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SOME RESULTS ON SOLVABLE FUZZY SUBGROUP OF A GROUP

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Abstract

In this paper we introduce an alternative definition of a solvable fuzzy group and study some of its properties . Many of new results are also proved, which is useful and important in fuzzy mathematics .

1. Introduction

The concept of fuzzy sets is introduced by [1] .Rosenfeld introduced the notion of a fuzzy group as early as 1971.

The technique of generating a fuzzy group (the smallest fuzzy group) containing an arbitrarily chosen fuzzy set was developed only in 1992 by [2] .

Then many research study properties of fuzzy group , fuzzy subgroup of group ,fuzzy coset , and fuzzy normal sub group of group . In this paper we introduce the definition of solvable fuzzy group and study some of its properties .

Now we introduce the following definitions which is necessary and needed in the next section :

Definition 1.1 [2], [3]

A mapping from a nonempty set X to the interval [0, 1] is called a fuzzy subset of X .

Next, we shall give some definitions and concepts related to fuzzy subsets of G.

Definition 1.2

Let μ, v be fuzzy subsets of G, if $\mu(x) \leq v(x)$ for every $x \in G$, then we say that μ is contained in v (or v contains μ) and we write $\mu \subseteq v$ (or $v \supseteq \mu$).

If $\mu \subseteq v$ and $\mu \neq v$, then μ is said to be properly contained in v (or v properly contains μ) and we write $\mu \subset v$ (or $v \supset \mu$).[4]

Note that: $\mu = v$ if and only if $\mu(x) = v(x)$ for all $x \in G$.[5]

Definition 1.3 [4]

Let μ, v be two fuzzy subsets of G. Then $\mu \cup v$ *and* $\mu \cap v$ are fuzzy subsets as follows:

(i) $(\mu \cup v)(x) = \max\{\mu(x), v(x)\}$

(ii) $(\mu \cap v)(x) = \min\{\mu(x), v(x)\}$, for all $x \in G$

Then $\mu \cup v$ *and* $\mu \cap v$ are called the union and intersection of μ and v, respectively.

Definition 1.4 [5],[6]

For μ, v are two fuzzy subsets of G, we define the operation $\mu \circ v$ as follows:

$(\mu \circ v)(x) = sup\{min\{\mu(a), v(b)\} \mid a, b \in G \;\; and \;\; x = a * b\}$ For all $x \in G$.

We call $\mu \circ v$ the product of μ and v.

Now, we are ready to give the definition of a fuzzy subgroup of a group :

Definition 1.5[2], [7]

A fuzzy subset μ of a group G is a fuzzy subgroup of G if:

(i) $min\{\mu(a), \mu(b)\} \leq \mu(a * b)$

(ii) $\mu(a^{-1}) = \mu(a)$, for all $a, b \in G$.

Theorem 1.6 [4]

If μ is a fuzzy subset of G, then μ is a fuzzy subgroup of G, if and only if, μ satisfies the following conditions:

(i) $\mu \circ \mu \subseteq \mu$

(ii) $\mu^{-1} = \mu$

where $\mu^{-1}(x) = \mu(x)$, $\forall$ x $\in$G.

Proposition 1.7 [7]

Let μ be a fuzzy group. Then $\mu(a) \leq \mu(e) \quad \forall a \in G$.

Definition 1.8 [8]

If μ is a fuzzy subgroup of G, then μ is said to be abelian ,if $\forall\ x, y \in G$, $\mu(x) > 0, \mu(y) > 0$, then $\mu(xy) = \mu(yx)$.

Definition 1.9 [9] , [10]

A fuzzy subgroup μ of G is said to be normal fuzzy subgroup if $\mu(x * y) = \mu(y * x)$, $\forall x, y \in G$.

Definition 1.10 [11]

Let λ and μ be two fuzzy subsets of G. The commutator of λ and μ is the fuzzy subgroup $[\lambda, \mu]$ of G generated by the fuzzy subset (λ, μ) of G which is defined as follows, for any x $\in$ G:

$$(\lambda, \mu)(x) = \begin{cases} \sup \{\lambda(a) \wedge \mu(b)\} & \text{if } x \text{ is a commutator} \\ x = [a, b] & \\ & \end{cases}$$

Now, we introduce the following theorems about the commutator of two fuzzy subsets of a group which are needed in the next section :

Theorem 1. 11[11]

If A, B are subsets of G, then $[\chi_A, \chi_B] = \chi_{[A,B]}$

where for all x $\in$ G: $\chi_A(a) = \begin{cases} 1, & if\ \ a \in A \\ 0, & if\ \ a \notin A \end{cases}$

Theorem 1.12[11]

If λ, μ, β and δ are fuzzy subsets of G such that $\lambda \subseteq \mu$ and $\beta \subseteq \delta$, then $[\lambda, \beta] \subseteq [\mu, \delta]$.

Definition 1.13 [12]

A fuzzy subgroup μ of G is said to be normal fuzzy subgroup if $\mu(x * y) = \mu(y * x)$, $\forall x, y \in G$.

Corollary 1.14[11]

If λ, β are normal fuzzy subgroups of μ and δ, respectively. Then $[\lambda, \beta]$ is a normal fuzzy subgroup of $[\mu, \delta]$.

Proposition 1.15 [3]

Let λ be a fuzzy subgroup of a fuzzy group μ, then λ is a normal fuzzy subgroup in μ if and only if λ_t is normal subgroup in $\mu_t, \forall t \in (0,1]$, where $\lambda(e) = \mu(e)$.

Now, we introduce an important concept about the fuzzy subset.

Definition 1.16[11]

Let λ be a fuzzy subset of G. Then the tip of λ is the supremum of the set $\{\lambda(x) | x \in G\}$.

Theorem 1. 17[11]

Let $\lambda and \mu$ be fuzzy subsets of G. Then the tip of $[\lambda, \mu]$ is the minimum of tip of λ and tip of μ.

Now, we are ready to define the concept of derived chain, which is of great importance in the next section :

Definition 1. 18[11]

Let λ be a fuzzy subgroup of G. We call the chain

$$\lambda = \lambda^{(0)} \supseteq \lambda^{(1)} \supseteq \ldots\ldots \supseteq \lambda^{(n)} \supseteq \ldots.$$

of fuzzy subgroups of G the derived chain of λ.

2. Solvable Fuzzy Subgroups of A Group

In this section, we propose an alternative definition of a solvable fuzzy group and study some of its properties :

Definition 2.1

Let λ be a fuzzy subgroup of G with tip α. If the derived chain $\lambda = \lambda^{(0)} \supseteq \lambda^{(1)} \supseteq \ldots\ldots\ldots\ldots \supseteq \lambda^{(n)} \supseteq \ldots\ldots$ of λ terminates finitely to $(e)_\alpha$, then we call λ a solvable fuzzy subgroup of G. If k is the least nonnegative integer such that $\lambda^{(k)} = (e)_\alpha$, then we call the series $\lambda = \lambda^{(0)} \supseteq \lambda^{(1)} \ldots\ldots\ldots \supseteq \lambda^{(k)} = (e)_\alpha$ the derived series of λ.

Now, we introduce another definition of solvable fuzzy groups.

Definition 2.2

Let λ be a fuzzy subgroup of G with tip α. We call a series $\lambda = \lambda_0 \supseteq \lambda_1 \supseteq \ldots\ldots\ldots\ldots \supseteq \lambda_n = (e)_\alpha$ of fuzzy subgroups of G a solvable series for λ if for $0 \le i < n$, $[\lambda_i, \lambda_i] \subseteq \lambda_{i+1}$. This is equivalent to saying that :

min $\{\lambda_i(a), \lambda_i(b)\} \leq \lambda_{i+1}([a,b]), \quad for\ 0 \leq i < n.$

To prove the equivalence between definitions (2.1) and (2.2), we need firstly the following proposition .

Proposition 2.3

If $\lambda = \lambda^{(0)} \supseteq \lambda^{(1)} \supseteq \ldots\ldots\ldots\ldots \supseteq \lambda^{(n)} = (e)_\alpha$ be a derived series and $\lambda = \lambda_0 \supseteq \lambda_1 \supseteq \ldots\ldots\ldots \supseteq \lambda_n = (e)_\alpha$ be a solvable series , then : $\lambda^{(i)} \subseteq \lambda_i, \ \ i = 0,1\ldots\ldots\ldots, n.$

Proof

We show by induction that $\lambda^{(i)} \subseteq \lambda_i, i = 0,1,\ldots.n$ .we have $\lambda^{(0)} = \lambda = \lambda_0$, and we have $\lambda^{(1)} = [\lambda^{(0)}, \lambda^{(0)}] = [\lambda_0, \lambda_0] \subseteq \lambda_1$.

Therefore, the result holds for $n = 1$.

Now, let $\lambda^{(i)} \subseteq \lambda_i$ for some $i = 0,1,\ldots.n-1$ then by theorem (1.12), $\lambda^{(i+1)} = [\lambda^{(i)}, \lambda^{(i)}] \subseteq [\lambda_i, \lambda_i] \subseteq \lambda_{i+1}$

Hence, $\lambda^{(i)} \subseteq \lambda_i$ for $i = 0,1,\ldots\ldots n$.

Now, we are ready to prove the equivalence between definitions (2.1) and (2.2)

Proposition 2.4

Definition (2.1) $\Leftrightarrow$ Definition (2.2).

Proof

Let λ be a fuzzy subgroup of G with tip α. First, suppose definition (2.1) is hold then the derived series of λ is a solvable series for λ .That is definition (2.2) hold . Conversely, suppose definition (2.2) is hold that is, λ have a solvable series $\lambda = \lambda_0 \supseteq \lambda_1 \supseteq \ldots\ldots\ldots\ldots \supseteq \lambda_n = (e)_\alpha$ such that $[\lambda_i, \lambda_i] \subseteq \lambda_{i+1}$ for some $0 \leq i < n$.

Form proposition (2.3), $\lambda^{(i)} \subseteq \lambda_i$, for 0 ≤ i ≤ n .

Therefore, we get $(e)_\alpha \subseteq \lambda^{(n)} \subseteq \lambda_n = (e)_\alpha$. Consequently, λ is solvable.

That is, definition (2.1) is hold.

Now ,we introduce a nontrivial example of a solvable fuzzy subgroup of the group S_4 (the group of all permutations on the set $\{1,2,3,4\}$) .

Example 2.5

Let $D_4 = \{(1),(12)(34),(13)(24),(14)(23),(24),(1234),(1432),(13)\}$ .Which is a dihedral subgroup of S_4 with center $C = \{(1),(13)(24)\}$.

Let λ be the fuzzy subset of S_4 defined by :

$$\lambda(x) = \begin{cases} 1 & \text{if } x \in C = \{(1),(13)(24)\} \\ ½ & \text{if } x \in \{\langle(1234)\rangle \setminus C\} \\ ¼ & \text{if } x \in D_4 \setminus \langle(1234)\rangle \\ 0 & \text{if } x \in S_4 \setminus D_4 \end{cases}$$

$$\begin{cases} 1 & \text{if } x = (1) \\ ¼ & \text{if } x = (13)(24) \\ 0 & \text{otherwise} \quad , x \in S_4 \end{cases}$$

Clearly, λ is a fuzzy subgroup of S_4 .The fuzzy subgroup $\lambda^{(1)}$ has the following definition:

$\lambda^{(1)}(x) =$

And ,

$$\lambda^{(2)}(x) = \begin{cases} 1 & \text{if } x = (1) \\ 0 & \text{otherwise} \quad , x \in S_4 \end{cases}$$

Thus, we have $\lambda = \lambda^{(0)} \supseteq \lambda^{(1)} \supseteq \lambda^{(2)} = (e)_1$

Hence, λ is a solvable fuzzy subgroup of S_4 .

We will give the following remark :

Remark 2.6

1) If G is a solvable group, then 1_G is a solvable fuzzy subgroup of G.

Proof

Let G be a solvable group, then $G = G^{(0)} \supseteq G^{(1)} \supseteq .... \supseteq G^{(n)} = (e)$. Now :

$$1_G = 1_{G^{(0)}} \supseteq 1_{G^{(1)}} \supseteq .... \supseteq 1_{G^{(n)}} \supseteq 1_{\{(e)\}} = (e)_1$$

Hence, from definition (2.1), 1_G is a solvable fuzzy subgroup of G.

2) If H is a subgroup of G, then for all n ≥ 1, $(\chi_H)^{(n)} = \chi_{H^{(n)}}$. [2]

Now, we can give the following example :

Example 2.7

$(S_3, \circ)$ is solvable group with $S_3 \supseteq A_3 \supseteq \{e\}$. Now ,let $\lambda(x) = 1_{S_3}$ that is $\lambda(x) = 1$ for all $x \in S_3$:

$$\lambda^{(1)}(x) = 1_{A_3} \qquad \text{that is } \lambda^{(1)}(x) = \begin{cases} 1 & \text{if } x \in A_3 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\lambda^{(2)}(x) = 1_{\{e\}} \qquad \text{that is } \lambda^{(2)}(x) = \begin{cases} 1 & \text{if } x = \{e\} \\ 0 & \text{otherwise} \end{cases}$$

then $\lambda = \lambda^{(0)} \supseteq \lambda^{(1)} \supseteq \lambda^{(2)} = (e)_1$

That is $1_{S_3} \supseteq 1_{A_3} \supseteq 1_{\{e\}} = (e)_1$

Thus , 1_{S_3} is a solvable fuzzy subgroup of S_3.

Now we introduce the following theorem :

Theorem 2.8

A subgroup A of G is solvable if and only if ,χ_A is a solvable fuzzy subgroup of G

Proof

Let A be any subgroup of G. Consider the derived chain of A

$$A = A^{(0)} \supseteq A^{(1)} \supseteq \ldots\ldots\ldots\ldots \supseteq A^{(n)} \supseteq \ldots\ldots\ldots\ldots$$

From Remark (2.6) , $(\chi_A)^{(n)} = \chi_{A^{(n)}}$ for each $n \geq 0$ and therefore, $(\chi_A)^{(n)} = (e)_1$ if and only if, $A^{(n)} = (e)$. The result now is clear .

Now, we have the following result :

Theorem 2.9

If $\lambda and \mu$ are two solvable fuzzy subgroups of G. Then $[\lambda, \mu]$ is a solvable fuzzy subgroup of G.

Proof

Let λ is solvable fuzzy subgroup of G with tip α_1, then :

$$\lambda = \lambda^{(0)} \supseteq \lambda^{(1)} \supseteq \ldots\ldots\ldots \supseteq \lambda^{(n)} = (e)_{\alpha_1}$$

Also, μ is solvable fuzzy subgroup of G with tip α_2, then :

$$\mu = \mu^{(0)} \supseteq \mu^{(1)} \supseteq ....\supseteq \mu^{(n)} = (e)_{\alpha_2}$$

From theorem (1.17), the tip of $[\lambda, \mu] = \alpha_1 \wedge \alpha_2$. Therefore ,

$$[\lambda, \mu] = [\lambda^{(0)}, \mu^{(0)}] \supseteq [\lambda^{(1)}, \mu^{(1)}] \supseteq .... \supseteq [\lambda^{(n)}, \mu^{(n)}] = (e)_{\alpha_1 \wedge \alpha_2}$$

Then $[\lambda, \mu]$ is solvable fuzzy subgroup of G.

Next, we shall state the following theorem :

Theorem 2.10

Let λ be a solvable fuzzy subgroup of G. Then λ_t is a solvable, $\forall t \in (0,1]$.

Proof

Suppose λ is a solvable fuzzy subgroup of G with tip α. Then λ has a solvable series

$$\lambda = \lambda^{(0)} \supseteq \lambda^{(1)} \supseteq ....\supseteq \lambda^{(n)} = (e)_{\alpha}$$

Now,

$$\lambda_t = \left(\lambda^{(0)}\right)_t \supseteq \left(\lambda^{(1)}\right)_t \supseteq .... \supseteq \left(\lambda^{(n)}\right)_t = \left((e)_{\alpha}\right)_t = \{e\}$$

Hence, λ_t is a solvable subgroup of G.

Now, we will give the following interesting theorem :

Theorem 2.11

Every fuzzy subgroup of a solvable group is solvable.

Proof

Let G be a solvable group. Then :

$$G = G^{(0)} \supseteq G^{(1)} \supseteq .... \supseteq G^{(n)} = (e)$$

Let λ be any fuzzy subgroup of G with tip α *for* $0 \le i \le n$. Define λ_i by

$$\lambda_i(x) = \begin{cases} \lambda(x) & \text{if } x \in G^{(i)} \\ 0 & \text{otherwise} \end{cases} \quad , x \in G$$

Then $\lambda = \lambda_0 \supseteq \lambda_1 \supseteq ....\supseteq \lambda_n$ is a finite chain of fuzzy subgroups of G, such that $S(\lambda_i) \subseteq G^{(i)}$ for each i, clearly $\lambda_n = (e)_{\alpha}$

Let $a, b \in G$ and $o \le i \le n-1$,if $a \notin G^{(i)}$ or $b \notin G^{(i)}$ then :

$\min\{\lambda_i(a), \lambda_i(b)\} = 0 \le \lambda_{i+1}([a,b])$. on the other hand, if $a, b \in G^{(i)}$, then $[a,b] \in G^{(i+1)}$. And therefore, $\min\{\lambda_i(a), \lambda_i(b)\} = \min\{\lambda(a), \lambda(b)\}$

$$\leq \lambda([a,b])$$
$$= \lambda_{i+1}([a,b])$$

Thus ,

$\lambda = \lambda_0 \supseteq \lambda_1 \supseteq .... \supseteq \lambda_n = (e)_\alpha$ is a solvable series for λ, and λ is solvable.

Now, we can obtain the following result :

<u>Corollary 2.12</u>

If λ, μ are fuzzy subgroups of a solvable group G, then $[\lambda, \mu]$ is solvable

Proof

λ, μ are solvable fuzzy subgroups of G from theorem (2.11).

Then $[\lambda, \mu]$ is solvable fuzzy subgroups of G from theorem (2.9).

<u>Theorem 2.13</u>

If λ, μ are fuzzy subsets of a solvable group G. Then $[\lambda, \mu]$is solvable.

Proof

Let G be a solvable group, then :

$G = G^{(0)} \supseteq G^{(1)} \supseteq .... \supseteq G^{(n)} = (e)$.

Now, if γ, β are the tips of λ, μ, respectively, then from theorem (1.17) $\alpha = \gamma \wedge \beta$ is the tip of $[\lambda, \mu]$.

Define $[\lambda_i, \mu_i]$ as follows:

$$[\lambda_i, \mu_i](x) = \begin{cases} [\lambda, \mu] \ (x) & \text{if } x \in G^{(i)} \end{cases}$$

Then $[\lambda, \mu] = [\lambda_0, \mu_0] \supseteq [\lambda_1, \mu_1] \supseteq .... \supseteq [\lambda_n, \mu_n]$ is a finite chain of fuzzy subgroups of G, such that $S([\lambda_i, \mu_i]) \subseteq G^{(i)}$ for each i .

Clearly $[\lambda_n, \mu_n] = (e)_\alpha$

Let $a,b \in G$ and $0 \le i \le n-1$, If $a \notin G^{(i)}$ or $b \notin G^{(i)}$, then $\min\{[\lambda_i, \mu_i](a), [\lambda_i, \mu_i](b)\} = 0 \le [\lambda_{i+1}, \mu_{i+1}]([a,b])$ on the other hand, if $a, b \in G^{(i)}$, then $[a,b] \in G^{(i+1)}$, and therefore,

$$\begin{aligned}\min\{[\lambda_i, \mu_i](a), [\lambda_i, \mu_i](b)\} &= \min\{[\lambda, \mu](a), [\lambda, \mu](b)\} \\ &\le [\lambda, \mu]([a,b]) \\ &= [\lambda_{i+1}, \mu_{i+1}]([a,b])\end{aligned}$$

Thus ,

$$[\lambda, \mu] = [\lambda_0, \mu_0] \supseteq [\lambda_1, \mu_1] \supseteq .... \supseteq [\lambda_n, \mu_n] = (e)_\alpha$$

is a solvable series for $[\lambda, \mu]$, and $[\lambda, \mu]$is solvable.

Now, we shall state and prove the following theorem :

Theorem 2.14

Every fuzzy subgroup of solvable fuzzy group is solvable.

Proof

Let λ be a solvable fuzzy subgroup of a group G with tip α and let μbe a fuzzy subgroup of G, such that $\mu \subseteq \lambda$ in view. By theorem (1.12), we get $\mu^{(n)} \subseteq \lambda^{(n)}$ for all $n \ge 0$. Also, λ is a solvable then $\lambda^{(n)} = (e)_\alpha$.

Thus we have $(e)_\alpha \subseteq \mu^{(n)} \subseteq \lambda^{(n)} = (e)_\alpha$, then $\mu^{(n)} = (e)_\alpha$

Hence, μ is a solvable fuzzy subgroup.

Now, we introduce the following theorem :

Theorem 2.15

Let λ be a fuzzy subgroup of G, and $t^* = \inf\{\lambda(x)/x \in G\}$, suppose that $t^* > 0$. Then the following are equivalent:

(i) G is solvable

(ii) λ is solvable

(iii) λ_t is solvable $\forall t \in (0,1]$.

Proof

Now, G is solvable if and only if, 1_G is solvable, but $\lambda \subseteq 1_G$ then from theorem (2.11) .

λ is solvable then (i)$\Rightarrow$(ii)

by theorem (2.10), (ii)$\Rightarrow$(iii).

Now, suppose (iii) holds, $G = \lambda_{t^*}$

Thus G is solvable, that is (iii) $\Rightarrow$ (i).

In the following proposition we obtain a sufficient condition for truth of the converse of theorem (2.14).

Proposition 2.16

Let λ, μ be fuzzy subgroups of G, such that $S(\lambda) = S(\mu), \mu \subseteq \lambda$ *and* μ is solvable. Then λ is solvable.

Proof

From theorem (2.10), μ_t is solvable, $\forall$ t $\in$ (0, 1]

Since $S(\mu) \subseteq \mu_t$, then by theorem (2.14), $S(\mu)$ is solvable and $S(\lambda)$ is solvable.

And from theorem (2.15), G is solvable .

Consequently, λ is solvable.

Theorem 2.17

Let $\lambda and \mu$ be fuzzy subgroups of G, such that μ is a normal fuzzy in λ . If λ is a solvable fuzzy group, then $(\lambda/\mu)^{(t)}$ is solvable fuzzy group, $\forall t \in (0,1]$.

Proof

Since λ is solvable fuzzy group, then from theorem (2.10), λ_t is solvable for all $t \in (0,1]$.

Also, by proposition (1.15), μ_t is normal in λ_t $\forall t \in (0,1]$.

Then (λ_t/μ_t) is solvable . But , $(\lambda_t/\mu_t) \cong (\lambda/\mu)^{(t)}$

Then $(\lambda/\mu)^{(t)}$ is solvable for all $t \in (0,1]$.

Also, we have the following theorem :

Theorem 2.18

Let $\lambda and \mu$ be fuzzy subgroups of G, such that μ is a normal fuzzy in λ. If λ is a solvable fuzzy group, then (λ/μ) is a solvable fuzzy semi-group.

Proof

Since λ is solvable fuzzy group. Then from theorem (2.10), λ_t is solvable for all $t \in (0,1]$.

Also, by proposition (1.15), μ_t is normal fuzzy subgroup in λ_t for all $t \in (0,1]$.

Then (λ_t/μ_t) is solvable, But $(\lambda_t/\mu_t) \cong (\lambda/\mu)_t$.

Hence (λ/μ) is solvable fuzzy semi-group.

Next , we introduce some important propositions :

Proposition 2.19

Let $\lambda, \mu and \gamma$ be fuzzy subgroups of G such that γ is normal fuzzy in $\lambda and \mu$. If $\lambda and \mu$ are solvable fuzzy then $[\lambda/\gamma, \mu/\gamma]$ is solvable.

Proof

Since λ, μ are solvable fuzzy subgroups of G and γ normal in $\lambda and \mu$, then from theorem (2.18), λ/γ and μ/γ are solvable fuzzy semi-group.
Also, by theorem (2.9). $[\lambda/\gamma, \mu/\gamma]$. Is solvable.

Proposition 2.20

Let α be a normal fuzzy subgroup of $\lambda and \beta$ be a normal fuzzy subgroup of μ. If λ, μ are solvable fuzzy subgroups of G, then $([\lambda,\mu] \;/\; [\beta,\alpha])$ is solvable fuzzy semi-group.

Proof

Let λ, μ two solvable fuzzy subgroups of G, then from theorem (2.9), $[\lambda, \mu]$ is solvable fuzzy subgroup of G.
And since β,α are normal fuzzy in λ, μ respectively then by corollary (1.14), $[\beta,\alpha]$ is normal fuzzy in $[\lambda,\mu]$
Thus from theorem (2.18), $([\lambda,\mu] \;/\; [\beta,\alpha])$ is solvable fuzzy semi-group.

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بعض النتائج حول الزمر الضبابية القابلة للحل

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الخلاصة :-

يتضمن البحث تعريف الزمرة الضبابية القابلة للحل بأكثر من صيغة ثم أثبات تكافئ الصيغ المختلفة للتعريف و دراسة خواصها وتقديم البراهين المهمة حول المفهوم .