Solve Multi Linear Programming Problem
By Taylor Polynomial solution

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الخلاصة

تضمن البحث حل مسائل البرمجة الخطية المتعددة باستخدام متسلسلة تايلور والحصول على الحل الأمثل . حيث تم مقارنة النتائج التي تم الحصول عليها باستعمال هذه الطريقه مع الحل الدقيق لهذه المسائل بالطرق الاعتيادية المستعملة لحل مسائل البرمجة الخطية اظهرت النتائج ان هذه الطريقه متقاربة وكفوءة .

Abstract:

We have proposed a solution to Multi Linear Programming Problem (MLPP) by expanding the order 1st Taylor polynomial series. The Taylor series is a series expansion that a representation of a function, these objective functions at optimal points of each linear objective functions in feasible region. Thus, the problem is reduced to a single objective. Numerical example is provided to demonstrate the efficiency and feasibility of the proposed approach.

Interdiction

In the modeling of the real word problems like financial and corporate planning, production planning, marketing and media selection, university planning and student admissions, health care and hospital planning, air force maintenance units, bank branches, etc. frequently may be faced up with decision to optimize dept/equity ratio, profit/cost, inventory/sales, actual cost/standard cost, output/employee, student/cost, nurse/patient ratio etc. respect to some constraints (Lai and Hwang, [1996]).[6]

In the literature, different approaches appear to solve different models of Linear Programming Problem (LPP). Because, programming solves more efficiently the above problems with respect to Linear Programming Problem (LPP).

In these papers LPP are discussed in details. It is showed that LPP can be optimised easily. But, in the great scale decision problems, there is more than one objective, which must be satisfied at the same time as possible. However, most of these are linear objectives. It is difficult to talk about the optimal solutions of these problems. The solutions searched for these problems are weak efficient or strong efficient. If required, one compromise solution can be reached by the affection of the models with the decision makers (DMs).

There exist several methodologies to solve multi objective linear programming problem (MoLPP) in the literature. Most of these methodologies are computationally burdensome (Chakraborty and Gupta, [2002]). Kornbluth and Steuer [198], Y.J.Lai and C.L.Hwang [1996] have developed an algorithm for solving the MoLPP for all weak-efficient vertices of the feasible region. Nykowski, Z.Zolkiewski [1978] and Dutta et all [1992] have proposed a compromise procedure for MoLPP. Choo and Atkins [1982] have given an analysis of the bicriteria LPP.[3]

1.1. Linear Programming [1],[2],[3],[4],[8]

A general model of crisp linear programming is formulated (Standard formulation):

$$\begin{aligned} &Max\ Z = cx \\ &\text{Subject to}\quad \boldsymbol{Ax} \le \mathrm{b} \qquad \ldots(1) \\ &\qquad\qquad\quad x \ge 0 \end{aligned}$$

where c and x are n dimensional vectors, b is an m dimensional vector, and A is (m * n) matrix.

1.2. Definition

$$let\ c^{(1)}, c^{(2)}, \ldots, c^{(p)} \in R^n \quad and \quad Z(x) = \max_{t=1,\ldots,p} \left\{ \left(c^{(t)}\right)^T x \right\}$$

The min − max $proplem$:

$$\min \{ Z(x) / Ax = b; x \geq 0 \} \qquad (1.1.1)$$

$$\min \left\{ x_0 / x_0 - \left(c^{(t)}\right)^T x \geq 0, t = 1, \ldots, p;\ Ax = b; x \geq 0 \right\} \qquad (1.1.2)$$

1.2.1. Theorem

If (xo* , x*) solve (1.1.2) then x* solve (1.1.1) and xo*= Z(x*)

Proof:

If x is a feasible solution to (1.1.1) then (Z(x) , x) is a feasible solution to (1.1.2)

(Since Ax = b ; x ≥ 0 and Z(x) – (c(t))^T x ≥ 0 ; t = 1 ,… ,p).

⇒ xo* ≤ Z(x)

⇒ xo* ≤ Z(x*) …(1)

(1.1.2) ⇒ xo* ≥ (c(t))^T x* ; t = 1 , … p

⇒ xo* ≥ Z(x*) … (2)

From (1) and (2) we have

xo* = Z(x*) and Z(x) ≥ xo* = Z(x*)
for any feasible solutions to (1.1.1)

2.1. Model Development

In this paper, we consider the Multi Objective Linear Programming Problem (MoLPP)

Max Zi (x) = Max { Z1 (x), Z2 (x), … , Zk (x) }
S.t.
x ∈ X = { x ∈ Rn ,Ax ≤ b , x ≥ 0 } 2.1.1
with b ∈ Rn , A ∈ Rm*n
and Zi(x) = ci x , where ci ∈ Rn
Let the maximum value ith objective function Max Zi (x) = Zi* , ∀i , on the feasible region it occurs when xi* = (xi1* , xi2* , … , xin*) ,
for i = 1,2,…, k.

Suppose that Z(x) and all of it's partial derivative of order less than or equal (n + 1) are continuous on the feasible region X , x* ∈ X .
By expanding the 1st order Taylor polynomial series for objective function Zi (x) about xi* , objective function Zi (x) is obtained from

Zi (x) =Pi1 (x) + Pi2(x)

Where function Pi1 (x) is called the first Taylor polynomial in n variables about xi* and Pi2 (x) is the remainder term associated with Pi1 (x) , we have :

$$Z_i(x) = p_{i1}(x) \cong Z_i(x_i^*) + \begin{bmatrix} (x_1 - x_{i1}^*)\frac{\partial Z_i(x_i^*)}{\partial x_1} + (x_2 - x_{i2}^*)\frac{\partial Z_i(x_i^*)}{\partial x_2} + \ldots + \\ (x_1 - x_{in}^*)\frac{\partial Z_i(x_i^*)}{\partial x_n} \end{bmatrix} + O(h^2) \quad \ldots \quad (2.1.2)$$

$$Z_i(x) = p_{i1}(x) \cong Z_i(x_i^*) + \sum_{j=1}^{n}\left[(x_j - x_{ij}^*)\frac{\partial Z_i(x_i^*)}{\partial x_j}\right] + O(h^2) \qquad for\ i = 1,2,\ldots,k$$

where O(h2) is order of the maximum error. This polynomial gives an accurate approximation to Zi (x) when x is close to x* .

By replacing Zi (x) = ci x

$$Z_i(x) = p_{i1}(x) \cong Z_i(x_i^*) + \sum_{j=1}^{n}\left[(x_j - x_{ij}^*)\frac{\partial Z_i(x_i^*)}{\partial x_j}\right]$$

In MOLPP ,all of the objective functions Zi (x) , i = 1,2,…,k, become the 1st order linear functions as

$$e_i + \sum_{j=1}^{n} a_j x_j \quad , e_i, a_j \in R, i = 1,2,\ldots,k$$

so MOLPP reduces the following :

$$Max\left\{ Z_1(x) = e_1 + \sum_{j=1}^{n} a_{1j} x_j,\ Z_2(x) = e_2 + \sum_{j=1}^{n} a_{2j} x_j, \ldots, Z_k(x) = e_k + \sum_{j=1}^{n} a_{kj} x_j \right\}$$

x ∈ X = { x ∈ Rn ,Ax ≤ b , x ≥ 0 } (2.1.3)

with b ∈ Rn , A ∈ Rm*n

If we assume that the weights of objective functions in problem (2.1.3) are equal, then problem (2.1.3) is written as follows:

$$Max\left\{ \sum_{j=1}^{n} a_{1j} x_j + \sum_{j=1}^{n} a_{2j} x_j + \ldots + \sum_{j=1}^{n} a_{kj} x_j + \sum_{1=1}^{k} e_i \right\} \qquad (2.1.4)$$

$x \in X = \{ x \in Rn , Ax \le b , x \ge 0 \}$
with $b \in Rn$, $A \in Rm*n$

In problem (2.1.4), set X is non-empty convex set having feasible points. The optimal solution of problem (2.1.4) gives the efficient solution of MoLPP (2.1.3). Because, weights of objective functions that are expanded Taylor series are equal and in (2.1.3) is considered the weighted objective function.

Example:

$$\begin{aligned} &\text{Max } \{ Z1 = x1 + x2 , Z2 = 3x1 - 2x2 \} \\ &\text{S.t.} \quad 2x1 + x2 \le 6 \\ &\qquad x1 + 4x2 \le 12 \\ &\qquad x1 , x2 \ge 0 \end{aligned}$$

Now to solve objection function (Z1),
The Standard form of objection function (Z1) is :

$$\begin{aligned} &Z1 - x1 - x2 = 0 \\ &2x1 + x2 + S1 = 6 \\ &x1 + 4x2 + S2 = 12 \\ &x1 , x2 \ge 0 \end{aligned}$$

We have the optimal solution of Z1 are:

x1 = 1.71 ,
x2 = 2.571429 ,
then Max Z1 = 4.28

Now to solve objection function (Z2)

$$\begin{aligned} &\text{Max } Z2 = 3x1 - 2x2 \\ &\text{S.t.} \quad 2x1 + x2 \le 6 \\ &\qquad x1 + 4x2 \le 12 \\ &\qquad x1 , x2 \ge 0 \end{aligned}$$

The Standard form of objection function (Z2) is:

$$\begin{aligned} &Z2 - 3x1 + 2x2 = 0 \\ &2x1 + x2 + S1 = 6 \\ &x1 + 4x2 + S2 = 12 \\ &x1 , x2 \ge 0 \end{aligned}$$

we have the optimal solution of Z1 are:

x1 = 3 ,
x2 = 0 ,

Max Z2 = 9

It's observed that Z1 & Z2 ≥ 0 , for each x

Z1Max (1.71, 2.57) = 4.28 &

Z2Max (3 , 0) = 9

By expending the 1st order polynomial series for objective function Z1(x) and Z2(x) about points (1.71, 2.57) and (3 , 0) is :

$$Z_1(x) \cong 4.28+(x_1-1.71)\frac{\partial Z_1(1.71,2.57)}{\partial x_1}+(x_2-2.57)\frac{\partial Z_1(1.71,2.57)}{\partial x_2}$$

$$\cong 4.28+(x_1-1.71)\times(1)+(x_2-2.57)\times(1)$$

$$\therefore Z_1(x) \cong x_1+x_2$$

$$Z_2(x) \cong 9+(x_1-3)\frac{\partial Z_2(3,0)}{\partial x_1}+(x_2-0)\frac{\partial Z_2(3,0)}{\partial x_2}$$

$$\cong 9+[(x_1-3)\times 3]+[(x_2-0)\times(-2)]$$

$$Z_2(x) \cong 3x_1-2x_2$$

So all objectives are transformed to the L.P. The obtained MoLP is equivalent the following:

Max { Z1(x) + Z2(x) } = 4x1 – x2
S.t.
2x1 + x2 ≤ 6
x1 + 4x2 ≤ 12
x1 , x2 ≥ 0

The optimal solution for this MoLP is:

Max { Z1(x) + Z2(x) } (3,0) = 12
x1 = 3 , and x2 = 0 ,
we have

Max Z1 = 3 and Max Z2 = 9

Conclusion

In this paper, we have proposed a solution to Multi Objective Linear Programming Problem (MoLPP) using Taylor polynomial series. With the help of the order 1st Taylor polynomial series at optimal points of each linear objective function in feasible region. The obtained MoLPP is solved assuming that weights of these linear objective are equal and considering the sum of the linear objective functions. The proposed solution to MoLPP always yields efficient solution, even a strong-efficient solution. Therefore, the complexity in solving MoLPP has reduced easy computational.

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