FUZZY PETRI NET MODEL ANALYSIS USING PIVOT AND INVARIANT METHOD

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Abstract

In this search, pivot, invariant and fuzzy logic are used for the analysis of Fuzzy Petri Net (FPN) model, in order to detect and eliminate the cyclic behavior which occurs in the fuzzy Petri net model.

Key words: Pivot, Invariant, Fuzzy logic, Petri nets.

1- Fuzzy Petri net

Petri Net models have emerged as a very promising performance modeling tool for systems that exhibit concurrency, synchronization, and randomness. This search uses Petri Nets as a modeling tool for knowledge refinement. Neural nets used as learning tool, because the weakness of Petri nets learning capability.

Petri Nets (or place-transition nets) are bipartite graphs and provide an elegant and mathematically rigorous modeling framework for discrete event dynamical systems [1]. They manipulate events according to certain rules. This section explains some of the main concepts of a Petri Net and how they can be used to analyze system coherency and coordination [3], [12].

Definition: A PN is a four-tuple (P, T, F, W, Mo) (1.1)
where

$P = \{p1, p2, p3, ..., pn\}$ is a set of places
$T = \{t1, t2, t3, ..., tm\}$ is a set of transitions
$F \subseteq (P \times T) \cup (T \times P)$ is the set of arcs from places to transitions and from transitions to places in the graph;
$W: I \cup O \rightarrow \{1, 2, 3,...\}$ is the weight function on the arcs;
$P \cap T = \phi$ and $P \cup T \neq \phi$
$M_0: P \rightarrow \{0, 1, 2, 3, ...\}$ is the initial marking.

The set of input (output) places of transition t_j:
$\bullet t = I(t_j) = \{ p_i \in P : (p_i,t_j) \in F \}$, $t\bullet = O(t_j) = \{ p_i \in P : (t_j, p_i) \in F \}$
Similar notation can be used I(pi),O(pi).

Pictorially, places are represented by circles and transitions by bars. In some models, places represent conditions or resource in the system while transitions model the activities in the system, while other models use the opposite representation. Places and transitions are connected by arcs. Places represent an "OR" situation, where only one of the arcs starting from it can be activated. Transitions represent an "AND" situation. When a transition fires, all the arcs starting from the transitions get activated. Figure (1.1) shows a Petri Net with five places and four transitions. In this Petri Net P = {p1, p2, p3, p4, p5} is the set of places, and T = {t1, t2, t3, t4} is the set of transitions [4],[16],[18].

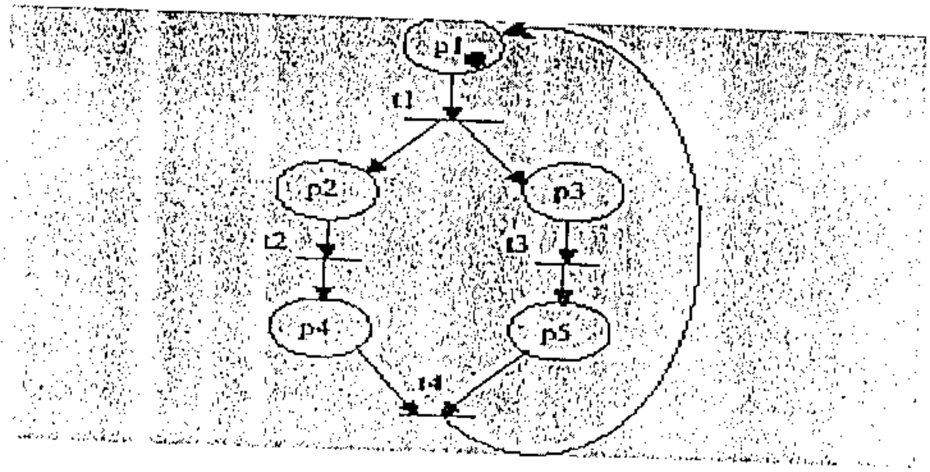

Figure (1.1): A Petri Net Model

. - Invariant Analysis:

One of the structural properties of Petri nets, i.e. properties that depend only on the topological structure of the Petri net and not on the net's initial marking, is the net invariants. There are two kinds of invariants: place invariants and transition invariants [2], [5], [9].

Place invariants are sets of places whose token count remains always constant. They are represented by an n-column vector x, where n is the number of places of the Petri net, whose non-zero entries correspond to the places that belong to the particular place invariant and zeros everywhere else. Every integer vector x which satisfies the following equation

$$\mu^T . x = \mu^T_o . x \quad (1.2$$

..ere μ_o is the net's initial marking, while μ represents any subsequent marking, ..fines a place invariant. Equation (1.2) means that the possibly weighted sum of the ..kens in the places of the invariant remains constant at all markings and this sum is ..etermined by the initial marking of the Petri net. The place invariants are defined by .l integer vectors which satisfy the following equation

$$A.x=0 \quad (1.3)$$

where A is the $(n \times m)$ composite change matrix of the Petri net, with n being .. number of places and m the number of transitions of the net. It is easily shown that ..ry linear combination of place invariants is also a place invariant for the net.

Transition invariant denotes which transitions must fire and how many times ..h, so that the initial marking is repeated. They are represented by an m-column ..ctor y which contains integers in the positions corresponding to the transitions ..onging to the transition invariant and zeros everywhere else. The integers denote ..w many times the corresponding transition must fire in order for the initial marking .. be repeated. They can be computed from the following equation

$$A.y = 0 \quad (1.4)$$

As with place invariants, any linear combination of transition invariants is ..o a transition invariant for the Petri net. The existence of transition invariants in the ..tri net denotes a *cyclic* behavior.

Place and transition invariants are important means for analyzing Petri nets ..ce they allow for the net's structure to be investigated independently of any ..namic process [7],[10],[13].

Another advantage of the invariants is that analyses can be performed on local subnets without considering the whole system. Invariants are also used for model verification [14],[11].

Net invariant analyses comprise all processes which aim at the solution of systems of final equations derived from the incidence matrix of the net.

A net is covered by place invariants, if there exists a p-invariant which assigns a positive value to each place [15], [17]. A net is covered by transition invariants, if there exists a t-invariant which assigns a positive value to each transition.

The place or transition invariants of a net are the integer solutions of the homogenous system of linear equations $A^T.x = 0$ or $A.y = 0$, respectively, where A is the incidence matrix of the net.

The components of *p*-invariants are interpreted as weights of the respective place [11]. The weighted amount of tokens is invariant with respect to firing. The component of *T*-invariants can be interpreted as firing numbers of the respective transmission (negative values correspond to the reverse firing).

Firing all transitions as many times as their firing number indicates leads to the same marking as before.

For the matrix shows below

	P_1	P_2	P_3	P_4	P_5
t_1	-1	1	0	0	0
t_2	-1	0	0	0	1
t_3	0	-1	-1	1	0
t_4	0	0	1	-1	-1
t_5	1	0	0	0	0

The entry *-1* means that the transition (t_i) subtracts a token from place (p_i). The entry *1* means, on the other hand, that firing the transition (t_j) adds a token into palace (p_j).

An n-vector x (m-vector y) of integers is called a *P*-invariant (*T*-invariant) if $A^Tx=0$ ($Ay= 0$). The set of places (transitions) corresponding to nonzero entries in a *P*-invariant $x > 0$ (*T*- invariant $y>0$) is called the support of the invariant. A support is said to be minimal, if it cannot be represented as a linear combination of other invariant. The support Sup (i) of a *P*-invariant is the set of places that have non-zero weights in the invariants. The weighted number of tokens on the support of a *p*-invariant is constant for all reachable marking.

3- Pivot Method

Analyzing a Place-Transition (*PT*) net by calculating its invariants is in most cases more time and space efficient than inspecting the reachability set, since the complexity[8],[1] of this kind of analyses depends only on the number of places and transitions and not on the size of the reachability set.

This section introduces the operation of pivoting and considers systems of linear equations which don't have unique solutions, [12], [13], [9], and [2].

Roughly speaking, the Gaussian elimination method applied to a matrix proceeds as follows: Consider the columns one at a time, from left to right. For each column use the elementary row operations to transform the appropriate entry to one and the remaining entries in the column to zeros (the "appropriate entry is the first entry in the first column, the second entry in the second column, and so forth) . This sequence of elementary row operations performed for each column is called pivoting. More precisely:

Method: To pivot a matrix about a given nonzero entry:

1-Transform the given entry into a one.

2-Transform all other entries in the same column into zeros.

Pivoting is the basis for the simplex method of solving linear programming problems.

Gaussian elimination method [9] transforms a system of linear equations into diagonal form

1. Write down the matrix corresponding to the linear system.
2. Make sure that the first entry in the first column is nonzero. Do this by interchanging the first row with one of the rows below it, if necessary.
3. Pivot the matrix about the first entry in the first column.
4. Make sure that the second entry in the second column is nonzero. Do this by interchanging the second row with one of the rows below it, if necessary.
5. Pivot the matrix about the second entry in the second column.
6. Continue in this manner.

4- Write the equations corresponding to the last matrix read;

5- Specify values (1, 0) for the independent variables, and compute the dependent variables; Form the invariants which are obtained from different values of the variables (dependent and independent); the obtained invariant must satisfy the following equation: $A \cdot y = 0$ {using T-Invariant} or

$A^T . x = 0$ {using P-Invariant} y and x are invariants.

Example :

In the figure (1.2) shown below find the sets of invariants.

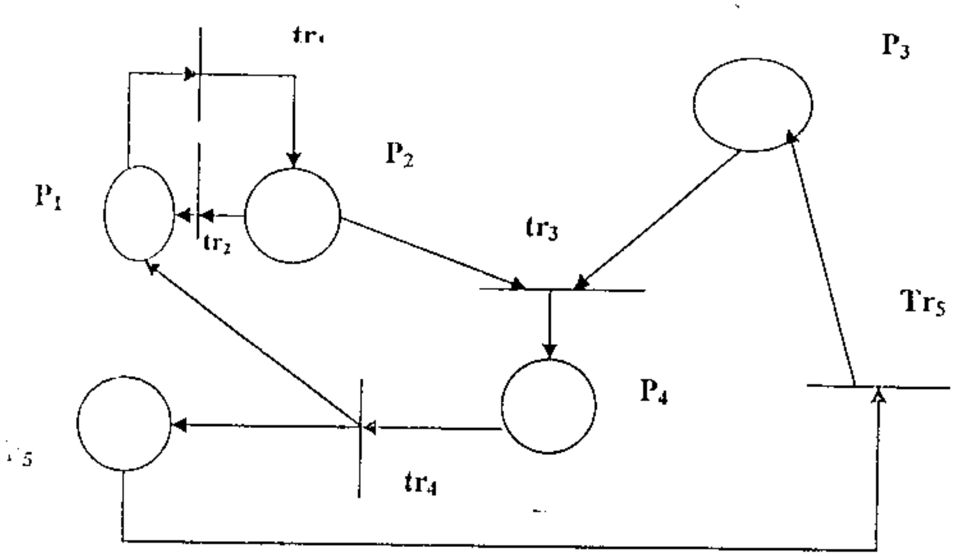

Figure (1.2): Fuzzy Petri net model analysis

Problem solving steps:

1-Define the transpose incidence matrix A^T

A^T =

	p_1	p_2	p_3	p_4	p_5
t_1	-1	1	0	0	0
t_2	1	-1	0	0	0
t_3	0	-1	-1	1	0
t_4	1	0	0	-1	1
t_5	0	0	1	0	-1

Steps of using pivot methods

Step$_1$: Find an element a (1, 1) $\neq$ 0, and use it as a pivot point;

Pivot element →

-1	1	0	0	0
1	-1	0	0	0
0	-1	-1	1	0
1	0	0	-1	1
0	0	1	0	-1

Step$_2$: Transform all other entries in the same column into zeros;

-1	1	0	0	0
0	0	0	0	0
0	-1	-1	1	0
0	1	0	-1	1
0	0	1	0	-1

Step$_3$: Repeat step$_1$ and step$_2$ until no other pivot element is obtained;

-1	1	0	0	0
0	-1	-1	1	0
0	1	0	-1	1
0	0	1	0	-1

Pivot

-1	0	-1	1	0
0	-1	-1	1	0
0	0	-1	0	1
0	0	1	0	-1

Pivot

-1	0	0	1	-1
0	-1	0	1	-1
0	0	-1	0	1

Step4: There are no other pivots obtained. Now construct the following equations:

$-x_1 - x_5 + x_4 = 0$ ....(1) $x_1 = x_4 - x_5$

$-x_2 - x_5 + x_4 = 0$ ....(2) $x_2 = x_4 - x_5$

$-x_3 + x_5 = 0$ .... (3) $x_3 = x_5$

Step5: Follow back tracking by using binary logic to give values (*0, 1*) to independent variables x_4, x_5 and compute value of dependent variables (x_1, x_2, x_3) as follows:

x_5	x_4
0	1
1	0
1	1

x_3	x_2	x_1
0	1	1
1	-1	-1
1	0	0

Step6: Finally the following three invariants are obtained:

1= [1 1 0 1 0]

2= [0 0 1 1 1]

3= [-1 -1 1 0 1]

The correctness of these invariants can be proved by the following equation:

$A^T . x = 0$

From invariant *x= {-1,-1, 1, 0, 1}*

The negative sign denotes that place p_1 and p_2 are on cycle behavior.

From the net given in figure (1.2), one can find the transitions which are on cycles as follow:

1-Define the incidence matrix A

A =

	t_1	t_2	t_3	t_4	t_5
p_1	-1	1	0	1	0
p_2	1	-1	-1	0	0
p_3	0	0	-1	0	1
p_4	0	0	1	-1	0
p_5	0	0	0	1	-1

By using pivot and invariant methods, the following three invariants are obtained:

1 = [1 1 0 0 0]

2 = [1 0 1 1 1]

3 = [2 1 1 1 1]

The correctness of these invariants can be proved by the following equation:

$A\,y = 0$

From invariant [*11000*] and the matrix *A*, one find from figure(1.2), that (t_1, t_2) are on cycle, such that from the matrix *A*, t_1 is input to p_2 and it is output to p_1 , t_2 is input to p_1 and it is out put to p_2 (p_1, t_1, p_2, t_2, p_1).

From invariant [*1 01 1 1*] and the incidence matrix *A* , one find from figure(1.2), that (t_4 t_1 t_3), (t_4 t_5 t_3) are on cycles, such that ($t_4 \rightarrow p_1 \rightarrow t_1 \rightarrow p_2 \rightarrow t_3 \rightarrow p_4 \rightarrow t_4$) forms the cycle net for (t_4, t_1, t_3), ($t_4 \rightarrow p_5 \rightarrow t_5 \rightarrow p_3 \rightarrow t_3 \rightarrow p_4 \rightarrow t_4$) forms the cycle net for (t_4, t_5, t_3).

Fuzzy Petri Net Model Analysis using T-Invariant

The incidence matrix (transition matrix), invariant solution for the model shows in figure (1.3) are shown in Figure (1.4).

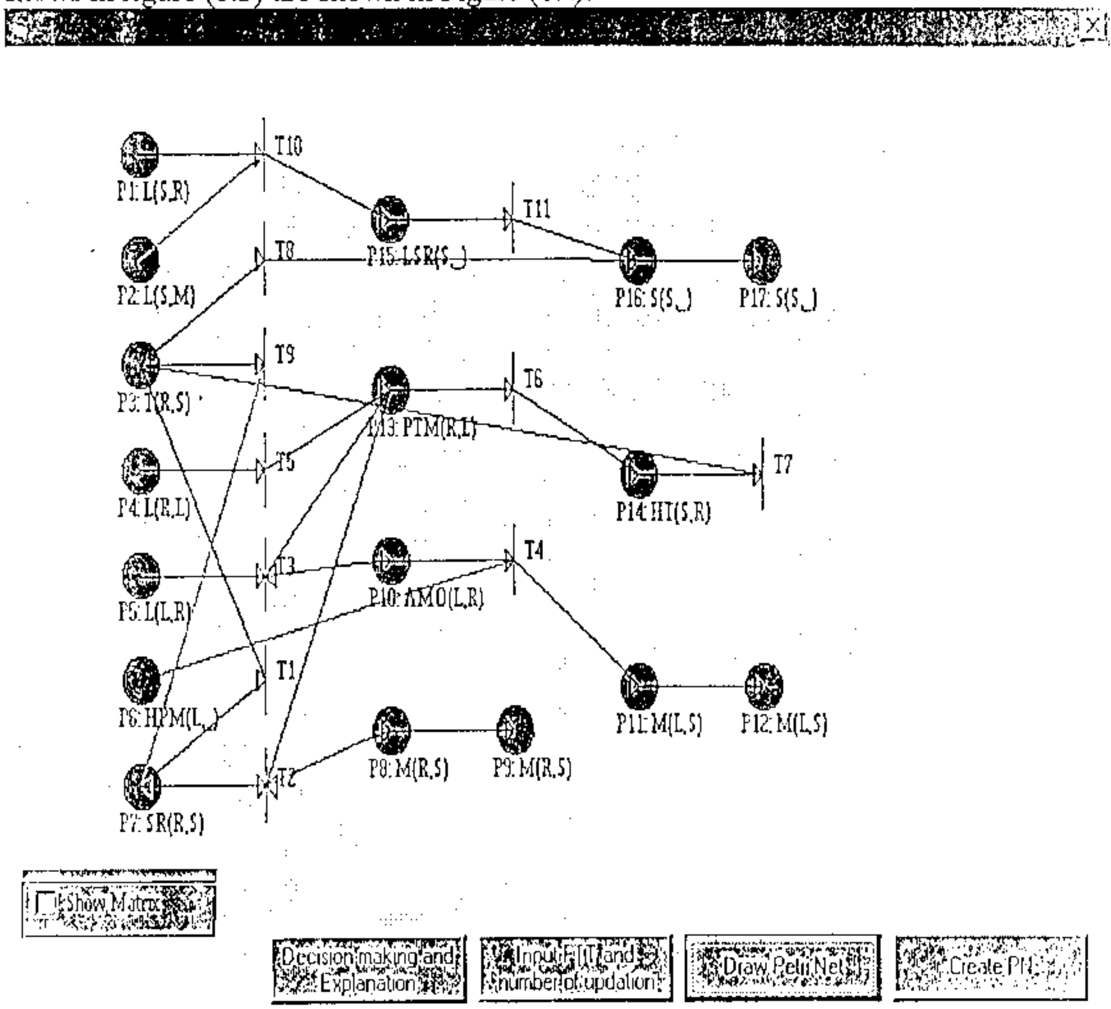

Figure (1.3): FPN model

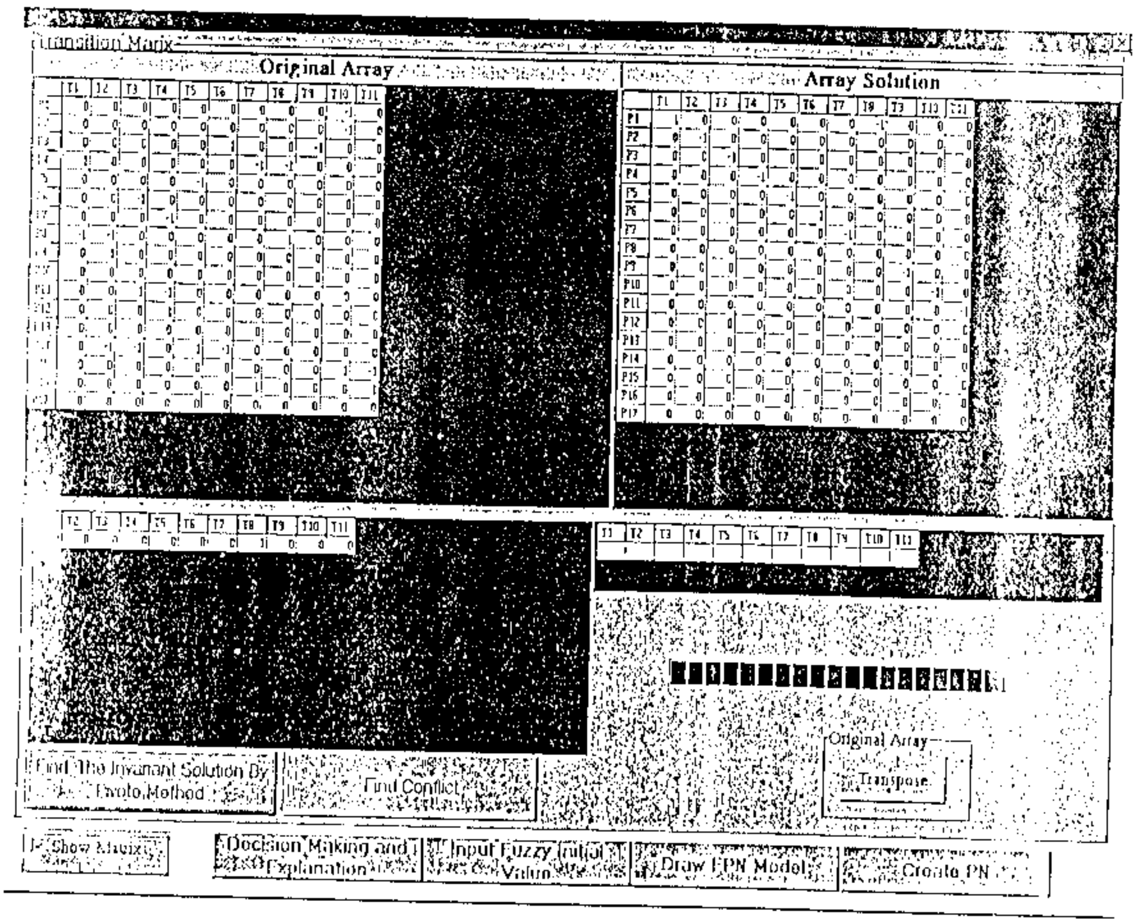

Figure (1.4): Incident matrix and invariant solution.

From figure (1.4), one can estimate that t_1 and t_9 are on cycle. The method to eliminate cycles is the adjustment of threshold of transitions on cycle. The threshold is chosen to be the largest fuzzy belief on the reasoning path from the axioms {L{r, l)} in our proposed system to the selected transition (tr_9).

This is achieved by setting the threshold for transition tr_9 to 0.9.

4- Conclusions

The advantage of analyzing a Fuzzy Petri Net (FPN) by calculating its invariants is in most cases more time and space efficient than inspecting the reachability set, since the complexity [8] of this kind of analysis depends only on the number of places and transitions and not on the size of the reachability set.

The major weakness of Petri nets is the complexity problem, i.e., Petri net-based model tends to become too large for analysis.

5- Future Work

1- Designing a reasoning model of fuzzy time Petri nets that can handle fuzziness of time tokens;
2- Designing properties of a fuzzy Petri net by which you can say that the net will exhibit limit cycle for some ranges of initial fuzzy beliefs;
3-Using pivot and invariant method in the analysis of fuzzy Petri net model.

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تحليل نموذج شبكة بتري المضببة باستعمال طريقة الثبات والصواب

الخلاصة

في هذا البحث تستخدم طريقة المنطق المضبب والثبات لتحليل نموذج شبكة بتري لكشف واختزال خصائص الدورات التي تحدث في نموذج شبكة بتري.

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