مجلة جامعة بابل / العلوم الهندسية/ المجلد ١٠/ العدد ٥ : ٢٠٠٥

PREWARPING TECHNIQUES FOR SWITCHED CAPACITOR FILTER DESIGN BASED ON OPTIMIZATION

Yahia Ali Lafta
Al- Kufa University, College of Engineering

Laith A. El-Anzy
Electronic Eng., Technical College-Mussaiab

Abstract

The warping effect due to S–Z transformation represents a significant limitation in the design of switched capacitor filter, especially at high frequency. One method to reduce the warping effect is the use of high sampling to signal frequency ratio, this method results a filter structure with high capacitance spread that require large fabrication area. This paper presents a new design technique based on Simpson S-Z transformation inconjuction with optimization to obtain the optimum transformation coefficients. This technique eliminates the warping effect in the filter response at low sampling frequency that preserves the low capacitance spread.

1. Introduction

Computer-aided circuit design (CACD) has played a significant role in the design of conventional filters. This method, which is an alternative to the classical synthesis approach, provides us with a powerful tool that enables the designer to deal with very complicated filters and is also capable of incorporating into the design all practical constraints with which straightforward synthesis cannot cope [1]; optimization is one of these tools.

With the aid of existing general-purpose circuit analysis programs such as PSPICE, after simulating the circuit, one can adjust certain parameter variable by simulating these variables, or simulating the parameters that affect these variables, until a required performance is obtained. This manual process is not efficient, particularly when it is necessary to manipulate the larger numbers of variable that occur in most design problems [2]. Therefore, an automatic design technique (numerical optimization) can be used to avoid inefficiency. Optimization in S-Z transformation is used to determine the optimum parameter values of a given transformation which is required to satisfy a set of specifications, possibly including practical constraints; in other words an iterative optimization technique is used to minimize a given error criterion by manipulating the coefficient of some S-Z transformations like Simpson, Third- and Fourth- order transformation[3].

The optimization of the coefficients of S-Z transformation will be used to eliminate the discrepancy between the filter frequency response and the realized counterpart caused by the optimization S-Z transformations. This technique eliminating the need for prewarping the S-domain transfers function. The optimization of the coefficients of the S-Z transformation is carried out by matching the realization frequency response of prewarped formal of this transformation with the ideal counter part. Another advantage of using optimization S-Z transformations is that it deals with the problem of capacitance spread; minimization of the capacitance spread will therefore reduce the total on-chip capacitance [4].

The general relationship between S and Z domain is given by:

$$\Omega = \frac{\sum_{i=0}^{n} a_i}{T}\left[\frac{1-Z^{-1}}{a_o + a_1 Z^{-1/n} + a_2 Z^{-2/n} + \dots + a_n Z^{-n/n}}\right] \quad \dots\dots(1)$$

Where $a_{n-i}=a_i$ for $i=0,1,2,\dots\dots,n$

n is the order of transformation, and $\mathbf{a_i}$ are the coefficients of transformation.

And the prewarping rule is given by:

$$\Omega = \frac{\sum_{i=0}^{n} a_i}{T}\left[\frac{1-Z^{-1}}{\sum_{k=0}^{m} a_{n-k}\cos\left[\left(\tfrac{1}{2}-\tfrac{k}{n}\right)\beta T\right]+b}\right] \quad \dots\dots(2)$$

Where,

$b = \frac{a_{n/2}}{2}$ For $m=\frac{n}{2}-1$ when n is even.

$b=0$ For $m=\frac{n-1}{2}$ when n is odd.

The weight of samples $\mathbf{a_k}$ in the general S-Z transformation which may be chosen according [illegible] a desired prewarping relationship, or to provide a desired component value [illegible] [4].

Simpson S-Z transformation which does not appear to have been applied [illegible] in filter design, it is derived from the parabolic integration method [5]. [illegible] and prewarping rules for this transformation are given by:

$$s = \frac{\sum_{i=0}^{2} a_i}{T}\,\frac{1-Z^{-1}}{a_o + a_1 Z^{-1/2} + a_2 Z^{-1}}$$

And $\Omega = \frac{\sum a}{T}$ [illegible] sin [illegible] βT [illegible]

Where $a_2 = a_0$

The digitization of [illegible] transformation is a [illegible] mapping [illegible] the sample a_i, and these were [illegible] in order to examine the [illegible] these coefficients can be [illegible] S-Z transformation, each pole [illegible] two poles (zeros) in the Z-plane, [illegible] other outside; naturally, the former is chosen [illegible]

Moreover, each root on the imaginary [illegible] separate roots on the unit circle in the Z-plane, [illegible] combination of the Z-plane poles and zeros [illegible] transfer function H(z). Whilst the alternative [illegible] amplitude frequency response with a scaling along the physical frequency axis and a significant enhancement of the attenuation in the stop-band. It should be mentioned that the Simpson mapping procedure satisfies the above relationships between the S and Z planes, and hence the frequency characteristics and stability properties of the continuous-time S-plane filter are preserved [8]. The prewarping effect for Simpson S-Z transformation for different weight of coefficient (a_0,a_1,a_2) is presented in fig(1).

A general S-Z transformation has been presented which enables sampling weight to be chosen to achieve different relationships between the physical frequency variable of the S-plane and its counterpart in the Z-plane. In order to illustrate the advantages of this degree of freedom, the following third and fourth-order S-Z transformation were studied [1, 2, and 3].

Mapping rule is:

$$s = \frac{\sum_{i=0}^{3} b_i}{T}\left[\frac{1-Z^{-1}}{b_0 + b_1 Z^{-1/3} + b_2 Z^{-2/3} + b_3 Z^{-1}}\right] \qquad \text{.........(5)}$$

Prewarping rule is:

$$\Omega = \frac{\sum_{i=0}^{3} b_i}{T}\left[\frac{\sin\left(\frac{WT}{2}\right)}{b_0\cos\left(\frac{WT}{2}\right) + b_1\cos\left(\frac{WT}{6}\right)}\right] \qquad \text{...........(6)}$$

Where b_0, b_1, b_2, b_3

And for fourth order S-Z transformation mapping rule is:

$$s = \frac{\sum_{i=0}^{4} a_i}{T}\left[\frac{1-Z^{-1}}{a_0 + a_1 Z^{-1/4} + a_2 Z^{-2/4} + a_3 Z^{-3/4} + a_4 Z^{-1}}\right] \qquad \text{.........(7)}$$

Prewarping rule is:

$$\Omega = \frac{\sum_{i=0}^{4} a_i}{T}\left[\frac{\sin\left(\frac{WT}{2}\right)}{a_0\cos\left(\frac{WT}{2}\right) + a_1\cos\left(\frac{WT}{4}\right) + \frac{a_2}{2}}\right] \qquad \text{...........(8)}$$

Where a_0, a_1, a_2, a_3

The frequency prewarping effect depend on labeled (b_0,b_1,b_2,b_3) and (a_0,a_1,a_2,a_3,a_4) [2,4]. The weights of the sample a_i, b_i were chosen to provide a variety of useful prewarping curves in order to examine the characteristics of various S-Z transformation as shown in fig (1) for different weight of coefficients of third-and fourth-order S-Z transformations.

2. Simpson Transformation based on optimization:

To determine the optimal value of these coefficients, the optimization routine is applied. The error function is constructed from the ideal prewarping relationship that is given as [7]:

And the Simpson prewarping relationship will be: $\Omega_s = W_d$ ......(9)

$$E_k = \Omega_l - \Omega_R = W_k - \frac{\sum_{i=0}^{2} a_i}{T} \frac{\sin\left(\frac{W_k T}{2}\right)}{a_0 \cos\left(\frac{W_k T}{2}\right) + \frac{a_1}{2}} \qquad \ldots\ldots\ldots\ldots\ldots$$

Where: k is the index of samples of error function and discrete freq...
The application of Hooke and Jeeves method [8] by optimization ... error function minimization. The optimum values of Simps... obtained (a_0, a_1, a_2).

The initial base point of this variable is (-10,-10,-10) and ... (a_2 a_0).

Table (1) presents the tabulation of optimum coefficie... optimization using different step length. Reducing the step s... increases the accuracy.

Table (1): The optimum values of Simpson transformat...

Step length	Optimum values for the coeff...		
h	a_0	a_1	a_2
1	2	7	2
0.1	1.1	4.1	1.1
0.05	1.55	5.5	1.55
0.025	1.025	3.825	1.025
0.01	2.06	4.06	2.0...

There is a good linearity between the continuous ... physical frequency variables (w,Ω), that are an achie... coefficients, especially for coefficients that were deter... length.

Fig (2) shows the warping effect using optimal va... (1), and a random value for these coefficients and com... It is clear that a significant improvement is obtain... Fig (3) shows the frequency response of 3rd Chebysh... kHz and pass band ripple r_p=0.5 db using Simps... values of coefficients that were shown in table ... frequency responses of these coefficients have be... frequency Fs=15 kHz. It is clear that, the application ... Simpson transformation improves the frequency resp... plane frequency response for the first step size h ... transformation formula is now as follow [7]:

$$s = \frac{\sum_{i=0}^{2} a_i}{T} \frac{1 - Z^{-1}}{2 + 7Z^{1/2} + 2Z^{-1}} \qquad \ldots\ldots\ldots\ldots$$

If the values of Summation substitute in equation (4 ...

$$s = \frac{11}{T} \frac{1 - Z^{-1}}{2 + 7Z^{1/2} + 2Z^{-1}}$$

The substitutions of equation (12) in the s-... transform it into Z-plane transfer function, then subst... the frequency response of Z-plane transfer functi... frequency about 3475Hz is obtained for the discrete f...

error is less than 0.7%, and when the coefficients of smallest step size h=0.01, (1.06, 4, 06, 1.06), the cutoff frequency was been about 3497Hz, and that means, the error is less than 0.08%, and this error can be reduced by increasing sampling frequency. Fig (3) achieves all that, and from this figure there is a certain similarity between the simulated frequency response of discrete transfer function for all optimal values of Simpson transformation, and the frequency response of continuous transfer function.

2. Third and Fourth Order S-To-Z Transformations based on optimization:

As mentioned in last section, we need to find the error function .The same procedure used in Simpson transformation will be used here; therefore, the error function is given by:

$$E_r = \Omega - \Omega_d = w_i - \frac{\sum_{i=0}^{3} a_i}{T}\left[\frac{\sin\left(\frac{w_i T}{2}\right)}{a_0\cos\left(\frac{w_i T}{2}\right)+a_1\cos\left(\frac{w_i T}{6}\right)}\right] \quad \text{.........................(13)}$$

Where $a_3 = a_0$ and $a_2 = a_1$.

The procedure of Hooke and Jeeves method is applied to equation (13) to minimization the error function. The point of the optimum coefficients (a_0, a_1, a_2, a_3) are therefore be found

The initial base point of these coefficients is (-10,-10,-10,-10), and we are used a fixed step length h for all coefficients, the procedure is applied using different step length as presented in table (2).

Table (2): The optimum values of third-order transformation coefficients

Step length h	Optimal values of the coefficients			
	a_0	a_1	a_2	a_3
1	1	3	3	1
0.1	3.1	8.8	8.8	3.1
0.05	3.25	9.55	9.55	3.25
0.025	2.325	6.925	6.925	2.325

Fig (4) shows a comparison between the warping effects for using third order transformation with optimized coefficients (2.325, 6.925, 6.925, 2325), and a random value of these coefficients (1, 6, 6, 1). It is clear that the optimized warping effect has good linearity in the interval (0-Π) when compare them with standard S-Z transformation ($\Omega = w$) curve. This interval is the important one from the range (0-2Π), because of (Π) here represents the half sampling frequency used, and this frequency represents the highest frequency that one can be designed discrete filter, because of the sampling theorem constrained.

A 3rd order Chebyshev LPF, with same specification was considered in previous section. Third order transformation with optimized coefficients will be used in order to obtain Z-plane transfer function, in same procedure that carries out in Simpson transformation .The frequency response of discrete transfer function was presented in fig (5), and for all optimized coefficients shown in table (2). It is clear from this figure; there is a significant improvement in discrete frequency responses. There is a very small error between any of optimized responses and continuous response, i.e. for optimized coefficients (1,3,3,1), the cutoff frequency for discrete frequency response was about 3512Hz with error about 0.3%, and for the best optimized coefficients, (1.06,4.06,4.06,1.06), the cutoff frequency is about 3499.5Hz, with error about 0.014%. All these cutoff frequencies for sampling to ... about (4.28) [7]. Increasing this ratio can eliminate this error.

The prewarping rules of fourth order transformation was gi... (14), it is required to find the error function. that will be minimize... procedure of Hooke and Jeeves method [9]. The error function of ... transformation is given by:

$$E_i = \Omega_i - \Omega_c = w_i - \frac{\sum a}{T}\left[\frac{\sin\left(\frac{w_i T}{2}\right)}{a_0\cos\left(\frac{w_i T}{2}\right) + a_1\cos\left(\frac{w_i T}{4}\right) + \frac{a_2}{2}}\right] \quad \ldots\ldots\ldots\ldots\ldots$$

Where $a_4 = a_0$ and $a_3 = a_1$.

The initial base point of coefficients of fourth order trans... 10,-10) and also used a fixed step length h for all coefficients, ... length as shown in table (3).

Table (3): The optimum values of fourth-order transformati...

Step length h	Optimal values of coefficients				
	a_0	a_1	a_2	a_3	
1	2	7	4	7	2
0.5	1.5	5.5	3	5.5	1.5
0.25	2.25	9.75	4	9.75	2.25
0.1	2.2	9.8	3.9	9.8	2.2

Fig (6) shows a comparison between the warping effe... transformation optimized coefficients, T_4(2.2, 9.8, 3.9, 9.8, 2.2... of these coefficients, T_4 (1,3,5,3,1), and compare these two ... transformation (linear curve $\Omega = w$). The optimized ... significant linearity in interval (0-11), so as the standard ... due to warping effect will be minimum, and less than other...

A 3[rd] order Chebyshev LPF, with same specificati... achieved in previous two sections is presented here. The ... will be applied to transform the continuous transfer func... function with same procedure used in Simpson transf... response of discrete transfer function will be calculated fo... of table (3). These responses have been clarified in fig (7)... that responses have a significant improvement, with ... optimized coefficients with smallest step length, is identica... response .The cutoff frequency of discrete response with ... (2,7,4,7,2) is about 3493 Hz, with error about 0.2%. W... coefficients, the cutoff frequency is 3500 Hz, and the error ...

4. Cases study:

The procedure presented in the previous section ... realization two type of SC filters at different frequency app...

4-1 SC LPF:

In this section, a LPF of 8[th] order, Chebyshev type with ... and 0.5db passband ripple, which is designed using the pr... with approximated Simpson transformation in this example, ... order are needed to realize this filter. After factorizing the ... and numerator to four sections of 2[nd] order types, the pole fre...

factor q_p are calculated for each section, and then transform each section transfer function to discrete one. The calculated coefficient of each sections that are presented in [7], after using approximation to term that can't realized in SC, after this approximation, the cascade sections are realized by SC technique, the frequency response of this cascade filter is calculated for different sampling frequencies as shown in fig (8). The frequency response in passband and stopband is illustrated in fig (8a). Passband frequency response is shown in fig (8b) that shows the frequency response in passband-enlarged view. From fig (8b) the cutoff frequency when Fs= 20MHz is about (3.535MHz) and for same sampling frequency, it is about (3.25MHz) for bilinear transformation and about (3.3MHZ) for using prewarped bilinear transformation. The program can calculate the capacitor value for each section and for each sampling frequency.

4-2. SC BPF:

A BPF of 4th order, chebyshev type, with center frequency 3.1MHz, and passband ripple 0.5 db which, presented to illustrate the interdependent procedure to design SC filter with approximate Simpson transformation. In this case, two second order sections like that presented in [7,10], are needed to the realization of this filter. The procedure is similar to that one for LPF, except the step that frequency transformation for normalized LPF to BPF, and the order BPF will be doubled the order of LPF. Then after frequency transformation step, the step of factorizing the polynomials of denominator and numerator, to the two sections of 2nd order filters, the pole (zero) frequency Ω_p (Ω_z) and quality factor $q_p(q_z)$ are calculated for each section, then transform the transfer function of the two sections, to discrete one using approximate Simpson transformation with optimal coefficients, with using of equations that shown in [7], after coefficients of two sections of the filter are calculated after using approximation to term $Z^{1/2}$,and $Z^{3/2}$ with best approximation, the frequency response of discrete filter would be calculated. and plotted, as shown in fig (9) for different sampling frequencies ,from this figure, the bandwidth is about 1MHz for continuous response. the frequency response of SC filter with Fs= 20MHz is shown in response No.(1), and the BW= 0.95MHz. and when increasing Fs= 40MHz, but with a response similar to that of continuous one, with a difference about 0.2db only, and when increase Fs= 80MHz, the BW= 0.98MHz. with a difference less than 0.1db. And when increasing Fs= 160 MHz, the difference was very small (less 0.05db).

From all these examples, it is clear that approximate Simpson transformation with optimal value of coefficient is better than all methods presented in this work, in frequency response when same sampling frequencies are used Therefore. one can use Simpson transformation with approximation $Z^{-1/2} = \left(1 + Z^{-1}\right)/2$ with optimal coefficients in low sampling frequency, and get same performance or better, then cost will be reduced because in using low sampling frequency, small chip area is needed.

5. Conclusion

An optimization method alternative to prewarping technique was presented in this paper to determine the parameter values of a certain transformation such as Simpson, Third-and Fourth-order transformations. The optimal values have been determined using Hooke and Jeeves methods; this technique minimizes a given error between ideal prewarping and the prewarping relationship response. When these optimized coefficients are used ,the resultant Z-domain transfer function will have a minimum error between the continuous transfer function and discrete transfer function. This is true for all transformation (Simpson, Third--and Fourth–order S-Z transformations). These transformations can be used in very low sampling frequency until the Nyquist rate.

The problem in these transformations is that it can't be implemented in SC-form; therefore, an approximation method for Simpson transformation was presented that approximate the $Z^{-1/2}$ term to a realizable term. The total capacitor area in SC filter is directly proportional to the sampling frequency, and also the non-ideal effects of the MOS elements become significant for high sampling frequency. Therefore, the lowest sampling frequency (Fs) has to be chosen in order firstly to satisfy the Nyquist condition and secondly with regard to the noise and leakage current generated in MOS device.

تأثير الانفتال المتعلق بتحويل S-Z واضافته في تصميم مرشح المتسعة المفتاحية

الخلاصة

تأثير الانفتال المتعلق بتحويل S-Z يمثل محددات واضحة في تصميم مرشح المتسعة المفتاحية ، خصوصا في الترددات العالية . إحدى الطرق لتقليل تأثير الانفتال هو استخدام تردد نمذجة عالي نسبة إلى تردد الإشارة . هذه الطريقة تنتج مرشح مع متسعة انتشار عالية والتي تتطلب مساحة تصنيع كبيرة . هذا البحث يقدم تقنية تصميمية جديدة تعتمد على تحويل سمبسون S-Z . طبقت تقنية مثالية للحصول على معامل التحويل المثالي والذي يقلل بصورة واضحة تأثير الانفتال .

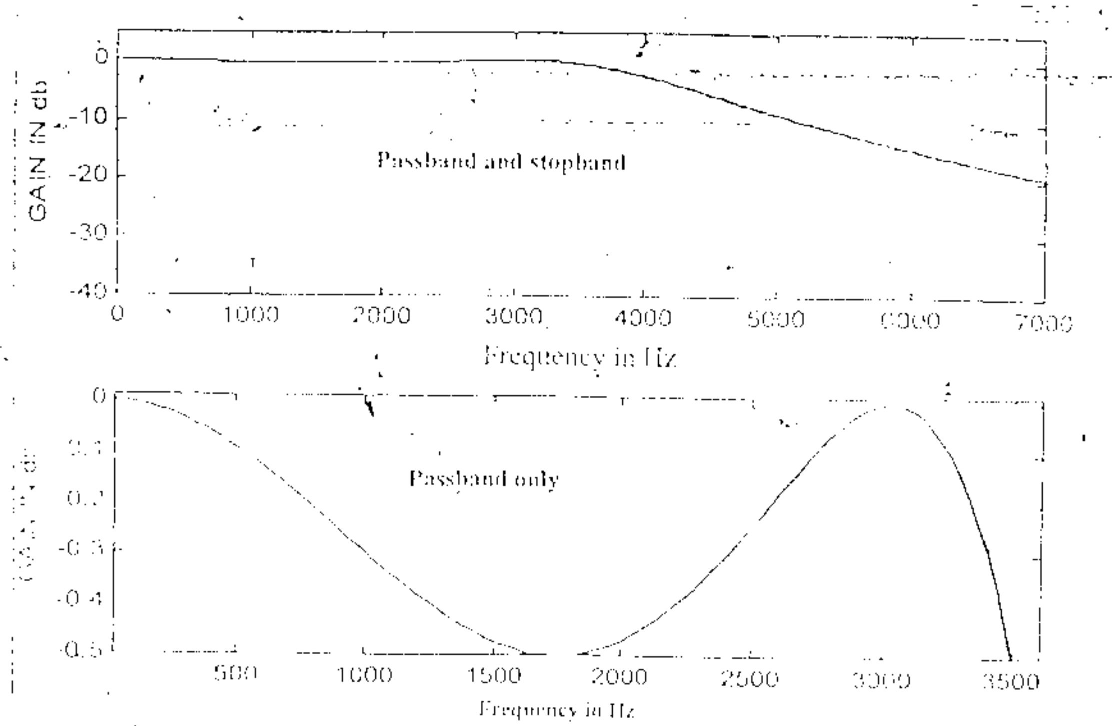

Fig (7): Frequency responses for 3rd order Chebyshev LPF with cutoff frequency using fourth order transformation with optimized coefficients of Table (3) at Fs=15 KHz.

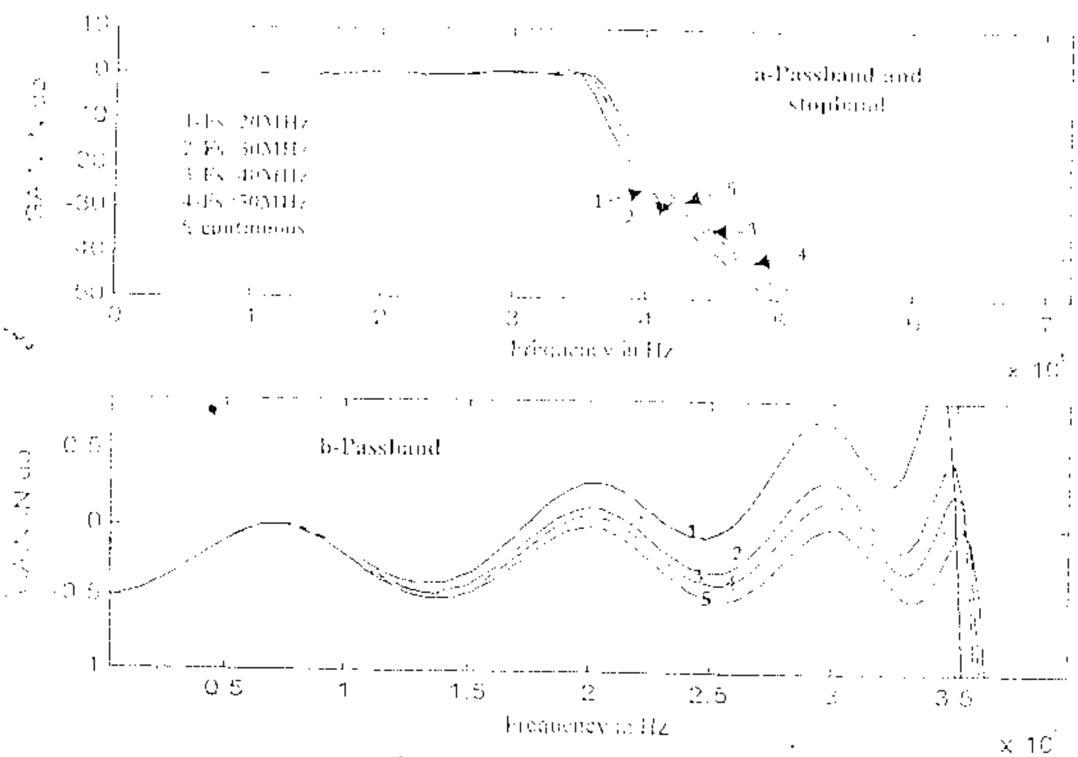

Fig (8): frequency response for 8th-order Chebyshev LPF with cutoff frequency 3.6MHZ using of Simpson transformation for different sampling frequencies of approximation $Z^{-1/2}=(1+Z^{-1})/2$.

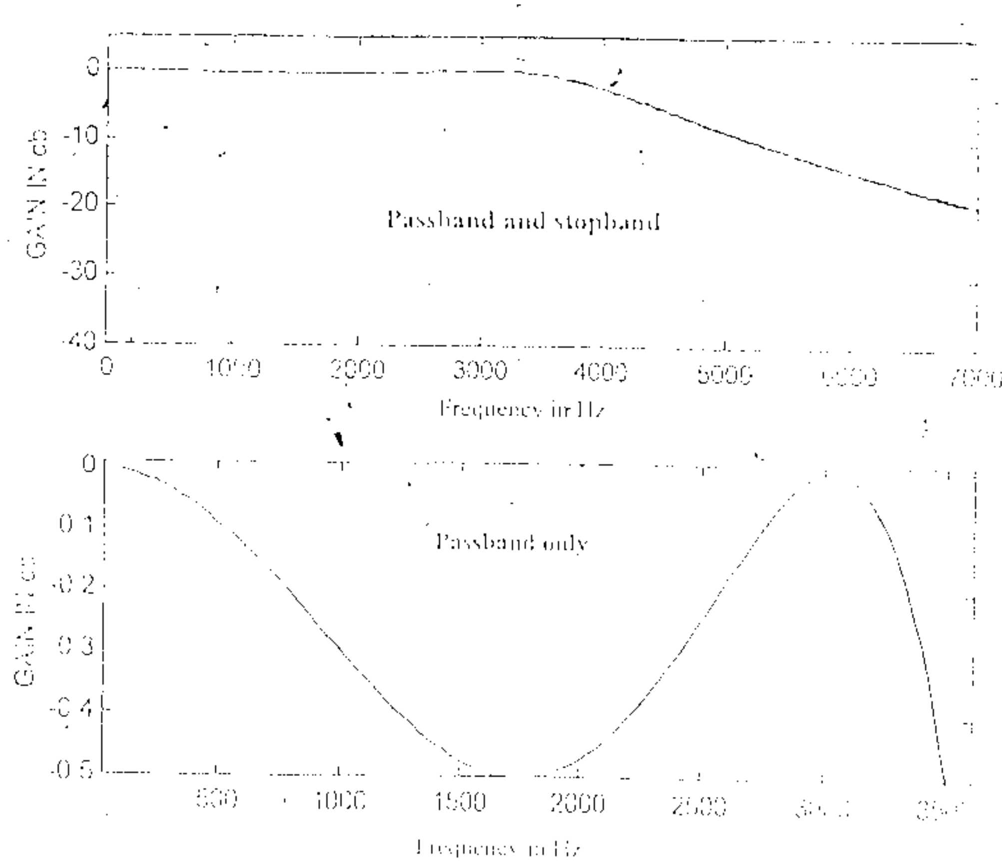

Fig (5). Frequency responses for 3rd order Chebyshev LPF with cutoff frequency ... third order transformation with optimized coeffecients of Table (2) at Fs ...

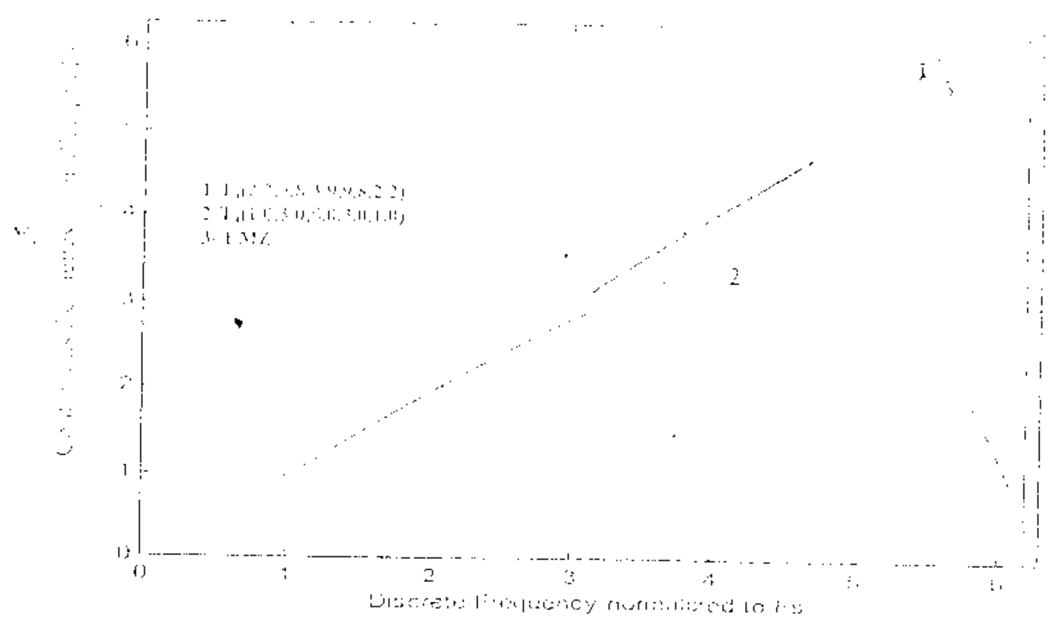

Fig (6) : prewarping effects for fourth S-Z transformation using 1-Optimized coefficients 2-random value 3-ideal warping

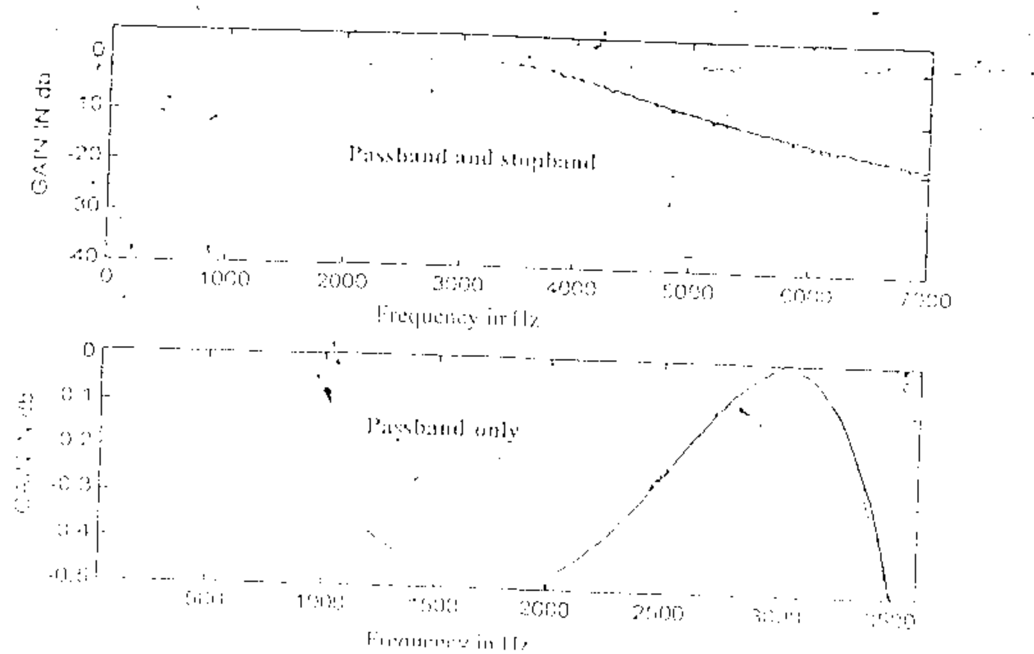

Fig. (3): Frequency responses for 3rd order Chebyshev LPF with cutoff frequency using Simpson transformation with optimized coefficients of Table (1) at Fs 15kHz.

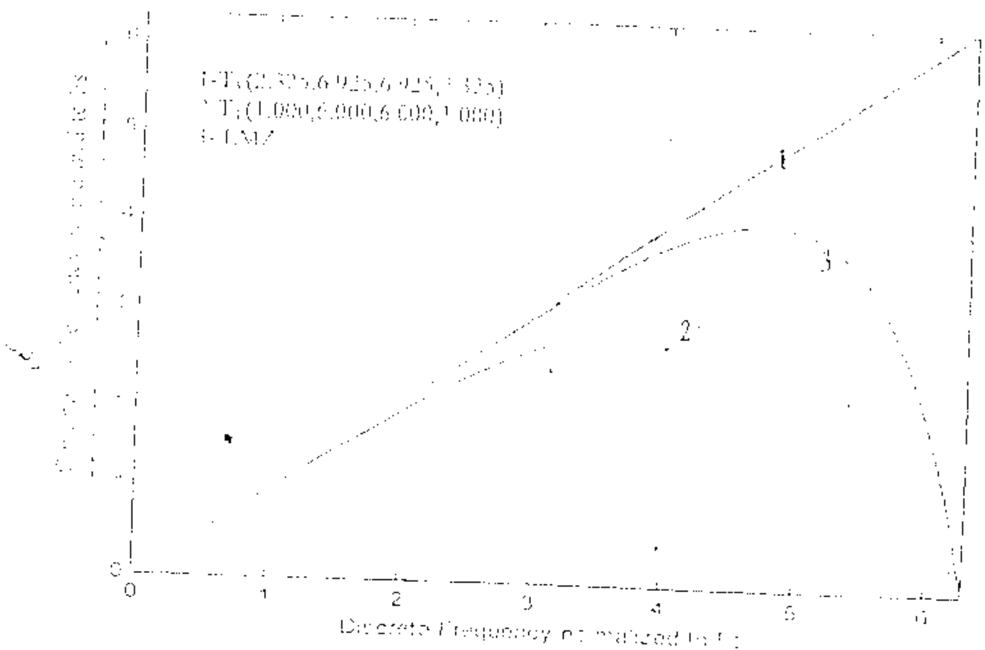

Fig (4): Prewarping effects for third S-Z transformation using 1-Optimized coefficients 2-random value 3-ideal warping

Fig (1) prewarping effects for Simpson, Third and Fourth order S-Z tra[illegible] different weights of coefficients.

Fig (2). Prewarping effects for Simpson S-Z transform [illegible] 1-Optimized coefficients 2-[illegible] value 3-[illegible] warping