OUT-OF-PLANE DYNAMICS OF CURVED – STRAIGHT PIPES CONVEYING FLUID

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Abstract

This work studies the dynamics of pipe system, which is constructed from curved and straight parts forming a semi-U bend pipe. The velocity of the fluid flowing in the pipe is considered to be unsteady.

The vibration and stability characteristics are studied using analytical mathematical methods.

It is indicated that the pipe loses its stability by flutter, and the regions of instability are enlarged with increasing the flow velocity and the excitation of flow velocity.

Introduction

The stability of pipe lines in power stations and heat exchanger pipes and other piping systems are of specific importance to certificate the structure design.

The vibration and stability problems of curved and straight pipes conveying steady state flow fluid are available in the literature. However, the vibration and stability characteristics of pipes combined of curved and straight parts conveying unsteady flow could not be found.

Al- Jumaily, A.M. and M.R. Ismaeel [1,2] in an experimental and theoretical investigations studied the dynamical behavior of multi-span pipe conveying fluid. They studied the effect of improper middle support positioning on the vibrations and stability of multi-span pipe, respectively. They concluded that severe vibrations as well as stability might occur if no significant attention is focused on the position of the intermediate support.

The dynamic response of curved beam vibrating in or out - of plane of curvature is of interest in many fields of engineering. Den Hartog [3] employed the Rayleigh Ritz method to find the lowest natural frequency of circular arcs with clamped edges.

The out of plane vibrations of continuous curved beams neglecting the effect of damping and rotary inertia was studied by Nettlton [4]. Natural frequencies for a two – span curved beam were determined. It was found that the natural frequency increases with increasing the central angle of the arc.

Chen [5-7] studied the dynamics and stability of curved pipes in the form of circular arcs conveying steady flow. It was assumed that the centerline of the tube is extensible. Also it was indicated that in the case of clamped - clamped and pinned – pinned boundary conditions, the tube loses stability by buckling when the flow velocity exceeds a certain critical value.

Hill and Davis [8] studies the dynamics and stability of clamped – clamped pipes. Their equations of motion had a significant difference from the previous studies. They included the effect of initial forces arising from the centrifugal effect and pressure of the fluid. They obtained an interesting result, which is if the initial forces are taken into account, then pipes with both ends supported do not lose stability no matter how high the velocity of flow may be.

V. V. Varadan, Jen Hwa Jeng, Liang chi chin, Xiao Qi Bao and V.K.Varadan [9] employed the finite element method to solve the eigenmodes of vibration of the transducer and an eigenmode superposition with unknown weighting coefficients is interfaced with the Floquet representation. Continuity at the boundary is used to solve for both sets of unknown coefficients. The effect of rod cross sections, concentration, material damping are studied as a function of frequency.

ALA'A ABBAS MAHDI [10] studied the effects of induced vibration on a one-span simply supported pipe conveying fluid with a restriction. The dynamics of pipe conveying fluid is described by means of transfer matrix approach, which provides a numerical technique for solving two-dimensional incompressible equations of forced and free vibration mode of a pipe conveying fluid. The governing partial differential equations are directly approximated by a finite difference scheme. The time-dependent two-dimensional Navier-Stokes equations is presented to show the structure of the flow and the numerical observations of both steady and unsteady two dimensional flow through a rigid walled pipe conveying fluid with different orifices to pipe porosity ratios. Experimental work was carried out and from the theoretical and experimental investigations, it is concluded that the fluid flow through a pipe with a restriction affects the dynamic behavior of the pipe in addition to the flow field structure due to induced vibration. Also it is concluded the importance of taking into consideration the fluid Coriolis force, fluid pressure and the mass ratio.

From the literature available and to our knowledge, there is no reference deals directly with curved - straight pipes, which convey fluctuating flowing fluid.

Theory

The equations of motion of the straight pipe in it's non dimensional form is [11]:

$$\frac{\partial^4 \xi_s}{\partial \mu^4} + v_s^2 \frac{\partial^2 \xi_s}{\partial \mu^2} + 2\beta^{\frac{1}{2}} v_s \frac{\partial^2 \xi_s}{\partial \mu \partial \tau_s} + \beta^{\frac{1}{2}} \frac{\partial v_s}{\partial \tau_s} \frac{\partial \xi_s}{\partial \mu} + \frac{\partial^2 \xi_s}{\partial \tau_s^2} = 0 \ldots\ldots\ldots\ldots\ldots (1)$$

Where, $\xi_s = \frac{\upsilon_s}{R}$, $\mu = \frac{x}{R}$, $\beta = \frac{m_f}{m_t + m_f}$, $v_s = (\frac{m_f}{EI})^{\frac{1}{2}} RU$, $\tau_s = (\frac{EI}{m_t + m_f})^{\frac{1}{2}} \frac{t}{R^2}$ ....(2)

Note that the subscript s denotes the straight segment of the pipe.

The derivation of the equation of motion for a curved pipe conveying unsteady flow fluid was not found in the literature. Using Hamilton's principle, the equation of motion of the curved pipe conveying unsteady flowing fluid in its final nondimensional form is:

$$\frac{\partial^6 \phi_c}{\partial \theta^6} + (2 + v_c^2) \frac{\partial^4 \phi_c}{\partial \theta^4} + 2\beta^{\frac{1}{2}} v_c \frac{\partial^4 \phi_c}{\partial \theta^3 \partial \tau_c} + \beta^{\frac{1}{2}} \frac{\partial^3 \phi_c}{\partial \theta^3} \frac{\partial v_c}{\partial \tau_c} + \frac{\partial^4 \xi_c}{\partial \theta^2 \partial \tau_c^2} + (1 - \frac{v_c^2}{k}) \frac{\partial^2 \phi_c}{\partial \theta^2} -$$

$$2 \frac{\beta^{\frac{1}{2}} v_c}{k} \frac{\partial^2 \phi_c}{\partial \theta \partial \tau_c} - \frac{\beta^{\frac{1}{2}}}{k} \frac{\partial v_c}{\partial \tau_c} \frac{\partial \phi_c}{\partial \theta} - \frac{1}{k} \frac{\partial^2 \phi_c}{\partial \tau_c^2} = 0 \ \ldots\ldots\ (3)$$

Where $k = \frac{\omega}{EI}$, $\xi_c = \frac{\upsilon_c}{R}$, $v_c = (\frac{m_f}{EI})^{0.5} RU$

$$\tau_c = (\frac{EI}{m_t + m_f})^{0.5} \frac{t}{R^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Note that the subscript c denotes the curved segment of the pipe.

To describe the function of the unsteady flow harmonically, Fourier series with one harmonic for the periodic velocity may be used [12]:

$$v = v_o(1 + \Delta\cos(\Omega\tau)) \qquad (5)$$

Substituting Eq.(5) into (1)& (3) yields:

$$\frac{\partial^4 \xi_s}{\partial \mu^4} + v_{so}^2 \varphi_s^2 \frac{\partial^2 \xi_s}{\partial \mu^2} + 2\beta^{\frac{1}{2}} v_{so}\varphi_s \frac{\partial^2 \xi_s}{\partial \mu \partial \tau_s} - \beta^{\frac{1}{2}} v_{so}\Omega_s \Delta \sin(\Omega_s \tau_s)\frac{\partial \xi_s}{\partial \mu} + \frac{\partial^2 \xi_s}{\partial \tau_s^2} = 0 \qquad (6)$$

and

$$\frac{\partial^6 \phi_c}{\partial \theta^6} + (2 + v_{co}^2 \varphi_c^2)\frac{\partial^4 \phi_c}{\partial \theta^4} + 2\beta^{\frac{1}{2}} v_{co}\varphi_c \frac{\partial^4 \phi_c}{\partial \theta^3 \partial \tau_c} - \beta^{\frac{1}{2}} v_{co}\Omega_c \Delta \sin(\Omega_c \tau_c)\frac{\partial^3 \phi_c}{\partial \theta^3}$$

$$+ (1 - \frac{1}{k} v_{co}^2 \varphi_c^2)\frac{\partial^2 \phi_c}{\partial \theta^2} - \frac{2\beta^{\frac{1}{2}}}{k} v_{co}\varphi_c \frac{\partial^2 \phi_c}{\partial \theta \partial \tau_c} + \frac{\beta^{\frac{1}{2}}}{k} v_{co}\Omega_c \Delta \sin(\Omega_c \tau_c)\frac{\partial \phi_c}{\partial \theta}$$

$$- \frac{1}{k}\frac{\partial^2 \phi_c}{\partial \tau_c^2} = 0 \qquad (7)$$

Where, $\varphi_s = 1 + \Delta\cos(\Omega_s \tau_s)$, $\varphi_c = 1 + \Delta\cos(\Omega_c \tau_c)$

Regions of Parametric Instabilities

The regions of unstable case of the pipe are divided into two main regions, namely the primary and secondary instability regions. These regions are separated when solving Eqs. 6 & 7 as:

$$\xi_s(\mu, \tau_s) = \sum_{q=0,1,2,3,\ldots} H_q(\mu)\sin(\frac{1}{2} q\Omega_s \tau_s) + h_q(\mu)\cos(\frac{1}{2} q\Omega_s \tau_s) \qquad (8)$$

and

$$\phi_c(\theta, \tau_c) = \sum_{q=0,1,2,3,\ldots} \Psi_q(\theta)\sin(\frac{1}{2} q\Omega_c \tau_c) + \psi_q(\theta)\cos(\frac{1}{2} q\Omega_c \tau_c) \qquad (9)$$

Primary instability regions are found when q=1,3,5,7,... while the secondary are determined when q=0,2,4,6,.... Note that H_q,h_q are unknown functions.
Substituting Eq. 8 into Eq. 6 yields:

$$\sum_{q=1,3,5,\ldots} \Big((\frac{d^4 H_q}{d\mu^4} + (v_{so}^2 + \frac{v_{so}^2 \Delta^2}{2})\frac{d^2 H_q}{d\mu^2} - (\frac{q}{2})^2 \Omega^2 H_q - \beta^{\frac{1}{2}} v_{so}\Omega_s q$$

$$\frac{dh_q}{d\mu})\sin(0.5 q\Omega_s \tau_s) + (\frac{d^4 hq}{d\mu^4}$$

$$+ (v_{so}^2 + \frac{v_{so}^2 \delta^2}{2})\frac{d^2 hq}{d\mu^2} - (\frac{q}{2})^2 \Omega_s^2 h_q + \beta^{\frac{1}{2}} v_{so}\Omega_s q \frac{dH_q}{d\mu})\cos(\frac{1}{2} q\Omega_s \tau_s)$$

$$+ (-\frac{q\Omega_s \beta^{\frac{1}{2}}}{2} v_{so}\Delta \frac{dh_q}{d\mu} + v_{so}^2 \Delta \frac{d^2 H_q}{d\mu^2} - (\frac{1}{2})\beta^{\frac{1}{2}} v_{so}\Delta\Omega_s \frac{dh_q}{d\mu})\sin(\frac{q+2}{2}\Omega_s \tau_s)$$

$$+(\frac{q\Omega_s}{2}\beta^{\frac{1}{2}}v_{so}\Delta\frac{dH_q}{d\mu}+v_{so}^2\Delta\frac{d^2h_q}{d\mu^2}+\frac{1}{2}\beta^{\frac{1}{2}}v_{so}\Delta\Omega_s\frac{dH_q}{d\mu})\cos(\frac{q+2}{2}\Omega_s\tau_s)$$
$$+(-\frac{q\Omega_s}{2}\beta^{\frac{1}{2}}v_{so}\Delta\frac{dh_q}{d\mu}+v_{so}^2\Delta\frac{d^2H_q}{d\mu^2}+\frac{1}{2}\beta^{\frac{1}{2}}v_{so}\Delta\Omega_s\frac{dh_q}{d\mu})\sin(\frac{q-2}{2}\Omega_s\tau_s)$$
$$+(\frac{q\Omega_s}{2}\beta^{\frac{1}{2}}v_{so}\Delta\frac{dH_q}{d\mu}+v_{so}^2\Delta\frac{d^2h_q}{d\mu^2}-\frac{1}{2}\beta^{\frac{1}{2}}v_{so}\Delta\Omega_s\frac{dH_q}{d\mu})\cos(\frac{q-2}{2}\Omega_s\tau_s)$$
$$+\frac{v_{so}^2\delta^2}{4}\frac{d^2H_q}{d\mu^2}(\sin(\frac{q+4}{2}\Omega_s\tau_s)+\sin(\frac{q-4}{2}\Omega_s\tau_s))$$
$$+\frac{v_{so}^2\delta^2}{4}\frac{d^2h_q}{d\mu^2}(\cos(\frac{q+4}{2}\Omega_s\tau_s)+\cos(\frac{q-4}{2}\Omega_s\tau_s)))=0 \quad ......(10)$$

for the primary instability regions with respect to the straight part, and

$$\sum_{q=0,2,4,...}((\frac{d^4H_q}{d\mu^4}+(v_{so}^2+\frac{v_{so}^2\Delta^2}{2})\frac{d^2H_q}{d\mu^2}-(\frac{q}{2})^2\Omega_s^2H_q-\beta^{\frac{1}{2}}v_{so}\Omega_s q\frac{dh_q}{d\mu})\sin(\frac{1}{2}q\Omega_s\tau_s)$$
$$+(\frac{d^4h_q}{d\mu^4}+(v_{so}^2+\frac{v_{so}^2\delta^2}{2})\frac{d^2h_q}{d\mu^2}-(\frac{q}{2})^2\Omega_s^2h_q+\beta^{\frac{1}{2}}v_{so}\Omega_s q\frac{dH_q}{d\mu})\cos(\frac{1}{2}q\Omega_s\tau_s)$$
$$+(-\frac{q\Omega_s}{2}\beta^{\frac{1}{2}}v_{so}\Delta\frac{dh_q}{d\mu}+v_{so}^2\Delta\frac{d^2H_q}{d\mu^2}-\frac{1}{2}\beta^{\frac{1}{2}}v_{so}\Delta\Omega_s\frac{dh_q}{d\mu})\sin(\frac{q+2}{2}\Omega_s\tau_s)$$
$$+(\frac{q\Omega_s}{2}\beta^{\frac{1}{2}}v_{so}\Delta\frac{dH_q}{d\mu}+v_{so}^2\Delta\frac{d^2h_q}{d\mu^2}+\frac{1}{2}\beta^{\frac{1}{2}}v_{so}\Delta\Omega_s\frac{dH_q}{d\mu})\cos(\frac{q+2}{2}\Omega_s\tau_s)$$
$$+(-\frac{q\Omega_s}{2}\beta^{\frac{1}{2}}v_{so}\Delta\frac{dh_q}{d\mu}+v_{so}^2\Delta\frac{d^2H_q}{d\mu^2}+\frac{1}{2}\beta^{\frac{1}{2}}v_{so}\Delta\Omega_s\frac{dh_q}{d\mu})\sin(\frac{q-2}{2}\Omega_s\tau_s)$$
$$+(\frac{q\Omega_s}{2}\beta^{\frac{1}{2}}v_{so}\Delta\frac{dH_q}{d\mu}+v_{so}^2\Delta\frac{d^2h_q}{d\mu^2}-\frac{1}{2}\beta^{\frac{1}{2}}v_{so}\Delta\Omega_s\frac{dH_q}{d\mu})\cos(\frac{q-2}{2}\Omega_s\tau_s)$$
$$+\frac{v_{so}^2\delta^2}{4}\frac{d^2H_q}{d\mu^2}(\sin(\frac{q+4}{2}\Omega_s\tau_s)+\sin(\frac{q-4}{2}\Omega_s\tau_s))$$
$$+\frac{v_{so}^2\delta^2}{4}\frac{d^2h_q}{d\mu^2}(\cos(\frac{q+4}{2}\Omega_s\tau_s)+\cos(\frac{q-4}{2}\Omega_s\tau_s)))=0 \quad ....(11)$$

for the secondary instability regions with respect to the straight too.
Now the regions of the curved part can be found by substituting Eq.(9) into Eq.(7) which gives the primary and secondary regions of instability respectively.

$$\sum_{q=1,3,5,..}((\frac{d^6\Psi_q}{d\theta^6}+(\frac{4+2v_{co}^2+v_{co}^2\Delta^2}{2})\frac{d^4\Psi_q}{d\theta^4}+v_{co}\beta^{\frac{1}{2}}q\Omega_c\frac{d^3\Psi_q}{d\theta^3}+(1-\frac{v_{co}^2}{k}-\frac{v_{co}^2\Delta^2}{2k}-$$
$$\frac{q^2\Omega_c^2}{4})\frac{d^2\Psi_q}{d\theta^2}+\frac{1}{4k}q^2\ \Omega_c^2\Psi_q)\sin(\frac{1}{2}q\Omega_c\tau_c)+(\frac{d^6\psi_q}{d\theta^6}+(\frac{4+2v_{co}^2+v_{co}^2\Delta^2}{2})\frac{d^4\psi_q}{d\theta^4}-$$

$$v_{co}\beta^{\frac{1}{2}} q\Omega_c \frac{d^3\psi_q}{d\theta^3} + \left(1 - \frac{v_{co}^2}{k} - \frac{v_{co}^2\Delta^2}{2k} - \frac{q^2\Omega_c^2}{4}\right)\frac{d^2\psi_q}{d\theta^2} + \frac{1}{4k}q^2\,\Omega_c^2\psi_q)\cos(\frac{1}{2}q\Omega_c\tau_c)$$

$$+(v_{co}^2\Delta\frac{d^4\Psi_q}{d\theta^4} + \frac{v_{co}\beta^{\frac12}\Omega_c q}{2}\frac{d^3\Psi_q}{d\theta^3} + \frac{v_{co}\Delta\beta^{\frac12}\Omega_c}{2}\frac{d^3\psi_q}{d\theta^3} - \frac{v_{co}^2\Delta}{k}\frac{d^2\Psi_q}{d\theta^2})\sin(\frac{(q-2)\Omega_c\tau_c}{2}) +$$

$$(v_{co}^2\Delta\frac{d^4\psi_q}{d\theta^4} - \frac{v_{co}\beta^{\frac12}\Omega_c q}{2}\frac{d^3\psi_q}{d\theta^3} - \frac{v_{co}\Delta\beta^{\frac12}\Omega_c}{2}\frac{d^3\Psi_q}{d\theta^3} - \frac{v_{co}^2\Delta}{k}\frac{d^2\psi_q}{d\theta^2})\cos(\frac{(q-2)\Omega_c\tau_c}{2}) +$$

$$(v_{co}^2\Delta\frac{d^4\Psi_q}{d\theta^4} + \frac{v_{co}\beta^{\frac12}q\Omega_c}{2}\frac{d^3\Psi_q}{d\theta^3} - \frac{v_{co}\Delta\beta^{\frac12}\Omega_c}{2}\frac{d^3\psi_q}{d\theta^3} - \frac{v_{co}^2\Delta}{k}\frac{d^2\Psi_q}{d\theta^2})\sin(\frac{(q+2)\Omega_c\tau_c}{2}) +$$

$$(v_{co}^2\Delta\frac{d^4\psi_q}{d\theta^4} - \frac{v_{co}\beta^{\frac12}\Omega_c q}{2}\frac{d^3\psi_q}{d\theta^3} + \frac{v_{co}\beta^{\frac12}q\Omega_c}{2}\frac{d^3\Psi_q}{d\theta^3} - \frac{v_{co}^2\Delta}{k}\frac{d^2\psi_q}{d\theta^2})\cos(\frac{(q+2)\Omega_c\tau_c}{2}) +$$

$$\frac{v_{co}^2\Delta^2}{4}\frac{d^4\Psi_q}{d\theta^4}\sin(\frac{(q-4)\Omega_c\tau_c}{2}) + \frac{v_{co}^2\Delta^2}{4}\frac{d^4\psi_q}{d\theta^4}\cos(\frac{(q-4)\Omega_c\tau_c}{2}) + \frac{v_{co}^2\Delta^2}{4}\frac{d^4\Psi_q}{d\theta^4}\sin(\frac{(q+4)\Omega_c\tau_c}{2}) + \frac{v_{co}^2\Delta^2}{4}\frac{d^4\psi_q}{d\theta^4}\cos(\frac{(q+4)\Omega_c\tau_c}{2}) -$$

$$\frac{v_{co}\beta^{\frac12}\Omega_c q}{4k}(\frac{d\Psi_q}{d\theta} - \frac{d\psi_q}{d\theta})\sin((q-1)\Omega_c\tau_c) + \frac{v_{co}\beta^{\frac12}\Omega_c q}{4k}(\frac{d\psi_q}{d\theta} - \frac{d\Psi_q}{d\theta})\sin((q+1)\Omega_c\tau_c) +$$

$$\frac{v_{co}\beta^{\frac12}\Omega_c q}{2k}(\frac{d\psi_q}{d\theta} - \frac{d\Psi_q}{d\theta})\sin(q\Omega_c\tau_c)) = 0 \quad \ldots\ldots(12)$$

$$\sum_{q=0,2,4,\ldots}((\frac{d^6\Psi_q}{d\theta^6} + (\frac{4+2v_{co}^2+v_{co}^2\Delta^2}{2})\frac{d^4\Psi_q}{d\theta^4} + v_{co}\beta^{\frac12}q\Omega_c\frac{d^3\Psi_q}{d\theta^3} + (1 - \frac{v_{co}^2}{k} - \frac{v_{co}^2\Delta^2}{2k} -$$

$$\frac{q^2\Omega_c^2}{4})\frac{d^2\Psi_q}{d\theta^2} + \frac{1}{4k}q^2\,\Omega_c^2\Psi_q)\sin(\frac12 q\Omega_c\tau_c) + (\frac{d^6\psi_q}{d\theta^6} + (\frac{4+2v_{co}^2+v_{co}^2\Delta^2}{2})\frac{d^4\psi_q}{d\theta^4} -$$

$$v_{co}\beta^{\frac12}q\Omega_c\frac{d^3\psi_q}{d\theta^3} + (1 - \frac{v_{co}^2}{k} - \frac{v_{co}^2\Delta^2}{2k} - \frac{q^2\Omega_c^2}{4})\frac{d^2\psi_q}{d\theta^2} + \frac{1}{4k}q^2\,\Omega_c^2\psi_q)\cos(\frac12 q\Omega_c\tau_c)$$

$$+(v_{co}^2\Delta\frac{d^4\Psi_q}{d\theta^4} + \frac{v_{co}\beta^{\frac12}\Omega_c q}{2}\frac{d^3\Psi_q}{d\theta^3} + \frac{v_{co}\Delta\beta^{\frac12}\Omega_c}{2}\frac{d^3\psi_q}{d\theta^3} - \frac{v_{co}^2\Delta}{k}\frac{d^2\Psi_q}{d\theta^2})\sin(\frac{(q-2)\Omega_c\tau_c}{2}) +$$

$$(v_{co}^2\Delta\frac{d^4\psi_q}{d\theta^4} - \frac{v_{co}\beta^{\frac12}\Omega_c q}{2}\frac{d^3\psi_q}{d\theta^3} - \frac{v_{co}\Delta\beta^{\frac12}\Omega_c}{2}\frac{d^3\Psi_q}{d\theta^3} - \frac{v_{co}^2\Delta}{k}\frac{d^2\psi_q}{d\theta^2})\cos(\frac{(q-2)\Omega_c\tau_c}{2}) +$$

$$(v_{co}^2\Delta\frac{d^4\Psi_q}{d\theta^4} + \frac{v_{co}\beta^{\frac12}q\Omega_c}{2}\frac{d^3\Psi_q}{d\theta^3} - \frac{v_{co}\Delta\beta^{\frac12}\Omega_c}{2}\frac{d^3\psi_q}{d\theta^3} - \frac{v_{co}^2\Delta}{k}\frac{d^2\Psi_q}{d\theta^2})\sin(\frac{(q+2)\Omega_c\tau_c}{2}) +$$

$$(v_{co}^2\Delta\frac{d^4\psi_q}{d\theta^4} - \frac{v_{co}\beta^{\frac12}\Omega_c q}{2}\frac{d^3\psi_q}{d\theta^3} + \frac{v_{co}\beta^{\frac12}q\Omega_c}{2}\frac{d^3\Psi_q}{d\theta^3} - \frac{v_{co}^2\Delta}{k}\frac{d^2\psi_q}{d\theta^2})\cos(\frac{(q+2)\Omega_c\tau_c}{2}) +$$

$$\frac{v_{co}^2\Delta^2}{4}\frac{d^4\Psi_q}{d\theta^4}\sin(\frac{(q-4)\Omega_c\tau_c}{2})+\frac{v_{co}^2\Delta^2}{4}\frac{d^4\psi_q}{d\theta^4}\cos(\frac{(q-4)\Omega_c\tau_c}{2})+\frac{v_{co}^2\Delta^2}{4}\frac{d^4\Psi_q}{d\theta^4}\sin$$
$$(\frac{q+4)\Omega_c\tau_c}{2})+\frac{v_{co}^2\Delta^2}{4}\frac{d^4\psi_q}{d\theta^4}\cos(\frac{q+4)\Omega_c\tau_c}{2})-$$
$$\frac{v_{co}\beta^2\Omega_c q}{4k}(\frac{d\Psi_q}{d\theta}-\frac{d\psi_q}{d\theta})\sin((q-1)\Omega_c\tau_c)+\frac{v_{co}\beta^2\Omega_c q}{4k}(\frac{d\psi_q}{d\theta}-\frac{d\Psi_q}{d\theta})\sin((q+1)\Omega_c\tau_c)+$$
$$\frac{v_{co}\beta^2\Omega_c q}{2k}(\frac{d\psi_q}{d\theta}-\frac{d\Psi_q}{d\theta})\sin(q\Omega_c\tau_c)))=0 \quad ....(13)$$

Boundary Conditions

The pipe is assumed to be clamped at both ends. At point (A), $\theta_1=0$ and at point (D), x=L, (Fig.1)

$$\xi_c(0,t)=\xi_s(L,t)=0$$
$$\frac{\partial\xi_c}{\partial\theta}(0,t)=\frac{\partial\xi_s}{\partial\mu}(L,t)=0 \qquad ....(14a)$$
$$\phi_c(0,t)=\phi_s(L,t)$$

At the straight – curved junction the following equations may be imposed to fulfil the requirements of continuity of displacement, slope, twist angle, bending moment, twisting moment and shear force as:

$$\xi_c(\theta_2,t)=\xi_s(0,t)$$
$$\frac{\partial\xi_s}{\partial\mu}(0,t)=\frac{\partial\xi_c}{\partial\theta}(\theta_2,t)$$
$$\phi_s(0,t)=\phi_c(\theta_2,t)$$
$$\frac{\partial^2\xi_s}{\partial\mu^2}(0,t)=\frac{\partial^2\xi_c}{\partial\theta^2}(\theta_2,t)-\phi_c(\theta_2,t) \qquad ....(14b)$$
$$k\frac{\partial\phi_s}{\partial\mu}(0,t)=\frac{\partial\xi_c}{\partial\theta}(\theta_2,t)+\frac{\partial\phi_c}{\partial\theta}(\theta_2,t)-\frac{R}{L}\phi_c(\theta_2,t)$$
$$\frac{\partial^3\xi_s}{\partial\mu^3}(0,t)=\frac{\partial^2\xi_c}{\partial\theta^2}(\theta_2,t)-k\frac{\partial\xi_c}{\partial\theta}(\theta_2,t)-(1+k)\frac{\partial\phi_c}{\partial\theta}(\theta_2,t)$$

Results and Discussions

The effect of the flow parameters, namely, flow velocity, mass ratio and excitation parameter on the dynamic characteristics of the system under consideration are discussed.

The effect of the ratio of the fluid mass per unit length to that of the tube plus fluid mass per unit length, β, on the natural frequencies may be observed in Fig. 2. It is clearly indicated that the fluid tends to reduce the natural frequencies within the stable range of operation. This fact agrees well with other findings such as that by Chen [7].

and Paidoussis [11]. Furthermore, Fig. (2) indicates that the fluid velocity also tends to reduce the natural frequency until a critical velocity is reached where buckling instability likely to occur is expected. This fact is attributed to the Coriolis term, namely, $(2\beta^{\frac{1}{2}} v_c \frac{\partial^2 \xi_c}{\partial\theta \partial\tau_c})$ for the curved segment and $(2\beta^{\frac{1}{2}} v_s \frac{\partial^2 \xi_s}{\partial\mu \partial\tau_s})$ for the straight segment which has the tendency to damp the system and reduce the natural frequencies.

For fluctuating flow, the effect of the fluid flow velocity [v_o] on the regions of the instabilities is indicated in Figs.(3) and (5) while Fig.(4) shows the variation of the regions of instabilities with the excitation parameter [Δ]. It is clear that the size of unstable region increase with increasing the flow velocity for both the primary and secondary regions. To elaborate on this behavior, it may be noticed that the time period of harmonic fluctuation in the fluid velocity occurs at 2T and T for the two regions respectively. The resonance appears at these two periods due to:

1- The time duration through the reduction in flow velocity from the higher limit [v_{om}] to the mean velocity [v_o] increases with increasing the flow velocity; therefore the size of the unstable region becomes wider with increasing [v_o].

2- The inertia forces delivered from the fluctuation of the fluid flow increase with increasing [v_o], hence this force will be an auxiliary factor in resonance appearance.

The excitation parameter has an identical effect on the regions of instability as that [v_o], Fig. (4). It is shown that the regions of instability are enlarged with increasing the excitation parameters. This is due to the difference between the maximum and minimum values of flow velocity, which has an effect similar to sinusoidal force.

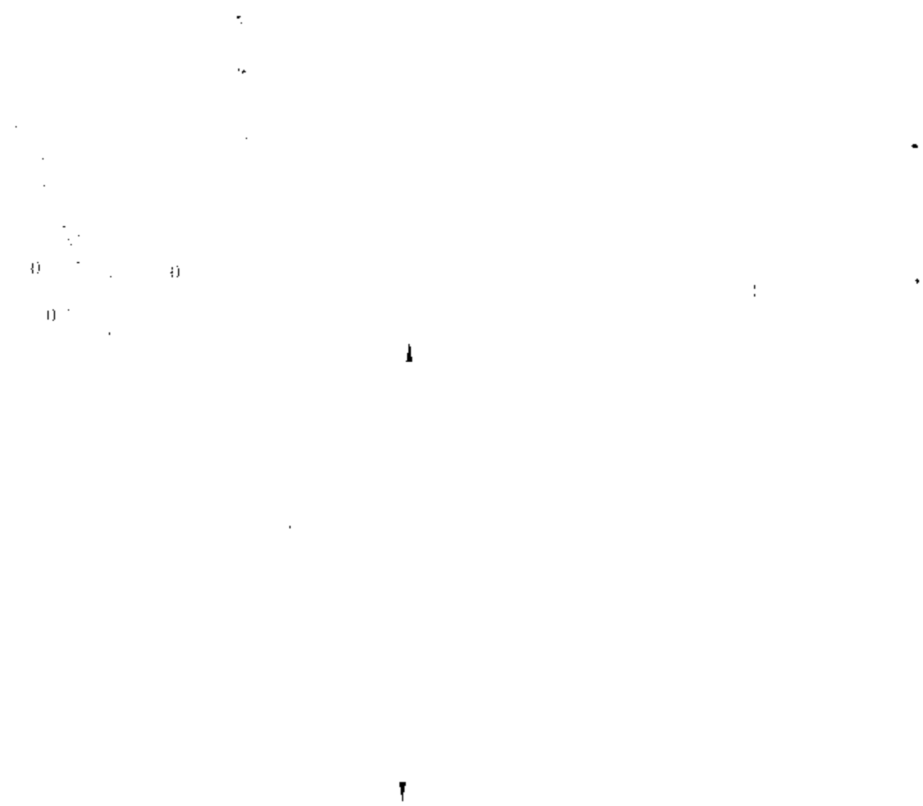
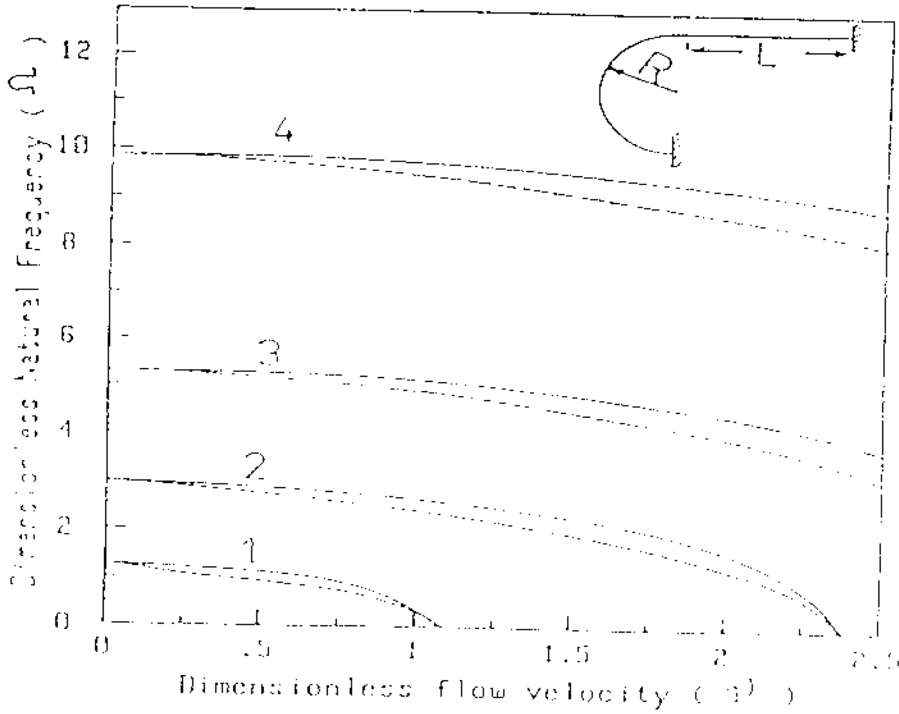
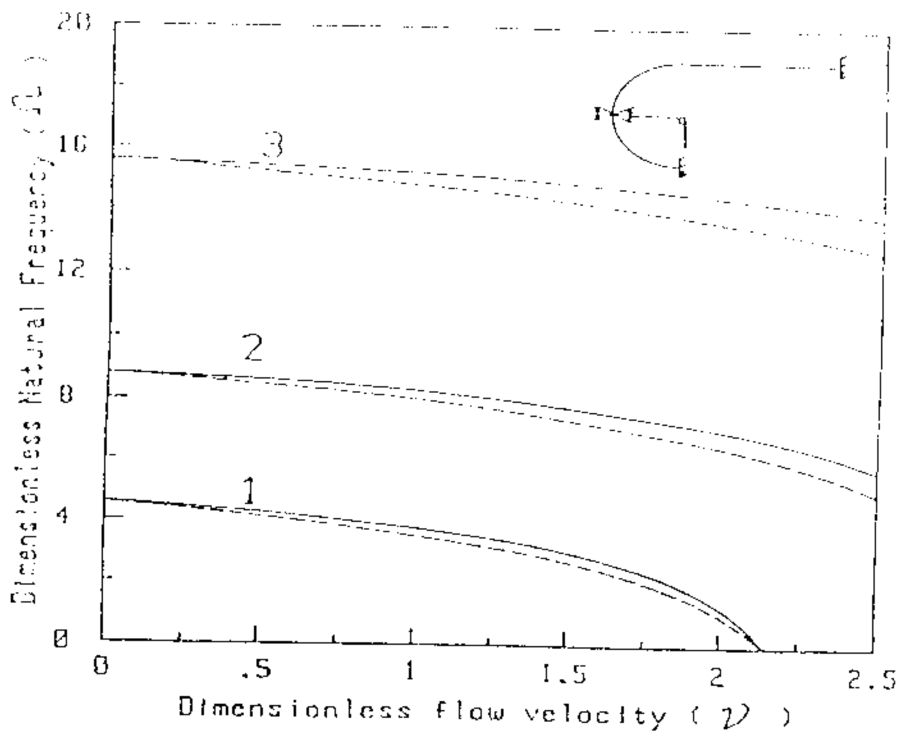


Fig.1 : Model of Curved –Staight Tube with Clamed Ends Condition .

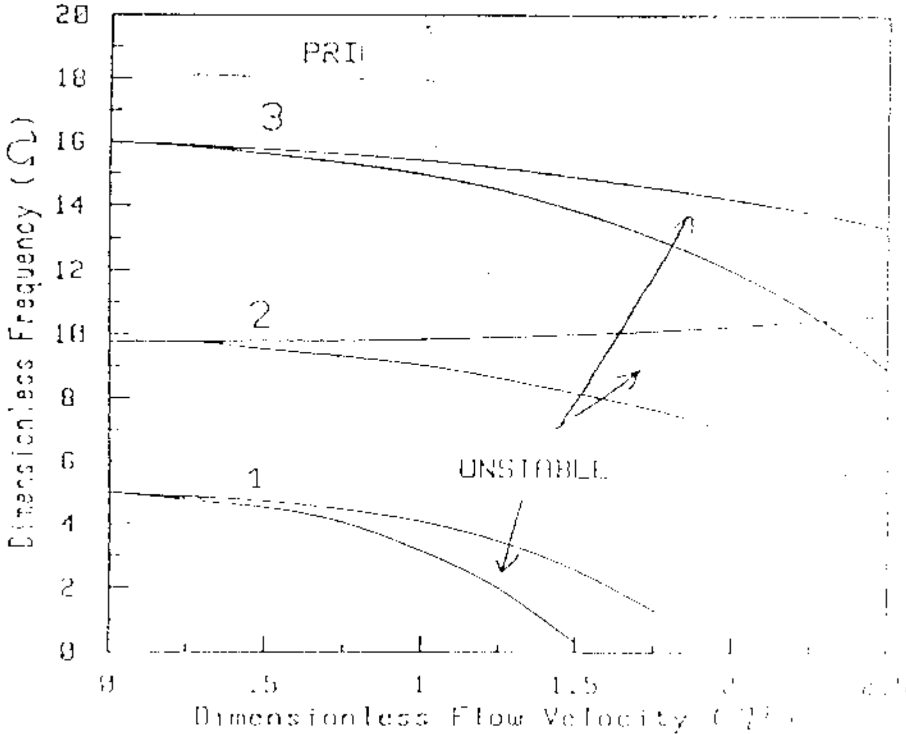


(A)

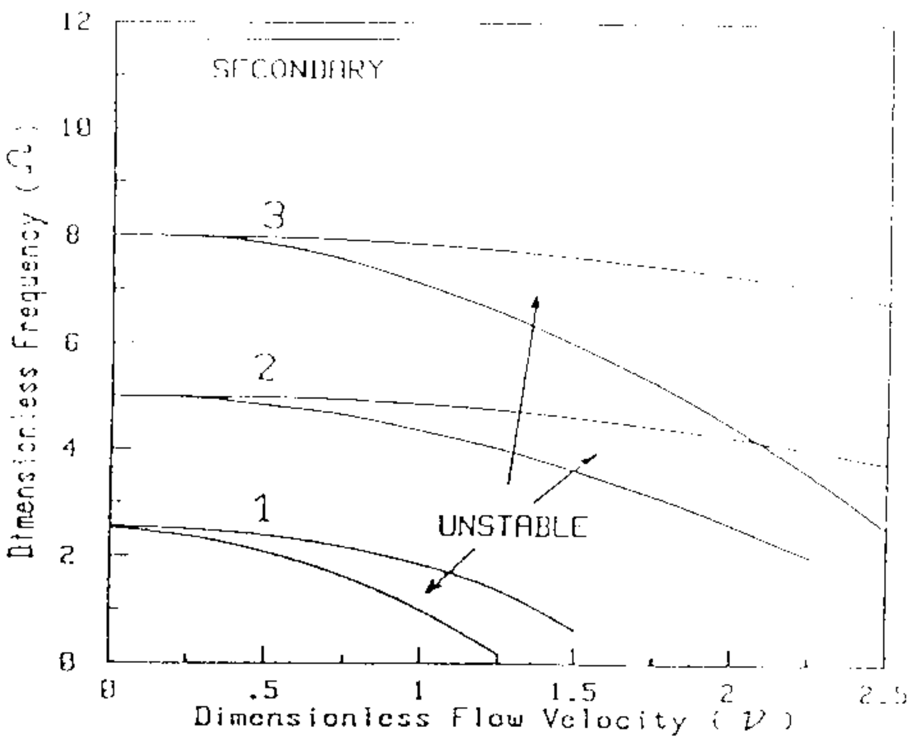


(B)

Fig 2 : Effect or Flow Velocity on the Natural Frequency for $\frac{L}{R}=1$.(A) without Intermediate Support , (B) with Intermediate Support———, for β=0.25,………, for β=0.75 .



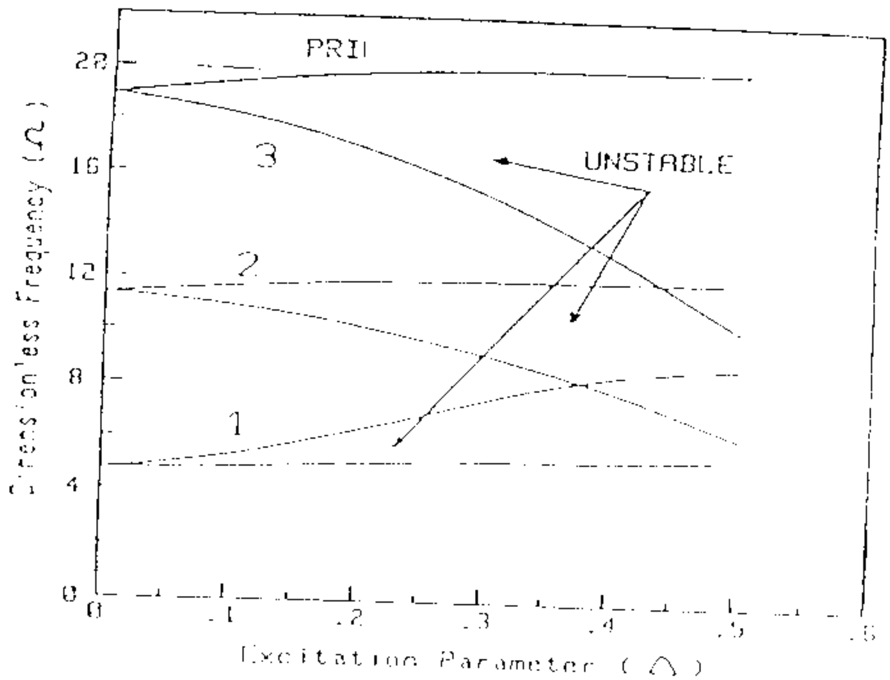
(A)



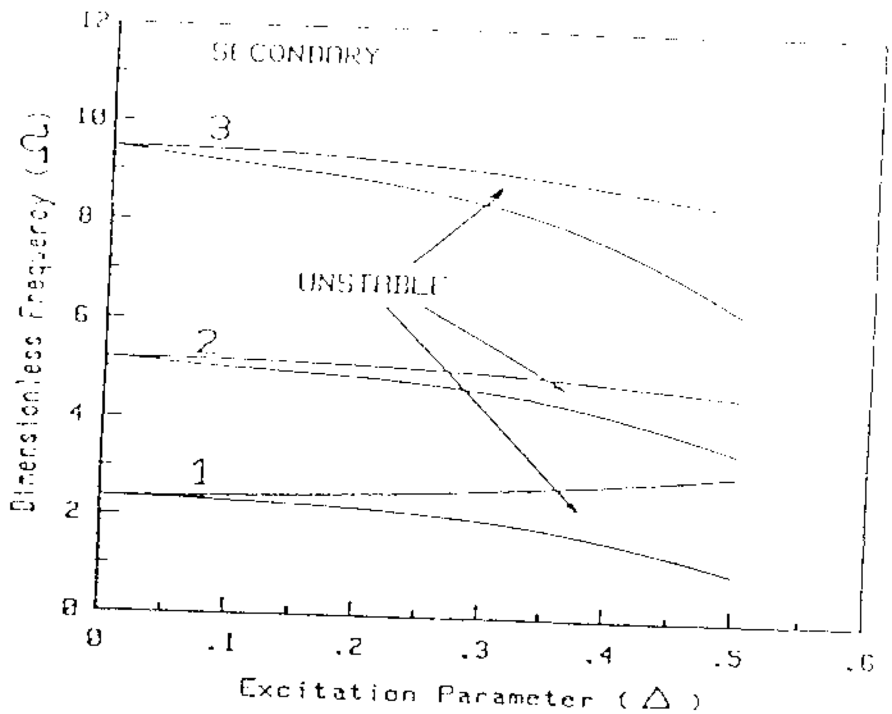
(B)

Fig 3 : Effect of Flow Velocity on Region of Instability for $\frac{L}{R} = 1$ and $\Lambda = 0.5$.

(A) : PRIMARY (B) : SECONDARY



(A)



(B)

Fig 4 : Effect of Excitation Parameter on Region of Instability , $\frac{L}{R} = 1, \nu_0 = 2$.

(A) : PRIMARY (B) : SECONDARY

Nomenclature

E: Young's modulus of elasticity of the pipe material (N/m^2).
I: Second moment of area of the pipe cross section (m^4).
m_f : Fluid mass per unit length (Kg/m).
m_t :Tube mass per unit length (Kg/m).
R: Mean radius of curvature of the curved part (m).
t : Time (Sec)
U: Uniform flow velocity (m/sec)
v: Dimensionless velocity.
v_o : Mean dimensionless velocity.
β : Mass ratio.
ω: Circular natural frequency (Hz).
Ω : Dimensionless frequency parameter, $\Omega = (\frac{m_f + m_t}{EI})^{\frac{1}{2}} R^2 \omega$
Λ : Excitation parameter $\Lambda = \frac{v_{max} - v_o}{v_o}$
τ : Dimensionless time.
μ : Dimensionless length of the straight segment.
υ: Transverse displacement (m).
ξ : Dimensionless transverse displacement.
ϕ : Angle of twist (rad).

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الخصائص الديناميكية الخارجة عن المستوى للأنابيب المنحنية – المستقيمة الناقلة للموائع

الخلاصة

يتناول هذا البحث دراسة الخصائص الديناميكية لأنبوب منحني - مستقيم يمر خلاله مائع ذو سرعة غير منتظمة. الأنبوب مكون من جزأين الأول منحني والأخر مستقيم، بحيث تشكل المجموعة أنبوب على شكل حرف (U) غير كامل.

لقد تمت دراسة خصائص اهتزازات و استقرارية الأنبوب باستخدام طرق رياضية تحليلية.

لقد وجد أن الأنبوب يفقد استقراريته بالرنين (الارتعاش) وأن المناطق غير المستقرة تزداد سعة مع زيادة سرعة الجريان وزيادة مقدار إثارة سرعة الجريان.