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The Effects of Atmospheric Drag and Zonal Harmonic on LEO Satellite Orbits

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Abstract

The effects of atmosphere drag and zonal harmonics J2 of the gravitational potential of the Earth on low earth orbit satellite have been investigated .Computer simulation of the equation of motion with perturbations using step-by-step integration (Cowell's method) is designed by mathlab a 7.4. position, velocity and range of the satellite with respect to tracking ground station proposed in Kufa holly city .

الخلاصة

تم في هذا البحث دراسة التأثيرات التوافقية الكروية الموقعية لجهد الجاذبية الأرضية J2 وقوة كبح الغلاف الغازي علـى حركـة الأقمار الاصطناعية واطئة المدار. حيث تم محاكاة معادلات الحركة مع الاضطرابات المدارية بالحاسـوب باسـتخدام Matlab a 7.4 وتكاملها خطوة - خطوة (Cowell's method) ، ومن ثم حساب موقع وسرعة ومدى القمر الاصطناعي نسبة إلى محطة تتبع أرضية مفترضة في مدينة الكوفة المقدسة .

1. Introduction

There are several sources of perturbations affecting satellite orbital motion from injection point until the end of it's lifetime. In general orbit perturbations can be divided into gravitational and non-gravitational. The gravitational are those due to oblateness of the Earth, the zonal, tesseral, and sectorial spherical harmonics and effect of sun/moon attraction.The non-gravitational perturbations include atmospheric drag force (the dominant for low earth orbits), solar radiation pressure(effective for geosynchronous satellites), magnetic forces (due to the interaction of the earth magnetic field with the dipole moment induced in the satellite), etc. The gravitational potential of the nonspherical earth models was initiated by [Kozai, 1959],short period and long period perturbations. Details of the gravitational potential theory and atmospheric drag force effect can found in [Chobotov, 1996]. Numerical simulation of the equation of orbital motion was performed by [Metris *etal* ,1995].

In this paper, numerical simulation of the equation of motion of two body problem under the combined effects of atmospheric drag force and J2 spherical harmonics of the oblateness of the gravitational potential have been performed using Matlab a 7.4. The atmospheric density was calculated as exponential function of the satellite's altitude from the earth surface and scale height as a function of solar activity indices F10.7 and Ap. The lifetime of low earth orbit satellite was evaluated by calculation of orbit decay.

In section 2, the orbital dynamic model of the satellite is presented under the perturbation forces. The atmospheric drag force and atmospheric density in section 3. The J2 spherical harmonic of the oblate Earth is shown in section 4. The simulation results, discussion and conclusion in section 5.

2. Orbital dynamic model

Motion of a body or object in space is an integral part of the preliminary orbit determination process .The Stark problem represents the motion of a test particle ,about a fixed Newtonian force center (i.e., the Kepler problem) subject in addition to a uniform force of constant magnitude and direction[Kozai ,1959].The equations of motion of two-body problem can be written in the form [Escopal ,1984 and Robert , 2003]

$$\frac{d^2\vec{r}}{dt^2} = -\frac{u}{r^3}\vec{r}..............................(1)$$

Where r is the position vector of the satellite from earth , and µ is the earth gravitational constant (38601.2 km^3/sec^2),

Eq.(1) can be written in cartezian coordinates as follows

$$\ddot{x} = -\frac{u}{r^3}x$$
$$\ddot{y} = -\frac{u}{r^3}y..............................(2)$$
$$\ddot{z} = -\frac{u}{r^3}z$$

3. Atmospheric Drag force and density model

Orbital perturbations in general may be defined as the external torques acting about the center of mass of the satellite such that the orbital elements will be deviated from their mean values. These torques arise from space environment . Orbital perturbations can be classified according to the geometry and mass distribution of satellite into two broad categories ,gravitational and non- gravitational. The gravitational perturbations depend on the masses of the attracting bodies like the gravitational field between earth and attraction of the sun and moon while ,the non- gravitational ones depend on the geometry of the satellite surfaces such that atmospheric drag[Farah ,2000].In this paper the atmospheric drag force and it's impact on position and velocity of satellite and zonal harmonic and effect at gravitational potential.

3.1 Atmospheric drag force

Drag is the resistance offered by atmosphere to the satellite . Drag force acts in a direction opposite to the direction of its motion This drag is greatest during launch and reentry, however, even a satellite in low Earth orbit experiences some drag as it moves through the Earth's thin upper atmosphere. In time, the action of drag on a satellite will cause it to spiral back into the atmosphere, eventually to disintegrate or burn up. If a space vehicle comes within 120 to 160 km of the Earth's surface, atmospheric drag will bring it down in a few days, with final disintegration occurring at an altitude of about 80 km..The drag force F_D is given by[Frank ,2006 and John ,1999]

$$F_D = -\frac{1}{2}C_D \rho V^2 S_{Ref} ....(3)$$

where C_D is the drag coefficient, ρ is the air density, V is the flow velocity of atmospheric particles , and S_{Ref} is the cross sectional area of the body normal to the flow. The drag coefficient depends on the geometric form of the body and is generally determined by experiment ,and the reduction in the period due to atmospheric drag is given by[John, 1999]

$$\frac{dP}{dt} = -3\pi a\rho(\frac{C_D S_{Ref}}{m})...........(4)$$

Where a is the semi major axis the satellite

3.2 Atmospheric density model

The upper atmospheric models, the thermosphere and exosphere, some times called thermospheric drag models that depends on solar and geomagnetic activities are : J77 ,MSIS [Frank , 2006 and Zarrouati , 1987] , DTM78 and DTM94 models [Rowa , 2002].
In this paper the density of the upper atmosphere is expressed as exponential function of altitude given by [John , 1999]

$$\rho = \rho o \exp\left[-\frac{(H - 175)}{SH}\right] \text{..........................(5)}$$

Where ρo is the atmospheric density at the initial perigee point and SH is the scale height give by[John , 1999]

SH=(900+2.5*(F10.7-70))/(27-0.01*Ap+1.5*(H-200))….(6)

Where F10.7 is the solar flux index, Ap is the index of solar activity

4.The gravitational perturbation model

The oblateness of the gravitational potential of the earth consists of three part , the zonal harmonics (J2-J6) , tesseral and sectorial .

The zonal spherical harmonics model of the gravitational potential adopted in this work is given by [Petter , 2002,Sofie , 2006 and Mikkjal , 2003]

$$U = \frac{u}{r}\left[1 - \sum_{n=2}^{\infty}\left(\frac{a_e}{r}\right)^n J_2 P_n \sin\varphi\right] \text{.................(7)}$$

where zonal harmonic coefficient are given by [Robert,2003 and Sofie , 2006] $J_2 = (1.083 \pm 0.3)\times 10^{-6}$

This method is a straightforward step-by-step integration equation of the two body equation of motion with perturbation .The equation of motion may be given[Chobotov, 1996]

$$\ddot{\vec{r}} + \frac{u}{r^3}\vec{r} = \vec{a}_p \text{.........................(8)}$$

Which ,for numerical integration, would be reduced to first-order differential equations

$$\dot{\vec{r}} = \vec{v}$$

$$\dot{\vec{v}} = -\frac{u}{r^3}\vec{r} + \vec{a}_p \text{.........................(9)}$$

Where $\vec{a}_p$ is the vector sum of all the perturbing accelerations to be included in the integration. The equations of motion are first reduced into first-order differential equations as shown in the following matrix [Farah , 2000]

$$\frac{dy}{dt}=\begin{bmatrix} y(2) \\ -\frac{u}{r}y(1)+Fdrag+\frac{u}{r}\left(\frac{a_e}{r^2}\right)^2 J2.(3/2-y(6)/r\wedge 3) \\ y(4) \\ -\frac{u}{r}y(3)+Fdrag+\frac{u}{r}\left(\frac{a_e}{r^2}\right)^2 J2\left(5/2-y(6)^2/r^3\right) \\ y(6) \\ -\frac{u}{r}y(5) \end{bmatrix} \quad ....(10)$$

The radial distance and orbital velocity are given by

$$r=\sqrt{y^2(1)\ +y^2(3)+y^{2)}(5)}.............(11a)$$

$$v=\sqrt{y^2(2)+y^2(4)+y^2(6)}...............(11b)$$

5. The ground station coordinates

The coordinates of a ground tracking station proposed in the holly city of Kufa of geographic coordinates (32 27 N ,44.3 E) on oblate geod are given by [Escopal ,1984]

$$X_{gs}=-G1\cos\varphi\cos\theta$$

$$Y_{gs}=-G1\cos\varphi\sin\theta \quad ....(12)$$

$$Z_{gs}=-G2\sin\varphi$$

Where

$$G1=a_e/\sqrt{1-(2fl-fl^2)\sin^2\varphi}+h$$

$$G2=a_e(1-fl)^2/\sqrt{1-\left(2fl-fl^2\right)\sin^2\varphi}$$

The range of the satellite with respect to the ground station is given by

$$Range=\sqrt{(y(1)-X_{gs})^2+(y(3)-Y_{gs})^2+(y(5)-Z_{gs})^2} \quad ....(13)$$

6. Discussion of simulation results and conclusions:

A computer simulation has been developed to the equation of orbital motion of two body problem with perturbations due to atmospheric drag force and the J2 zonal spherical harmonics using Matlab a 7.4 .The range of the satellite with respect to ground station proposed in the holly city of Kufa (32 N, 44 E) was calculated. The lifetime of the low earth orbit satellite is calculated in days in terms of orbit decay .

The results of simulation are shown in the following figures .Fig(1) shows the geocentric radial distance variations of the satellite during initial orbit injection phase at perigee point for one orbit. The radial distance at injection point was rp=6530 km which was minimum distance. During half orbit it starts to increase gradually until apogee (the max. distance ra=6830 km. then starts to decline steadily until perigee point. The variations of orbital velocity v with time for one orbit is shown in fig(2). The velocity at injection point was 7.9 km/s which was maximum then decreases gradually to minimum value at apogee. (7..548 km/s) during half orbit then starts to increase to maximum value during the second half of the orbit. The range of the satellite with respect to the proposed ground tracking station in the holly city of Kufa is shown in fig(3). The injection point was at range 12000 km from the station then start to reduce during half orbit to minimum value at apogee (3800 km).

The effects of atmospheric drag force on the satellite orbit at altitude 500 km are shown in fig(4) and fig(5). Fig(4) shows the altitude decrease until re-entry to the dense atmosphere at 180 km , the lifetime was 1560 days. The decay of orbital period is shown in fig(5).where the initial period was 94 minutes then reduced to 88 minutes then re-entry into the dense atmosphere .

It can be concluded from these figures that , there is no pronounced effect of the J2 spherical harmonics of the gravitational potential during the early phase of injection and the dominant effect was the atmospheric drag force at perigee which causes reduction in satellite altitude until re-entry after 1560 days into the dense atmosphere.

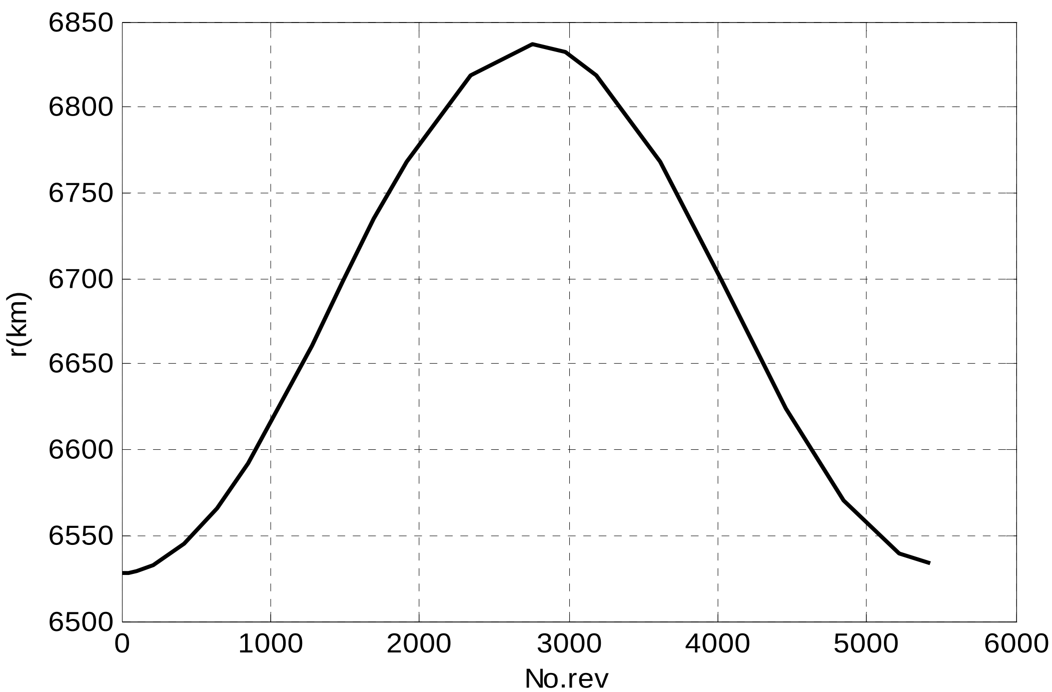

Fig.(1) The relationship between range of satellite and number of revolution it

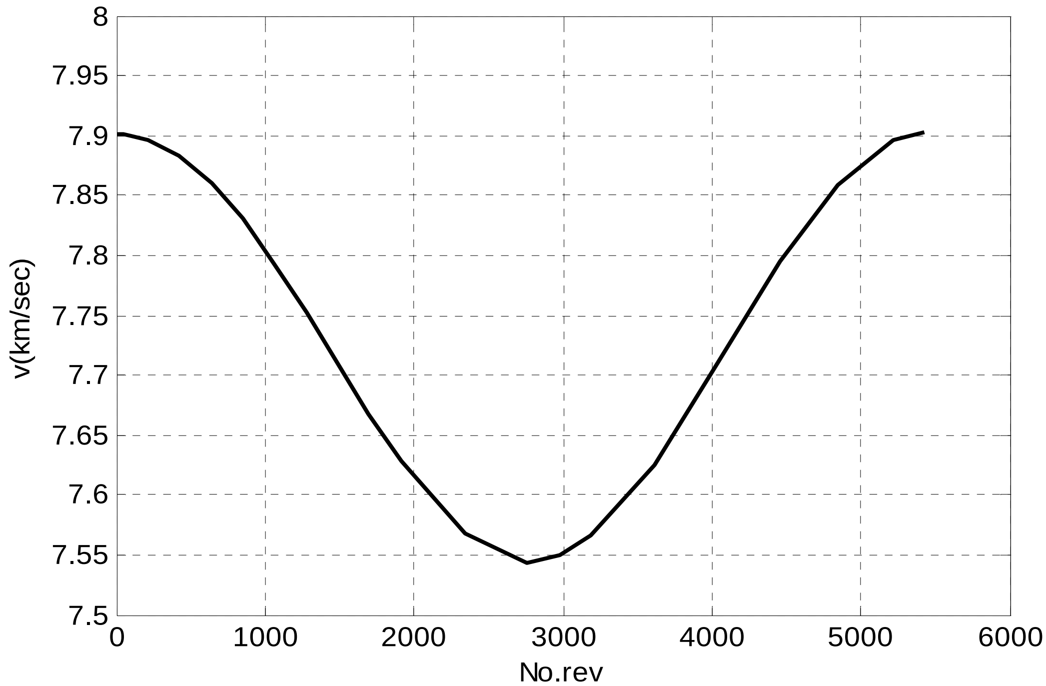

Fig.(2)The variations of orbital velocity with time for one orbit

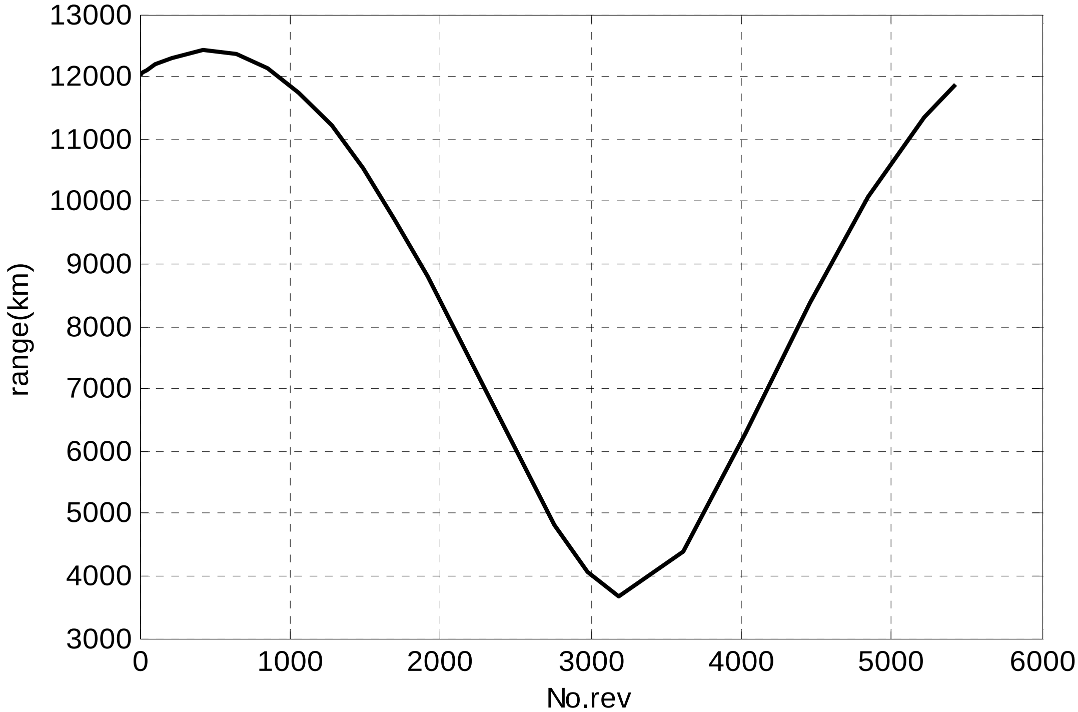

Fig.(3) The range of the satellite for one orbit

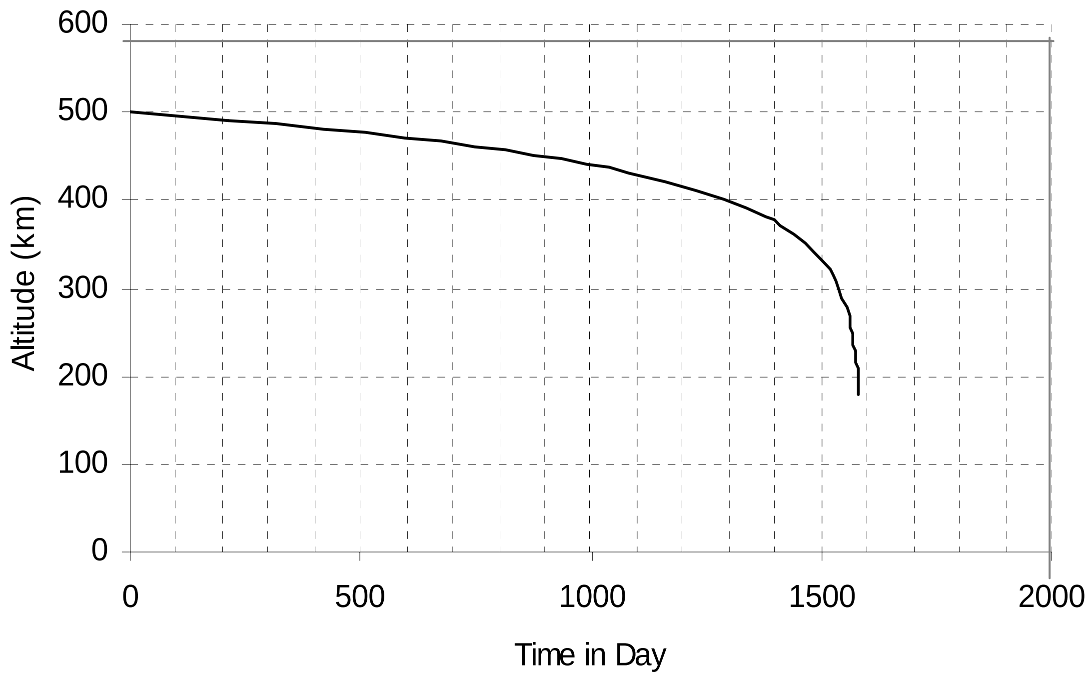

Fig.(4) The effects of atmospheric drag force on the satellite orbit at altitude

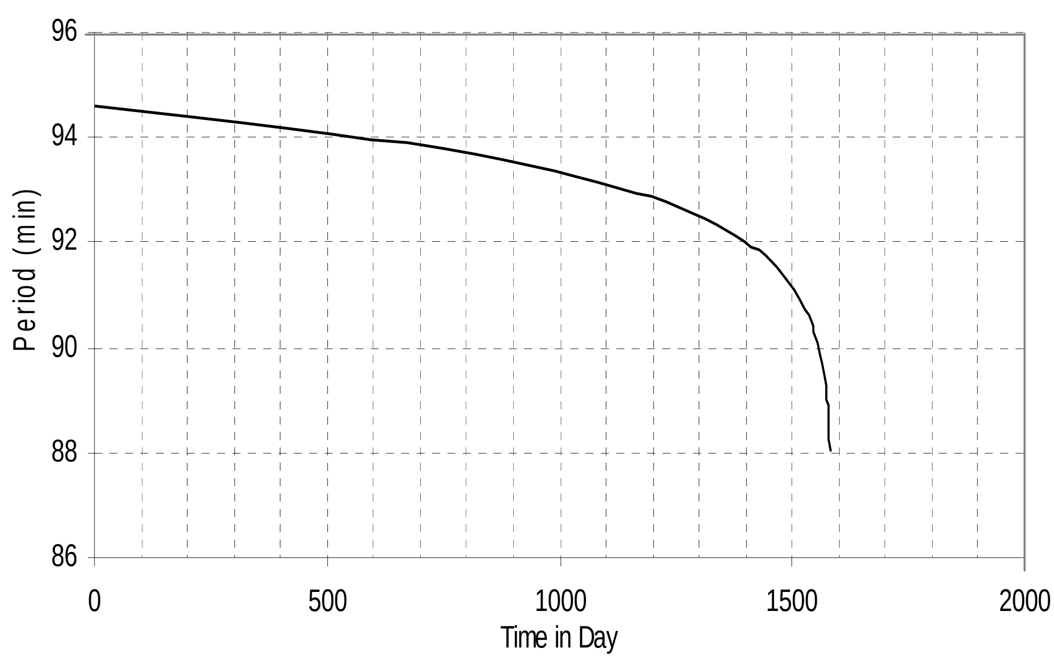

Fig.(5) . The decay of orbital period

References

Chobotov V. A.,1996,"Orbital Mechanics Second Edition", Air Force Institute of Technology, AIAA, Ohio.

Escopal P.R. 1984, "Methods of orbit determination, John-Wily and sons, INC. London.

Farah N.M., 2000, "Satellite Range Estimation Using the Extended Kalman Filter", M.Sc thesis, Al Nahrain University, College of Engineering, Satellite Engineering, Department of Electronic and Communication Engineering.

Frank A. Marcos, 2006, "New Satellite Drag Modeling Capabilities", Air Force Research Laboratory, Hanscom AFB, MA 01731-3010.

John K. ,1999 , "Satellite Orbital Decay Calculation ", IPS Radio and Space Services, The Australian Space Weather Agency,Sydney.

Kozai Y. 1959,"The Motion of Close Earth satellite", The Asronomical Journal,64,367-377.

Metris G. & Exertier, P. 1995, "Semi-Analytical Theory of Mean Orbital Motion", Astron, Astrophys, 294,278-286.

Mikkjal Gulklett ,2003, "Relativistic effects in GPS and LEO", University of Copenhagen, Department of Geophysics, The Niels Bohr Institute, Denmark.

Petter Eriksson, 2002," Orbital And Attitude Perturbation On A Small Satellite", ISSN 0280316X, www.particle.kth.se/group_docs/admin/docs/eriksson.pdfT.

Robert A.,2003, "A Global Positioning System (GPS) Primer" M.Sc thesis, Regents College, University of Colorado,USA.

Rowa M., 2002,"The Effect of Solar activity on the Orbits of Satellite of Low Altitude artificial "M.Sc. thesis, Babylon University.

Sofie L.,2006," Ørsted as a Gravity Mission Satellite", M.Sc.thesis, University of Copenhagen, Faculty of Science. Denmark.

Zarrouati O. ,1987,"Trajectory Specials",Cepadues-Edi,Center National D,Etude Spatials CNES ,France.