A discussion of mean –field theory in Ising spin models

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Abstracts :

IN this work an isospin system with spin values $S=\pm 1$ is studied through introducing improvement on a mean-field method for Ising spin system. The method is based on an extended Oguchi approximation (OA) . Within this scheme, three atoms are considered where the two nearest –neighbour spin correlation are taken into account when the Hamiltonian is presented .The result obtatined for the Curi temp. are more accurate compared with mean-field approximation (MFA) and Oguchi models Atable is stateed for comparison purposes , bet the present results and that of Curi temp. for approximation refeed earliear respectively.

مناقشة نظرية متوسط المجال في أنظمة أيزنك المغناطيسية

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الخلاصة:
في هذا البحث تم دراسة نظام مغناطيسي متماثل الذرات قيم البرم لها S=±1 من خلال إدخال تحسينات على نتائج تقريب متوسط المجال في انظمه ايزنك المغناطيسي .وقد أدخلت هذه التحسينات على أساس توسيع تمثيل اكوتشي . في هذا التمثيل الجديد درسنا ثلاث ذرات مركزية في النظام المغناطيسي المذكور اخذين بنظر الاعتبار دوال الارتباط بين اقرب الجارات الأولى والثانية عند صياغة الهاملتوني. إن النتائج التي تم الحصول عليها فيما يتعلق بدرجه حرارة كوري لمختلف السبائك البلورية كانت مشجعه جدا مقارنه بنتائج تقريب متوسط المجال وتقريب اكوتشي . وقد وضع جدولا لغرض المقارنة بين النتائج الحالية وقيم درجات حرارة كوري للتقريبات المذكورة .

1. Introduction :

The Ising model is one of best developed models in statistical physics. It has been solved in one-dimensional and certain two-dimensional lattice. Besides the considerable results based on the series –expansion and renormalization –group methods ,especially for the critical region of the models [1] ,A.bobak and M.jascur [2] potentially presented a new and helpful scheme for improving the values of the critical temperarture for a number of lattices in Ising spin system .The authors[3] presented another formwork based on the introduction of

differentional operator technique ,taking into accunt the effects of many –body static spin correlations .These method all used rigorous and complicated solutions which represent substation improvements of the standard mean- field approximation (MFA) . On the other hand , because of its simplicity and limitations ,the mean-field theory has played an important role for the description of coorperative phenomina, in which the effect of the ordering interactions is naively represented by that of a mean field proportional to the average net magnetic moment of a magnatic system .The MFA has some deficiencies ,due to the neglection of correlation; however, it is an adequate starting point [1,4,5].Improvment in this respect is sought by this paper .In this work , auseful scheme for obtaining approximate solution of curie temperature in the two –move dimensional Ising system is studied .

2. Theory and numerical results:

our procedure considers three lattice sites(i ,j ,k) whose spin states denoted by (S_i),(S_j) and (S_k) respectively , together with their nearest neighbors as shown in Fig .1.Accordingly , the Oguchi scheme to a new one is extended . The Hamiltonian for the spin -1/2 Ising system ,in zero field ,is defined by :[2]

$$S_i = \pm 1 \qquad \textbf{(1)}\ H = -\sum_{i,j} J_{ij} S_i S_j$$

Where the sum runs N identical spins , and J_{ij} is the exchange interaction between spins at sites I and j. According to our scheme considered (see Fig .1) , we can decompose the Hamiltonian (1) as follows :

$$H = -J_{ij} S_i S_j - J_{jk} S_j S_k - h_m S_i - h_n S_j - h_l S_k \quad \textbf{(2)}$$

Where ,

$$h_l = \sum_{l}^{Z-1} J_{Kl} S_l \qquad h_n = \sum_{n}^{Z-2} J_{jn} S_n \qquad h_m = \sum_{m}^{Z-1} J_{im} S_m$$

and z is the coordination number . Using equation (1) , it is readily calculated the average magnetic moment of the magnetic system which is considered , that :

$$m = \frac{1}{3} \cdot \frac{3\sinh(2h_1 + h_2) + e^{-4t}\sinh(2h_1 - h_2) + 2e^{-2t}\sinh(h_2)}{\cosh(2h_1 + h_2) + e^{-4t}\cosh(2h_1 - h_2) + 2e^{-2t}\cosh(h_2)} . \textbf{(3)}$$

$h_1 = tm(z-1)$ ‘ $t = \frac{J}{K_B T}$ $h_2 = tm(z-2)$

which yields the following equation for a critical temperature:

$$e^{-4t_c} + 2e^{-2t_c} + 1 = \frac{4}{3}e^{-4t_c} + \frac{4}{3}t_c e^{-2t_c} + 8t_c \quad \textbf{(4)}$$

Equation (4) show , for example , that the critical temperature for the square lattices (z = 4) is:

$\frac{K_B T_C}{J} = 3.6878$

hence , we have followed this procedure for the simple cubic lattice (Table(1)) . By way of comparison , we have listed the results of the MFA , 0A , and the exact or high-temperature series results [1,2].

Table (1) show the clearly that our valuse improve on those of the MFA and 0A respectively .

Table (1) : results of the critical energy $k_B T_c$ of the square (Z = 4) and simple cubic (z = 6) lattices respectively .

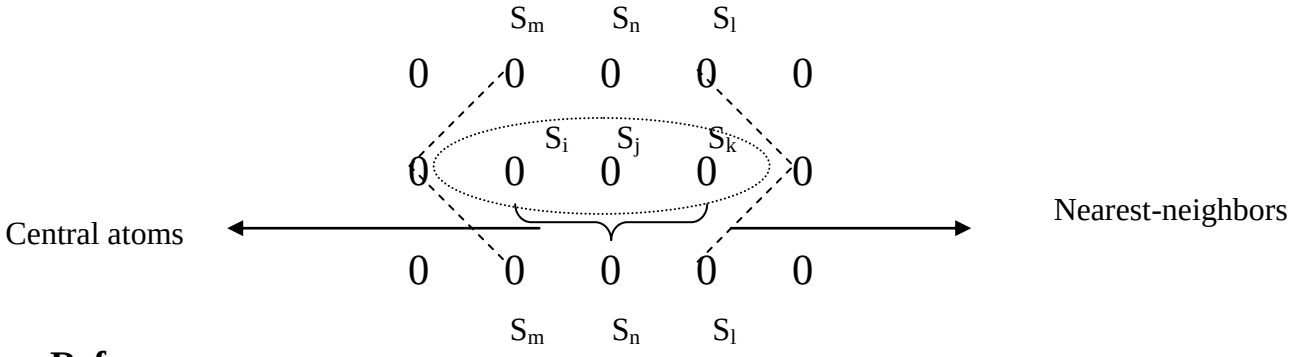
Method	Square $k_B Tc$	Simple cubic kBTc
MFA	4.0000	6.0000
O A	3.7764	5.8469
present work	3.6878	5.7855
Exact	2.2692	4.5108

3. conclusion :

In the present work , we assume that the correlations between the first and second nearest-neighbor spins and mean-field are existed in the system. The oguchi approximation is extended to get remarkably accurate results for critical temperatures for Ising spin lattices comparing to the mean-field approximation.

Figure caption :

Fig . 1: A square lattice of N magnetic atoms having spins (S_i) , (S_j) , (S_k) , together with the nearest neighbors (S_m) , (S_n) and (S_l) respectively .



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