EVALUATION OF THE SPILLWAY OF AL-ADHEEM DAM BY RUNGE- KUTTA METHOD

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Abstract

Runge- kutta method of third order scheme is used to route the probable maximum flood (PMF) hydrograph through AL- Adheem reservoir. The maximum outflow discharge is 1126.88 m^3/s. This discharge is less than the allowable maximum outflow discharge which is 1500 m^3/s. As a result, the spillway of AL- Adheem dam need not to be gated.

1. Introduction

Evaluation of the spillway of AL- Adheem dam means two things. The first is that the maximum flood can be routed through the reservoir and overtopping failure shouldn't occur. The second is that the necessary quantity of water downstream the dam is supplied without exceeding 1500 m^3/s. AL- Adheem dam is constructed to protect Baghdad from flooding by controlling the downstream releases through it to Tigris river during the flood flow to the allowable discharge, not more than 1500 m^3/s [1, 2, 6, 7].

The outflow discharge from a dam through a spillway can be under control if the spillway is gated. The spillway of AL- Adheem dam is ungated; therefore, the outflow discharge through the spillway is not under control. If the maximum outflow discharge exceeds 1500 m^3/s, the spillway must be gated.

AL- Adheem dam was tested against overtopping failure type by AL- Gazali [4]. He used level pool routing method and he found that overtopping failure may not occur. In this research, Runge- kutta method is used to obtain the maximum outflow discharge by routing the probable maximum flood (PMF) hydrograph through AL- Adheem reservoir. Then, the maximum outflow discharge is compared with the allowable maximum outflow discharge which is 1500 m^3/s. If the maximum outflow discharge is larger than the allowable outflow discharge, the suggestion is that the spillway must be gated.

2. Flow Routing

Flow routing is of two types. The first is the reservoir routing, and the second is the river routing. The reservoir routing method is applied to a reservoir with a horizontal water surface. Such reservoirs have a pool that is wide and deep compared with its length in the direction of flow. The velocity of flow in the reservoir is very low. AL- Adheem reservoir is of a horizontal water surface type [6]; therefore, the reservoir routing method is needed to apply to obtain the outflow hydrograph.

2.1 Level Pool Routing

Level pool routing method is a procedure to calculate the outflow hydrograph from a reservoir with a horizontal water surface, given its inflow hydrograph and storage- outflow characteristics.

In this method, the variation of inflow and outflow over the time interval is approximated to be linear. To develop the storage- outflow function, elevation-storage and elevation- outflow data must be available. Clearly, if elevation- storage data is not available storage- outflow function cannot be developed. As a result, level pool method cannot be applied. When storage- outflow function cannot be developed, Runge- Kutta method, which is described in this research, can be applied.

2.2 Runge- Kutta Method

This method is an alternative method for level pool method. It does not require the computation of the storage- outflow function and is more closely related to the hydraulics of flow through the reservoir.

In this method, the continuity equation is solved numerically. Various order of Runge- Kutta schemes can be adopted [3]. A third order scheme is chosen in this research. This scheme involves breaking each time interval into three increments and calculating successive values of water surface elevation and reservoir discharge for each increment.

The continuity equation is expressed as:

$$\frac{ds}{dt} = I(t) - Q(t) + (P(t) - E(t)) * A \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where

S (t) = the storage

I (t) = the inflow hydrograph

Q (t) = the outflow hydrograph

P (t) = the precipitation onto the reservoir surface

E (t) = the evaporation from the reservoir surface

A = the water surface area at elevation H.

The quantities of evaporation from AL- Adheem reservoir are frequently higher than the quantities of precipitation onto it as shown in table (1). The last term in the continuity equation is neglected to represent the worst case and the continuity equation becomes:

$$\frac{ds}{dt} = I(t) - Q(t) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The change in volume, ds, due to a change in elevation, can be expressed as:

$$ds = A(H)\,dH \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Eq. (2) is then rewritten as:

$$\frac{dH}{dt} = \frac{I(t) - Q(H)}{A(H)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

The solution is extended forward by small increments of the independent variable, time, using known values of the dependent variable H. For a third order scheme, there are three such increments in each time interval Δt, and three successive approximations are made for the change in head elevation, dH.

$$\Delta H_1 = \frac{I(t_j) - Q(H_j)}{A(H_j)} \Delta t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$$\Delta H_2 = \frac{I(t_j + \frac{\Delta t}{3}) - Q(H_j + \frac{\Delta H_1}{3})}{A(H_j + \frac{\Delta H_1}{3})} \Delta t \quad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

$$\Delta H_3 = \frac{I(t_j + \frac{2\Delta t}{3}) - Q(H_j + \frac{2\Delta H2}{3})}{A(H_j + \frac{2\Delta H2}{3})} \Delta t \quad \ldots\ldots\ldots\ldots\ldots(7)$$

$$\Delta H = \frac{\Delta H_1}{4} + \frac{3\Delta H_3}{4} \quad \ldots\ldots(8)$$

$$H_{j+1} = H_j + \Delta H \quad \ldots\ldots(9)$$

3. Application of Runge- Kutta method on AL- Adheem Reservoir

In order to apply Runge- Kutta method on AL- Adheem reservoir, the following must be determined:

1. The water surface area versus elevation relationship.
2. The initial conditions.
3. The inflow hydrograph.
4. The elevation- outflow relationship.

Then calculations proceed.

3.1 The water surface area vs. elevation relationship

The water surface area- elevation data is available and presented in table (2). Nonlinear correlation method was used by Ali Ahmed [1] and found the following equation:

$$A = 1.624 \times 10^{-4} (Elv. - 83.2)^{3.49} + 12.69 \quad \ldots\ldots(10)$$

Where

A = water surface area at elevation (Elv.) in sq. km.

Elv. = water level in the reservoir with m asl.

The coefficient correlation of this equation is R = 0.9986.

3.2 The initial conditions

a. Inflow = outflow = 0

b. Water surface level in the reservoir = 131.5m asl which is the normal operation level.

c. The outflow from AL- Adheem dam during routing is assumed to be through the spillway. The other outlets are assumed to be blocked for any reason. This procedure is normally followed since it does not depend on any human regulation during routing.

3.3 The inflow hydrograph

The probable maximum flood (PMF) presented in table (3) is chosen to be the inflow hydrograph. This choice is made since it has the maximum peak discharge.

3.4 The elevation- outflow relationship

The outflow discharge is assumed to be only through the spillway. The spillway discharge equation is [2]:

$$Q = 29.49\, H^{1.5} \quad \ldots\ldots(11)$$

Where

Q = the discharge in m^3/s

H = The water surface elevation above the sill of the spillway in m.

3.5 Calculations

According to the available data of the PMF hydrograph, the routing interval is

$\Delta t = 6$ hrs = 21600 sec

$I(t_j)$, $I(t_j + \Delta t/3)$, and $I(t_j + 2\Delta t/3)$ are found by linear interpolation between $I(t_j)$ and $I(t_{j+1})$ as follows:

$I(t_j + \Delta t/3) = I(t_j) + (I(t_{j+1}) - I(t_j))/3$

$I(t_j + 2\Delta t/3) = I(t_j) + (I(t_{j+1}) - I(t_j)) \times 2/3$

$I(t_j) = 0$, $I(t_{j+1}) = 20\ m^3/s$

$I(t_j + \Delta t/3) = I(0 + 6/3) = I(2)$

$I(t_j + 2\Delta t/3) = I(0 + 2 \times 6/3) = I(4)$

$I(2) = 0 + (20 - 0)/3 = 6.67\ m^3/s$

$I(4) = 0 + (20 - 0) \times 2/3 = 13.33\ m^3/s$

The discharge is found by applying Eq. (11)

$Q(H_j) = Q(H_o) = Q(0) = 0$

The area is found by applying Eq. (10)

$Elv. = H_j + 131.5 = 131.5m\ asl$

$A(131.5) = 135 \times 10^6\ m^2$

Then

$\Delta H_1 = (0 - 0) \times 21600/(135 \times 10^6) = 0\ m$

$H_j + \Delta H_1/3 = 0 + 0/3 = 0\ m$

$Q(H_j + \Delta H_1/3) = Q(0) = 0\ m^3/s$

$Elv. = H_j + \Delta H_1/3 + 131.5 = 131.5\ m\ asl$

$A(131.5) = 135 \times 10^6\ m^2$

$\Delta H_2 = (6.67 - 0) \times 21600/(135 \times 10^6) = 0.0011m$

$H_j + 2\Delta H_1/3 = 0 + 2 \times 0.0011/3 = 0.0007\ m$

$Q(0.0007) = 0.0005\ m^3/s$

$Elv. = 0.0007 + 131.5 = 131.5007\ m\ asl$

$A(131.5007) = 135 \times 10^6\ m^2$

$\Delta H_3 = (13.33 - 0.0005) \times 21600/(135 \times 10^6) = 0.0021m$

$\Delta H = \Delta H_1/4 + 3\Delta H_3/4$

$= 0/4 + 3 \times 0.0021/4 = 0.0016\ m$

$H_{j+1} = H_j + \Delta H$

$= 0 + 0.0016 = 0.0016\ m$

$Q(0.0016) = 0.0019\ m^3/s$

To approximate the results to two figures, $Q \approx 0.00\ m^3/s$

The routing calculations for subsequent periods follow the same procedure and the results are presented in table (4). Graph (1) shows the inflow and outflow hydrographs.

4. Conclusion

From table (4) and graph (1), the following are concluded:

1. The maximum outflow discharge is (1126.88) m^3/s. It is less than that of the design requirement which is 1500 m^3/s. As a result, the spillway needn't to be gated.
2. The peak inflow of (12780) m^3/s at (210) hr is reduced to (1126.88) m^3/s occurring at (276) hr.
3. The range of application of Eq. (10) is from (100) m asl to (143) m asl and it is found that it is sufficient since maximum elevation is (142.84) m asl.
4. Overtopping failure shouldn't occur since the elevation of the dam crest is (146) m asl and the maximum water surface elevation is (142.84) m asl.

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Appendix 1: Graphs and Tables

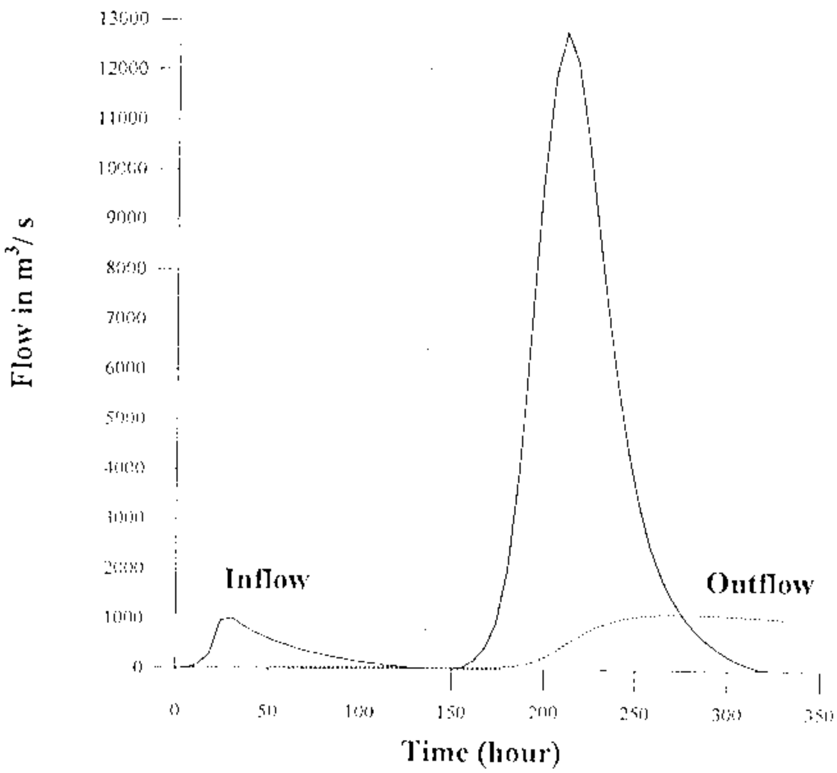

Fig. (1): Inflow and Outflow Hydrographs

Table (1): Mean Monthly Precipitation and Free- Water Surface Evaporation from Adhaim Reservoir [8].

Months	Precipitation (mm)	Evaporation (mm)
Oct.	7	145
Nov.	34	79
Dec.	48	37
Jan.	63	40
Feb.	49	61
Mar.	62	110
Apr.	39	151
May	16	236
Jun.	0	328
Jul.	0	362
Aug.	0	324
Sep.	0	227
Total	318	2100
Mean	26.5	175

Table (2): Topographic Features of Adhaim Reservoir (Based on State Commission for Survey) [2, 6].

Elevation (m above msl)	Area (sq. km.)
100.0	3
110.0	28
115.0	41
118.0	52
120.0	60
125.0	85
130.0	122
131.5	135
135.0	170
140.0	233
143.0	270

Table (3): "Probable Maximum Flood at Adhaim- Mouth" six- hour ordinate in cumecs [5]

Date	PMF	Date	PMF	Date	PMF	Date	PMF
Mar. 26	-		210		930	Apr. 6	1860
	20				3020		1420
	90	30	170		3970		1070
	290		140				800
			110	3	6740		
27	970		90		9700	7	600
	1020				11880		430
	870	31	60		12780		280
	740		40				170
			20	4	12170		
28	640		-		10720	8	90
	550				8960		20
	480	Apr. 1	-		7200		10
	400		10				-
			50	5	5620		
29	340		170		4320		
	300				3290		
	250	2	430		2450		

Table (4): The routing results of PMF hydrograph through AL-Adheem reservoir by Runge- Kutta method

Time (hr.)	Inflow (m^3/s)	Depth (m)	Elevation (m asl)	Outflow (m^3/s)
0	0	0.00	131.50	0.00
6	20	0.00	131.50	0.00
12	90	0.01	131.51	0.03
18	290	0.04	131.54	0.24
24	970	0.14	131.64	1.56
30	1020	0.30	131.80	4.78
36	870	0.44	131.94	8.72
42	740	0.57	132.07	12.58
48	640	0.67	132.17	16.69
54	550	0.76	132.26	19.48
60	480	0.83	132.33	22.45
66	400	0.90	132.40	25.04
72	340	0.95	132.45	27.24
78	300	0.99	132.49	29.15
84	250	1.03	132.53	30.78
90	210	1.06	132.56	32.12
96	170	1.08	132.58	33.19
102	140	1.10	132.60	34.03
108	110	1.11	132.61	34.66
114	90	1.12	132.62	35.11
120	60	1.13	132.63	35.39
126	40	1.13	132.63	35.49
132	20	1.13	132.63	35.45
138	0	1.13	132.63	35.27
144	0	1.12	132.62	35.03
150	10	1.12	132.62	34.82
156	50	1.12	132.62	34.79
162	170	1.13	132.63	36.31
168	430	1.17	132.67	37.17
174	930	1.26	132.76	41.78
180	2020	1.47	132.97	52.59
186	3970	1.89	133.39	76.71
192	6740	2.62	132.12	124.95
198	9700	3.67	135.17	207.50

Table (4): The routing results of PMF hydrograph through AL-Adheem reservoir by Runge- Kutta method (Cont'd)

204	11880	4.95	136.45	324.98
210	12780	6.29	137.79	465.45
216	12170	7.53	139.03	609.69
222	10720	8.58	140.08	741.04
228	8960	9.41	140.91	851.13
234	7200	10.05	141.55	938.87
240	5620	10.51	142.01	1004.31
246	4320	10.84	142.34	1052.44
252	3290	11.06	142.56	1085.40
258	2450	11.21	142.71	1106.73
264	1860	11.29	142.79	1119.24
270	1420	11.34	142.84	1125.45
276	1070	11.34	142.84	1126.88
282	800	11.33	142.83	1124.58
288	600	11.30	142.80	1119.51
294	430	11.25	142.75	1112.30
300	280	11.19	142.69	1103.25
306	170	11.11	142.61	1092.27
312	90	11.04	142.54	1081.22
318	20	10.95	142.45	1068.93
324	10	10.87	142.37	1056.30
330	0	10.78	142.28	1043.69

Appendix 2: Program

```
CLS
DEFDBL A-Z: DEFSNG I-L
DELT = 21600
N = 56
DIM T(N), I(N), H(N), EL(N), O(N)
T(1) = 0: H(1) = 0: EL(1) = 131.5: O(1) = 0
FOR J = 1 TO N - 1
        T(J + 1) = T(J) + 6: NEXT

OPEN "I", 1, "H.DAT"
FOR J = 1 TO N
        INPUT #1, I(J): NEXT
CLOSE #1
B$ = "### #####     ##.##    ###.##   ####.##"
OPEN "O", 2, "RES.OUT"
PRINT #2, "TIME   INFLOW   DEPTH  ELEVATION   OUTFLOW"
PRINT #2, USING B$; T(1); I(1); H(1); EL(1); O(1)
```

```
FOR J = 1 TO N - 1
          I1 = I(J) + (I(J + 1) - I(J)) / 3
          I2 = I(J) + 2 * (I(J + 1) - I(J)) / 3
          Z = H(J)
          GOSUB CAL
    H1 = (I(J) - Q) * DELT / A
    Z = H(J) + H1 / 3
    GOSUB CAL
      H2 = (I1 - Q) * DELT / A
      Z = H(J) + 2 * H2 / 3
      GOSUB CAL
          H3 = (I2 - Q) * DELT / A
    DELH = .25 * H1 + .75 * H3
    H(J + 1) = H(J) + DELH
    EL(J + 1) = H(J + 1) + 131.5
    O(J + 1) = 29.49 * (H(J + 1)) ^ 1.5
PRINT #2, USING B$; T(J + 1); I(J + 1); H(J + 1); EL(J + 1); O(J + 1)
NEXT J
CLOSE
END
'*************************************
CAL: 'COMPUTING AREA AND DISCHARGE
'*************************************
Q = 29.49 * (Z) ^ 1.5
Z1 = (Z + 131.5 - 83.2) ^ 3.49
A = Z1 * .0001624 + 12.69
A = A * 10 ^ 6
RETURN
```

الخلاصـــة

طريقة رونج - كتا ذات المخطط من الدرجة الثالثة قد استخدمت لاستتباع اقصى فيضـــان محتمـــل خلال خزان السد العظيم. اقصى قيمة للتصريف الخارج هي (1126.88 م^3/ثا). ان هذا التصريف هو اقـــل من التصريف الاقصى المسموح والذي هو (1500 م^3/ثا). لذا فانه لاحاجة لان يبوب مفيض السد العظيم.