beta(0)>\beta(y)$ for some x , y $\in$X . Then

$(\mu\times\beta)(x,y) = \max\{\mu(x), \beta(y)\} < \max\{\mu(0), \beta(0)\} = (\mu\times\beta)(0,0)$. This is a contradiction and we obtain (i).

(ii) Assume that there exist x , y $\in$X such that $\beta(0) > \mu(x)$ and $\beta(0) > \beta(y)$. Then

$(\mu\times\beta)(0,0) = \max\{\mu(0),\beta(0)\} = \beta(0)$ it follows that

$(\mu\times\beta)(x,y) = \max\{\mu(x), \beta(y)\} < \beta(0) = (\mu\times\beta)(0,0)$ which is a contradiction. Hence (ii) holds.

(iii) Is by similar method to part (ii).

(iv) Suppose $\beta(0) \leq \beta(x)$ by (i), then form (iii) either $\mu(0) \leq \mu(x)$ or $\mu(0) \leq \beta(x)$ for all $x \in X$.

If $\mu(0) \leq \beta(x)$, for any $x \in X$, then $(\mu\times\beta)(0,x) = \max\{\mu(0), \beta(x)\} = \beta(x)$. Let (x_1,x_2), (y_1,y_2), $(z_1,z_2) \in X\times X$, since $(\mu\times\beta)$ is an anti-fuzzy AT-ideal of $X\times X$, we have

$(\mu\times\beta)(x_1*z_1, x_2*z_2) \leq \max\{(\mu\times\beta)((x_1*(y_1*z_1)),(x_2*(y_2*z_2))), (\mu\times\beta)(y_1,y_2)\}$---- (A)

If we take $x_1 = y_1 = z_1 = 0$, then

$\beta(x_2*z_2) = (\mu\times\beta)(0,x_2*z_2)$

$\leq \max\{(\mu\times\beta)(0,(x_2*(y_2*z_2))), (\mu\times\beta)(0,y_2)\}$

$= \max\{\max\{\mu(0), \beta((x_2*(y_2*z_2)))\}, \max\{\mu(0), \beta(y_2)\}\}$

$= \max\{\beta((x_2*(y_2*z_2))), \beta(y_2)\}$

This prove that β is an anti – fuzzy AT – ideal of X .

Now we consider the case $\mu(0) \leq \mu(x)$ for all $x \in X$.

Suppose that $\mu(0) > \mu(y)$ for some $y \in X$. then $\beta(0) \leq \beta(y) < \mu(0)$.

Since $\mu(0) \leq \mu(x)$ for all $x \in X$, it follows that $\beta(0) < \mu(x)$ for any $x \in X$.

Hence $(\mu\times\beta)(x,0) = \max\{\mu(x), \beta(0)\} = \mu(x)$

taking $x_2 = y_2 = z_2 = 0$ in (A), then

$\mu(x_1*z_1) = (\mu\times\beta)(x_1*z_1, 0)$

$\leq \max\{(\mu\times\beta)((x_1*(y_1*z_1)),0), (\mu\times\beta)(y_1,0)\}$

$= \max\{\max\{\mu(x_1*(y_1*z_1)), \beta(0)\}, \max\{\mu(y_1), \beta(0)\}\}$

$= \max\{\mu(x_1*(y_1*z_1)), \mu(y_1)\}$

Which proves that μ is an anti – fuzzy AT – ideal of X . Hence either μ or β is an anti – fuzzy AT – ideal of X . ⌂

Theorem 6.12. Let β be a fuzzy subset of an AT – algebra X and let R_β be the strongest fuzzy relation on X , then β is an anti – fuzzy AT – ideal of X if and only if R_β is an anti – fuzzy AT – ideal of X×X .

Proof: Assume that β is an anti – fuzzy AT – ideal of X . By proposition (6.7) ,we get, $R_\beta(0,0) \leq R_\beta(x,y)$, for any $(x,y) \in X\times X$.

Let (x_1,x_2), (y_1,y_2), $(z_1,z_2) \in X\times X$, we have from (AAT_2) :

$R_\beta(z_1 * x_1, z_2 * x_2) = \max\{\beta(z_1 * x_1), \beta(z_2 * x_2)\}$

$\leq \max\{\max\{\beta(z_1 * (y_1 * x_1)), \beta(y_1)\}, \max\{\beta(z_2 * (y_2 * x_2)), \beta(y_2)\}\}$

$= \max\{\max\{\beta(z_1 * (y_1 * x_1)), \beta(z_2 * (y_2 * x_2))\}, \max\{\beta(y_1), \beta(y_2)\}\}$

$= \max\{R_\beta((z_2 * (y_2 * x_2)), (z_2 * (y_2 * x_2))), R_\beta(y_1, y_2)\}$

Hence R_β is an anti – fuzzy AT – ideal of $X \times X$.

Conversely, suppose that R_β is an anti – fuzzy AT – ideal of $X \times X$, by proposition (6.8) $\beta(0) \leq \beta(x)$ for all $x \in X$, which prove (AAT_1).

Now, let $(x_1,x_2), (y_1,y_2), (z_1,z_2) \in X \times X$. Then,

$\max\{\beta(z_1 * x_1), \beta(z_2 * x_2)\} = R_\beta(z_1 * x_1, z_2 * x_2)$

$\leq \max\{R_\beta((z_1,z_2) * ((y_1, y_2) * (x_1, x_2)), R_\beta(y_1,y_2)\}$

$= \max\{R_\beta((z_1 * (y_1 * x_1)), (z_2 * (y_2 * x_2))), R_\beta(y_1,y_2)\}$

$= \max\{\max\{\beta((z_1 * (y_1 * x_1)), \beta(z_2 * (y_2 * x_2))\}, \max\{\beta(y_1), \beta(y_2)\}\}$

In particular if we take $x_2 = y_2 = z_2 = 0$, then $\beta(z_1 * x_1) \leq \max\{\beta(z_1 * (y_1 * x_1)), \beta(y_1)\}$. This proves (AAT_2) and β is an anti – fuzzy AT – ideal of X . ⌂

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المثاليات الضبابية المضاد (AT) في الجبريات (AT)

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المستخلص :

في هذا البحث ، نقدم مفهوم المثاليات الضبابية المضاد "AT " في AT-الجبر ، وفيه يتم تقديم العديد من الأمثلة المناسبة ويتم التحقيق في خصائصها. يتم تعريف الصورة والصورة المعكوسة للمثالي الضبابي المضاد "AT " في الجبر -AT وكيف يتم دراسة خواص الصورة والصورة العكسيّة للمثاليات الضبابية المضادة AT في -ATالجبر. علاوة على ذلك ، يتم إعطاء الضرب الديكارتي للمثالي الضبابي المضاد AT.