THE NORMAL -OF –PLANE DYNAMICAL BEHAVIOR OF CURVED PIPES CONVEYING PULSATING FLUID.

Haithem H. Al-Daamy
University Of Babylon
College Of Engineering

Ala M. Hosain M.
University Of Babylon
College Of Engineering

Abdul Karim Al-Humdany
University Of Babylon
College Of Engineering

Abstract

This paper investigate the dynamics of curved pipes conveying pulsating flowing fluid. The pipe is considered to oscillate normal to the plane of curvature . Clamped ends conditions are imposed at the extreme ends and pinned condition is considered at the intermediate support.

The regions of instability are examined , using conventional methods to identify conditions of unstable operation of the pipe . The intermediate support is changed to show the effect of the intermediate support position on regions of instability. The regions of instability have the higher values when the intermediate support at the middle position .

The effect of excitation parameter and flow velocity on the dynamical behavior of the pipe are also examined .

Nomenclature :-

E : Young's modulus (N/m^2)
G: Shear modulus (N/m^2)
I: Second moment of area (m^4)
J: Polar moment of area (m^4)
K: Stiffness ratio (GJ / EI)
m_f : Fluid mass per unit length (kg /m)
m_p: Tube mass per unit length (kg /m)
R: Radius of pipe curvature (m)
t : Time (sec.)
V: Transverse displacement along z- axis .
U: Fluid velocity (m/ s)
α : Total angle of curvature of the pipe (rad)
ω : Circular frequency (Hz)

Introduction:-

The problem of stability of pipe lines in thermal power stations , fuel lines in combustion engines , heat exchangers and other piping systems indicates that specific attention should be taken into consideration for the dynamical behavior . A good selection of the parameters controlling the dynamics of piping systems which ensure stable operation, leads to safe operation for these systems. The stability characteristics of curved tubes as well as straight tubes conveying steady flowing fluid are available in the literature. However , the stability characteristics of curved tubes moves normal to the plane of curvature conveying pulsating flow could not be found .

The dynamical behavior of curved tubes conveying steady flow has received a considerable attention in the literature . For example the general theory which accounts for circular arcs vibrations was presented by Chen[1]. It was assumed that the centerline of the tube is inextensible and he found that the tubes lost their stability by static buckling when the flow exceeds a certain critical value .

Hill and Davis [2] studied the dynamics and stability of clamped - clamped pipes conveying fluid, shaped as circular arcs as well as (S & L) shaped and spiral configurations. They obtained that when the initial forces are taken into account, then the pipes supported at ends do not lose stability , it doesn't matter how the flow velocity may be. Long [3], Benjamin [4] and Gregory [5] have extensive theoretical and experimental investigation for the stability of pipes with different end conditions. The effect of harmonic fluctuation of the fluid flow on cantilever pipes was studied by Paidoussis and Issid [6]. They concluded that instability occurs by resonance appearance in two regions called "Primary " and " Secondary " regions of instability..

Theoretical analysis :-

The system under investigation is a uniformly curved pipe containing pulsating flowing fluid, see Fig. (1) for representation of coordinates and displacements of the pipe. The curved pipe is subjected to bending and twisting actions during the normal to – plane motion. The flexural rigidity of the pipe is EI, mass per unit length is m_p , mass of fluid per unit length is m_f and and velocity of U.

The equation of motion of the pipe is derived according to Hamilton's principle. The equation of motion is [1] : -

$$\frac{EI}{R^3}\left(\frac{\partial^4 V}{\partial\theta^4} - R\frac{\partial^2\varphi}{\partial\theta^2}\right) - \frac{GJ}{R^3}\left(\frac{\partial^2 V}{\partial\theta^2} + R\frac{\partial^2\varphi}{\partial\theta^2}\right) + \frac{m_f U^2}{R}\frac{\partial^2 V}{\partial\theta^2} + 2m_f U\frac{\partial^2 V}{\partial\theta\,\partial t} + m_f\frac{\partial U}{\partial t}\frac{\partial V}{\partial\theta}$$

$$+ R(m_p + m_f)\frac{\partial^2 V}{\partial t^2} = 0 \text{.....................(1)}$$

and

$$\frac{EI}{R^2}\left(R\varphi - \frac{\partial^2 V}{\partial\theta^2}\right) - \frac{GJ}{R^2}\left(R\frac{\partial^2\varphi}{\partial\theta^2} + \frac{\partial^2 V}{\partial\theta^2}\right) = 0 \text{ ....(2)}$$

Equations (1) & (2) are the general coupled equations of motion for a curved tube in a dimensional form . Introducing the following non dimensional quantities :-

$$v = \frac{V}{R} \quad , \qquad \beta = \frac{m_f}{m_f + m_p} \quad ,$$

$$u = \left(\frac{m_f}{EI}\right)^{\frac{1}{2}} R\,U \qquad , \tau = \left(\frac{EI}{m_p + m_f}\right)^{\frac{1}{2}}\frac{t}{R^2} \qquad , K = \frac{GJ}{EI}$$

Equations (1) & (2) will take the form:-

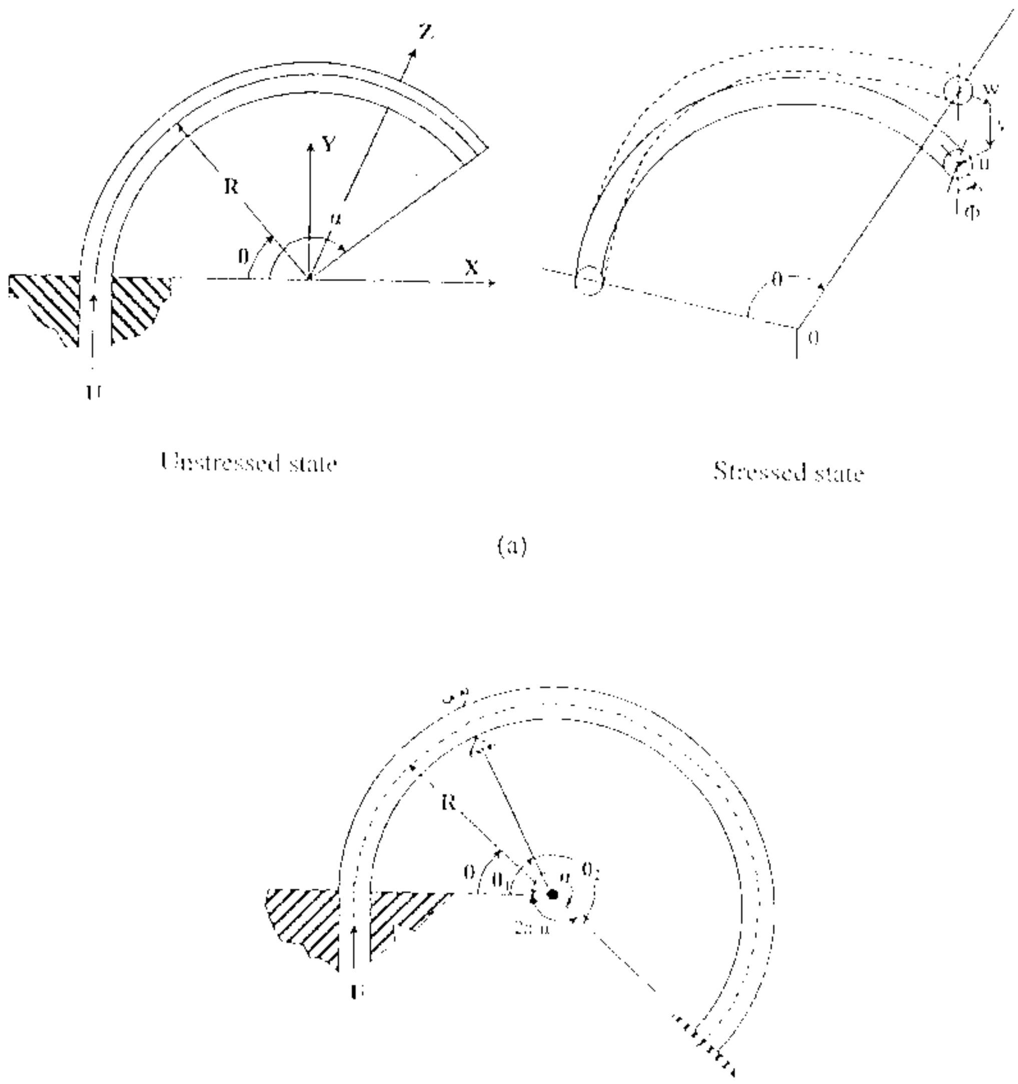

Figure (1): Representation of coordinates and displacements of uniformly curved pipe conveying fluid.

$$\frac{\partial^2 u}{\partial \theta^2}-\frac{\partial^2 \varphi}{\partial \theta^2}-K\left(\frac{\partial^2 u}{\partial \theta^2}+\frac{\partial^2 \varphi}{\partial \theta^2}\right)+\upsilon^2\frac{\partial^2 u}{\partial \theta^2}+2\beta^{\frac{1}{2}}\upsilon\frac{\partial^2 u}{\partial \theta\,\partial \tau}+\beta^{\frac{1}{2}}\frac{\partial \upsilon}{\partial \tau}\frac{\partial u}{\partial \theta}+\frac{\partial^2 u}{\partial \tau^2}=0 \quad \text{.......(3)}$$

and

$$\frac{\partial^2 u}{\partial \theta^2}-\varphi+K\left(\frac{\partial^2 \varphi}{\partial \theta^2}+\frac{\partial^2 u}{\partial \theta^2}\right)=0 \quad \text{........(4)}$$

Equations (3) &(4) are two coupled partial differential equations ; eliminating (u) yields :-

$$\frac{\partial^6 \varphi}{\partial \theta^6}+(2+\upsilon^2)\frac{\partial^4 \varphi}{\partial \theta^4}+2\beta^{\frac{1}{2}}\upsilon\frac{\partial^4 \varphi}{\partial \theta^3\partial \tau}+\beta^{\frac{1}{2}}\frac{\partial^3 \varphi}{\partial \theta^3}\frac{\partial \upsilon}{\partial \tau}+\frac{\partial^4 \varphi}{\partial \theta^2\partial \tau^2}+\left(1-\frac{\upsilon^2}{K}\right)\frac{\partial^2 \varphi}{\partial \theta^2}-\frac{2\beta^{\frac{1}{2}}\upsilon}{K}\frac{\partial^2 \varphi}{\partial \theta\,\partial \tau}$$

$$-\frac{\beta^{\frac{1}{2}}}{K}\frac{\partial \upsilon}{\partial \tau}\frac{\partial \varphi}{\partial \theta}-\frac{1}{K}\frac{\partial^2 \varphi}{\partial \tau^2}=0 \quad \text{.........(5)}$$

Since the flow is pulsating, the function describing the velocity is assumed to be harmonic which can be written as [9] :-

$$\upsilon=\upsilon_o[1+\delta\cos(\omega\tau)] \quad \text{........(6)}$$

Substituting Eq. (6) into (5) gives :-

$$\frac{\partial^6 \varphi}{\partial \theta^6}+[2+\upsilon_o^2\gamma^2]\frac{\partial^4 \varphi}{\partial \theta^4}+2\beta^{\frac{1}{2}}\upsilon_o\gamma\frac{\partial^4 \varphi}{\partial \theta^3\partial \tau}-\beta^{\frac{1}{2}}\upsilon_o\Omega\delta\sin\omega\tau\frac{\partial^3 \varphi}{\partial \theta^3}+\left[1-\frac{1}{K}\upsilon_o^2\gamma^2\right]\frac{\partial^2 \varphi}{\partial \theta^2}$$

$$-\frac{2\beta^{\frac{1}{2}}}{K}\upsilon_o\gamma\frac{\partial^2 \varphi}{\partial \theta\,\partial \tau}+\frac{\beta^{\frac{1}{2}}}{K}\upsilon_o\delta\sin\omega\tau\frac{\partial \varphi}{\partial \theta}-\frac{1}{K}\frac{\partial^2 \varphi}{\partial \tau^2}=0 \quad \text{.........(7)}$$

$$where \quad \gamma=(1+\delta\cos\omega\tau) \quad \text{.............(8)}$$

Regions of Instability :-

The regions of instability can be found by solving Eq. (7) .The solution of (7) is :

$$\varphi(\theta,\tau)=\sum_{q=0}^{\infty}\psi_q(\theta)\sin\left(\frac{1}{2}q\omega\tau\right)+\psi_q(\theta)\cos\left(\frac{1}{2}q\omega\tau\right) \quad \text{..........(9)}$$

The primary regions of instability are found when q = 1,3,5,…….., while the secondary instability regions obtained when q = 0,2,4,……..

Truncating the series at q=1 for the primary regions and at q= 2 for the secondary regions , one get the principal primary and principal secondary regions of instability. Solving the equation corresponding to the principal primary regions , we get :-

$$\psi_1 = \sum_{i=1}^{6} B_i \, e^{\lambda_i \theta} \qquad ....(10a)$$

and

$$\psi_2 = \sum_{j=1}^{6} b_j \, e^{\lambda_j \theta} \qquad ....(10b)$$

where λ_i's & λ_j's are the roots of the following two equations respectively:-

$$\lambda_1^6 + (2 + \upsilon_o^2 + \frac{\upsilon_o^2 \delta^2}{2} - \upsilon_o^2 \delta)\lambda_1^4 + (1 - \frac{\upsilon_o^2}{K} - \frac{\upsilon_o^2 \delta^2}{2K} - \frac{\omega^2}{4} + \frac{\upsilon_o^2 \delta}{K})\lambda_1^2 - \frac{\omega^2}{4K} = 0 \; ....(11)$$

and

$$\lambda_2^6 + (2 + \upsilon_o^2 + \frac{\upsilon_o^2 \delta^2}{2} + \upsilon_o^2 \delta)\lambda_2^4 + (1 - \frac{\upsilon_o^2}{K} - \frac{\upsilon_o^2 \delta^2}{2K} - \frac{\Omega^2}{4} + \frac{\upsilon_o^2 \delta}{K})\lambda_2^2 + \frac{\omega^2}{4K} = 0 \; ......(12)$$

Equations (10 a) & (10 b) gives the upper and the lower limits of unstable regions in terms of ψ. To define the regions of instability in terms of the non- dimensional displacement (t), the relation between t & ψ is [1] :-

$$t = \frac{1}{1+K}\left(\iint \psi \, d\theta \, d\theta - K\psi \right) ....(13)$$

substituting Eq.(10) into (13) gives the upper and the lower limits of the principal regions of instability as:-

$$t = \frac{1}{1+K} \sum_{i=1}^{6} B_i \left(K - \frac{1}{\lambda_i^2}\right) e^{\lambda_i \theta} \; .....(14a)$$

$$t = \frac{1}{1+K} \sum_{j=1}^{6} b_j \left(K - \frac{1}{\lambda_j^2}\right) e^{\lambda_j \theta} \; ....(14b)$$

The upper and the lower limits of the principal secondary regions of instability can be found in the same procedure as that for the principal primary regions. The upper limit is given by:-

$$t = \frac{1}{1+K} \sum d_i \left(K - \frac{1}{\lambda_{3i}^2}\right) e^{\lambda_{3i} \theta} ....(15)$$

Where λ_3's are the roots of the polynomial :-

$$\lambda_3^6 + (2 + \upsilon_o^2 + \frac{\upsilon_o^2 \delta^2}{2} + \upsilon_o^2 \delta^2)\lambda_3^4 + (1 - \frac{\upsilon_o^2}{K} - \frac{\upsilon_o^2 \delta^2}{K} - \omega^2)\lambda_3^2 + \frac{\omega}{K} = 0 ......(16)$$

and the lower limit is given by the following two coupled equations :-

$$u_o = \frac{1}{1+K}\sum_{j=1}^{12} d_{oj}\,(K - \frac{1}{\lambda_{oj}^2})\,e^{\lambda_{oj}\theta}$$

$$u_2 = \frac{1}{1+K}\sum_{j=1}^{12} d_{2j}\,(K - \frac{1}{\lambda_{oj}^2})\,e^{\lambda_{oj}\theta} \qquad \text{.........(17)}$$

where λ_{oj}'s are the roots of the polynomial :

$$[\lambda_o^6 + (2 + \upsilon_o^2 + \frac{1}{2}\upsilon_o^2\delta^2)\lambda_o^4 + (1 - \frac{\upsilon_o^2}{K} - \frac{\upsilon_o^2}{2K}\delta^2)\lambda_o^2]^2 -$$

$$(\upsilon_o^2\delta\lambda_o^3 - \frac{\upsilon_o^2\delta}{K}\lambda_o^2)(2\upsilon_o^2\delta\lambda_o^4 - \frac{2\upsilon_o^2\delta^2}{K} - \lambda_o^2) = 0 \qquad \text{...........(18)}$$

Boundary conditions:-

1. At the clamped ends :-

$$u(0,t) = u(\alpha,t) = 0$$

$$\frac{\partial u}{\partial \theta}(0,t) = \frac{\partial u}{\partial \theta}(\alpha,t) = 0 \qquad \text{.........(19)}$$

$$\phi(0,t) = \phi(\alpha,t) = 0$$

2. At the intermediate support :-

$$u_1(\theta_1,t) = 0$$

$$\frac{\partial u_1}{\partial \theta}(\theta_1,t) = \frac{\partial u_2}{\partial \theta}(0,t)$$

$$\phi_1(\theta_1,t) = \phi_2(0,t)$$

$$\frac{\partial^2 u_1}{\partial \theta^2}(\theta_1,t) - \varphi(\theta_1,t) = \frac{\partial^2 u_2}{\partial \theta^2}(0,t) - \varphi_2(0,t) \qquad \text{........(20)}$$

$$\frac{\partial u_1}{\partial \theta}(\theta_1,t) + \frac{\partial \varphi_1}{\partial \theta}(\theta_1,t) = \frac{\partial u_2}{\partial \theta}(0,t) + \frac{\partial \varphi_2}{\partial \theta}(0,t)$$

$$u_2(0,t) = 0$$

Where subscripts 1 & 2 refer to the left and the right sides of the intermediate support respectively .

Limits of Instability Regions:-

To calculate the primary or secondary principal limits of instability , the solutions 10,14,15,and 17 are substituted into the boundary conditions 20, which result a set of algebraic equations for each limit. They can be written as :-

$$[A_{ij}]\{C_j\} = 0 \qquad \text{.......(21)}$$

The value of the frequency (ω) that makes the determinant of (21) equal zero , represent the natural frequency for the corresponding limits .

Results and Discussions :-

The effect of intermediate support positioning and the excitation parameter on the regions of instability is shown in figure (2). This figure shows that the size of the unstable region is unaffected by the position of intermediate support. This position shifts the whole region to higher or lower frequencies. Further it may be noticed that for the first mode, the two regions always shift to higher frequencies, however, they fluctuate to higher or lower values for others modes. This may be explained by a careful observation of the equations correspond to the upper and lower regions respectively. These equations indicate that the size of the unstable region depends on the flow velocity and the excitation parameter only and there is no dependence of the latter parameters on the intermediate support positioning.

The effect of fluid flow velocity (υ_o) on the regions of instabilities is indicated in figure (3). The excitation parameters has an effect on the area of the unstable regions. This behavior can be explained as:-

1. For the time effect , ΔT^* is proportional to ($\upsilon_{max} - \upsilon_o$) and υ_{max} depends on the amplitude of excitation , therefore ΔT is proportional to A.
2. The interia forces delivered from fluid fluctuation do not depend on υ_o only but also on the difference between υ_{max} and υ_o . This makes it clear that ($\upsilon_{max} - \upsilon_o$) is proportional to A, therefore, the intertia forces is proportional to A.

It is clear that the size of an unstable region increases with increasing the flow velocity for both the primary and secondary regions. To elaborate on this behavior, it may be noticed that the time period of harmonic fluctuation in the fluid velocity occurs at 2T and T for the two regions respectively .The resonance appears at these two periods due to:-

1. The time duration through the reduction in flow velocity from the higher limit (υ_{max}) to the mean velocity (υ_o) increases with increasing the flow velocity , therefore the size of the unstable region becomes wider with increasing υ_o .
2. The inertia forces delivered from the fluctuation of the fluid flow increases with increasing υ_o , hence this force will be an auxiliary factor in resonance appearance .

The variation of the regions of instability with the angle of curvature is shown in figure (4). The effect of this parameter is similar to that of the intermediate support position, i.e the angle of curvature lowers or lifts the whole region but it has no effect on the area of the region of instability.

References:-

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* Note : ΔT = (T at υ_{max}) – (T at υ_o)

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السلوك الديناميكي لانبوب منحني ناقل مائع نبضي يتحرك عموديا على مستوى الانحناء

الخلاصة

يتناول هذا البحث السلوك الديناميكي لانبوب منحني يمر خلاله مائع ذو سرعة غير منتظمة (سرعة نبضية)، هذا الانبوب يتحرك بالاتجاه العمودي على المستوي الذي يقع فيه الانبوب المنحني . وقد تم افتراض ان نهايتي الانبوب مثبتتين تثبيتاً محكماً ، اما المسند الوسطي فقد اعتبر انه مسند بسيط .

وقد استخدمت الاساليب الرياضية التحليلية لتشخيص المناطق التي يفقد فيها الانبوب استقراريته. وقد تم تغيير موقع المسند الوسطي لدراسة تاثيره على المناطق الغير مستقرة حيث وجد ان اعلى قيم للمناطق الغير مستقرة تحصل عندما يكون المسند الوسطي في المنتصف. وبالاضافة الى ذلك فان تاثير كل من سرعة الجريان وعامل الاثارة على المناطق الغير مستقرة قد رسمت على شكل منحنيات .

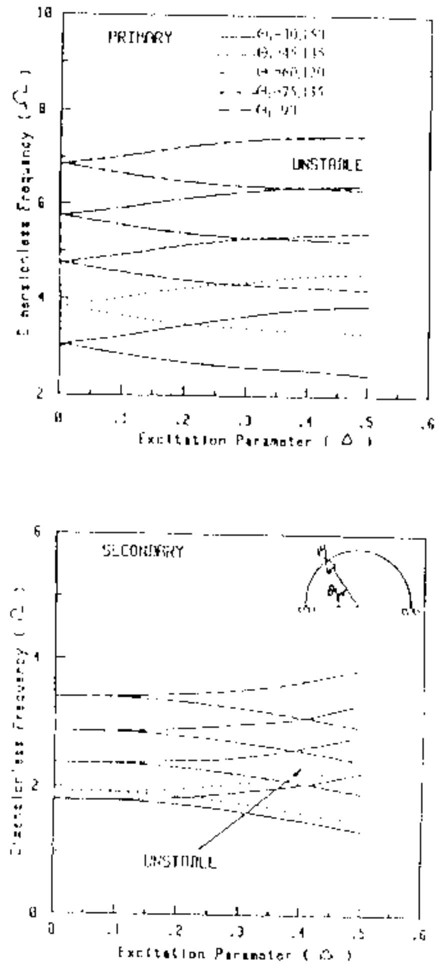

Fig. 2a: Effect of the intermediate supports positioning of curved pipe on regions of instability for the first mode.

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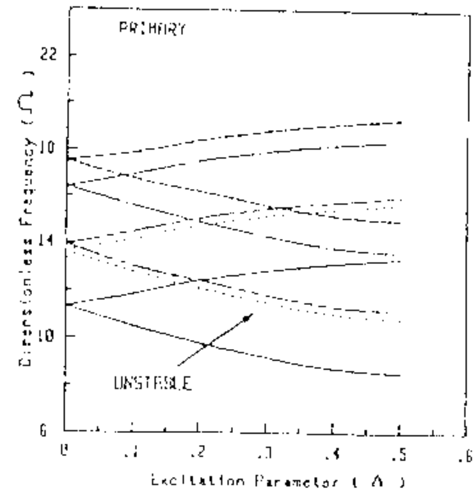

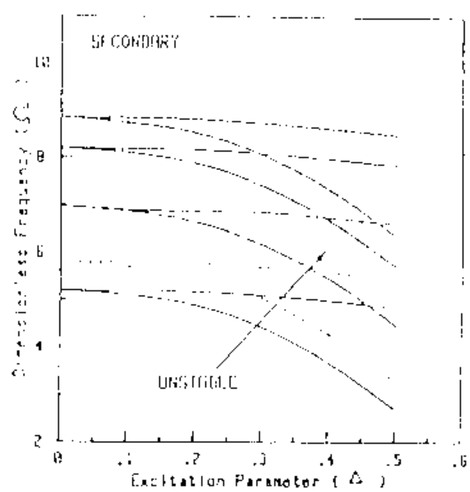

Fig.2b: Effect of the intermediate supports positioning of curved pipe regions of instability for the second mode.

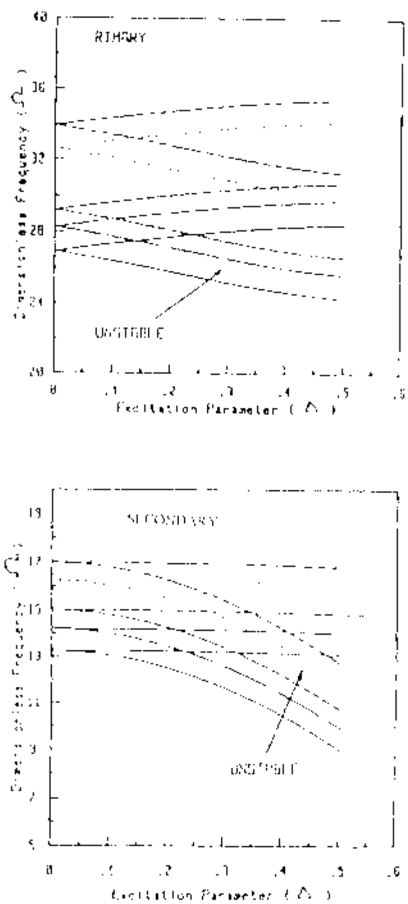

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Fig. 2c: Effect of the intermediate support positioning of curved pipe on regions of instability for the third mode.

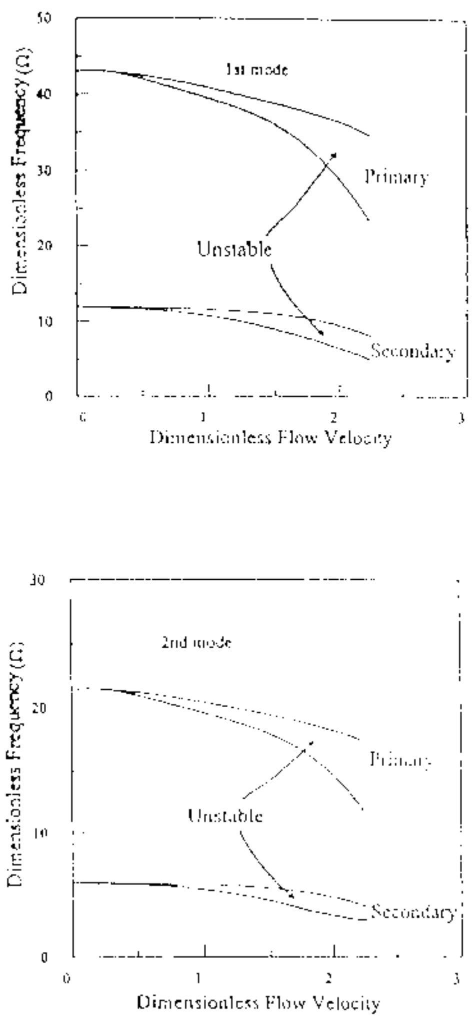

Fig. 3: Effect of flow velocity on regions of instability.

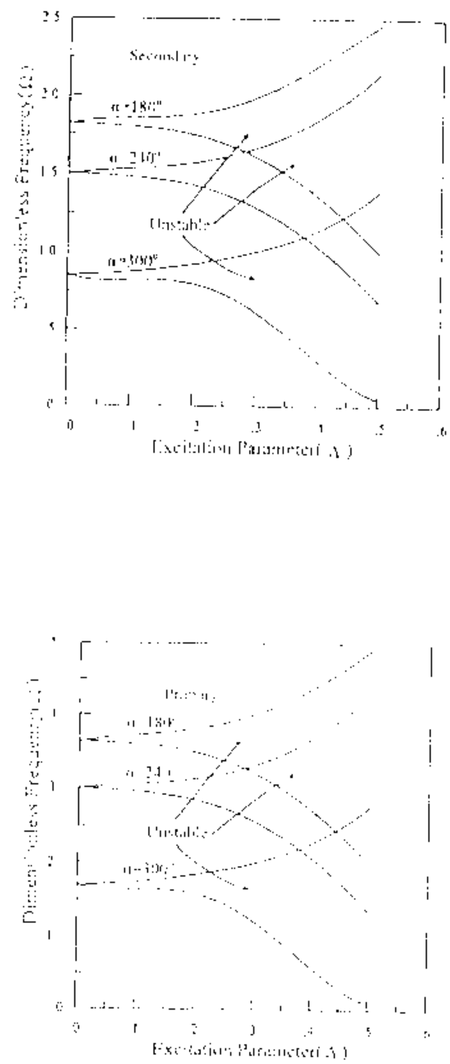

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Fig. 4: Effect of angle of curvature on regions of instability.