Optimum Solution of The Problem of Seepage Through Earth Dam

Salah Tawfeek Ali **Abdul-Hassan K. Shukur**
Zahra Abd Saleh
Department of Civil Engineering, College of Engineering, Babylon University

Abstract

Earth dams are often provided with dolomite core, grout curtain, and steel or concrete cut-off to control seepage through these dams. In the present study, a two-dimensional finite-element model is applied to Haditha dam to examine the separate and composite effects of these devices on seepage and pore pressure. Hence, a relationship between core width and cut-off depth corresponding to the design seepage value of 1.487 l/s per one meter length of the dam is developed. Other relationships for core width versus grout curtain width and core width versus cut-off depth and grout curtain width for different are also obtained. These relationships are consequently used as constraints in an optimization analysis to determine the width of each of the core and curtain and the length of the cut-off which minimize the cost. Although the method presented here is applied to Haditha dam, it may also be applied to other earth dams.

Key words: Optimum Solution, Seepage, Earth Dam.

الخلاصة

ان السد الترابي المستخدم يحتوي على لب طيني وستارة التحشية وقواطع رئيسة في المقدم والمؤخر للسيطرة على النضح خـلال السد. في هذا البحث تم دراسة نموذج ثنائي الابعاد باستخدام نظرية العناصر المحددة وتطبيقه على سد حديثة لمعرفة تـأثير تلـك العوامل على الجريان وضغط المسام. تم استخراج علاقة بين عرض اللب وعمق القاطع على النضح التصميمي والـذي مقـداره بحدود 1.487l/sec. لكل متر طول من السد. تم تحديد علاقة اخرى بين عرض اللب مع عمق القواطع وعرض سـتارة التحـشية ولابعاد مختلفة. ان العلاقات اعلاه استخدمت كمحددات لتحليل الامثلية وايجاد العرض لكل من اللب وستارة التحشية وطول القواطع والتي تعطي اقل كلفة. وقد تم التوصل الى امكانية تطبيق هذه الطريقة على سداد اخرى.

1. The Finite Element Model

For the analysis of seepage flow through and under earth dams, the flow may be considered two-dimensional and the following equation is applicable:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \qquad \ldots(1)$$

The field equation describing an approximate variation of piezometric head within a finite element is:

$$H^e = \sum_{i=1}^{n} N_i H_i \,, \qquad \ldots(2)$$

where:

H_i: Nodal value of head

n: Number of nodes per element;

N_i: Shape function of the element, at node i.

Equation (2) may also be written in matrix form as [**Zienkiewicz** *etal.*, (1966)]:

$$H^{PPe} = [N_{BBi}]\{H_i\} \qquad \ldots(3)^P$$

The approximate solution for head variation, H, over the whole domain is given as follows:

$$H = \sum_{e=1}^{n_e} H^e = \sum_{e=1}^{n_e} \sum_{1}^{n} N_i H_i \quad \ldots(4)$$

or

$$H = \sum_{e=1}^{n_e} [N_i]\{H_i\} \quad \ldots(5)$$

where:

$n_{BBBBBBBBBBBBBBBBBBBBBBBBBe}$: is the total number of elements in the problem domain.

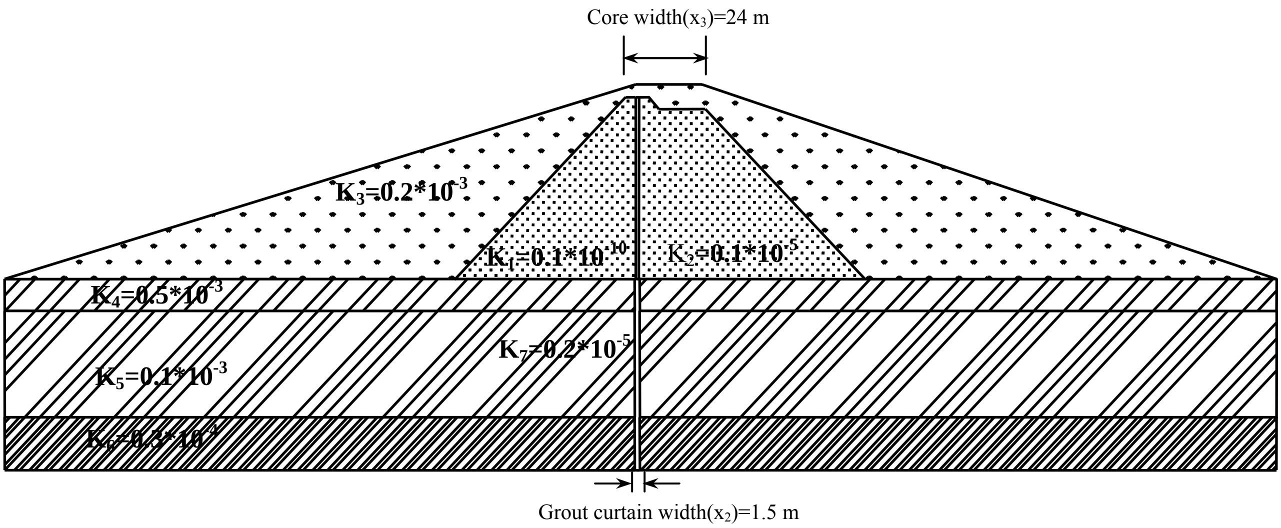
2. Haditha Dam

Haditha dam was constructed in 1988 on the Euphrates River in the Middle West of Iraq 7km upstream from Haditha town. The project generates (660 Mw) of electrical power a side from performing its flood control function. Central and southern parts of Iraq get the benefit of irrigation water from its reservoir. [**Hydroprojekt**, (1988)].

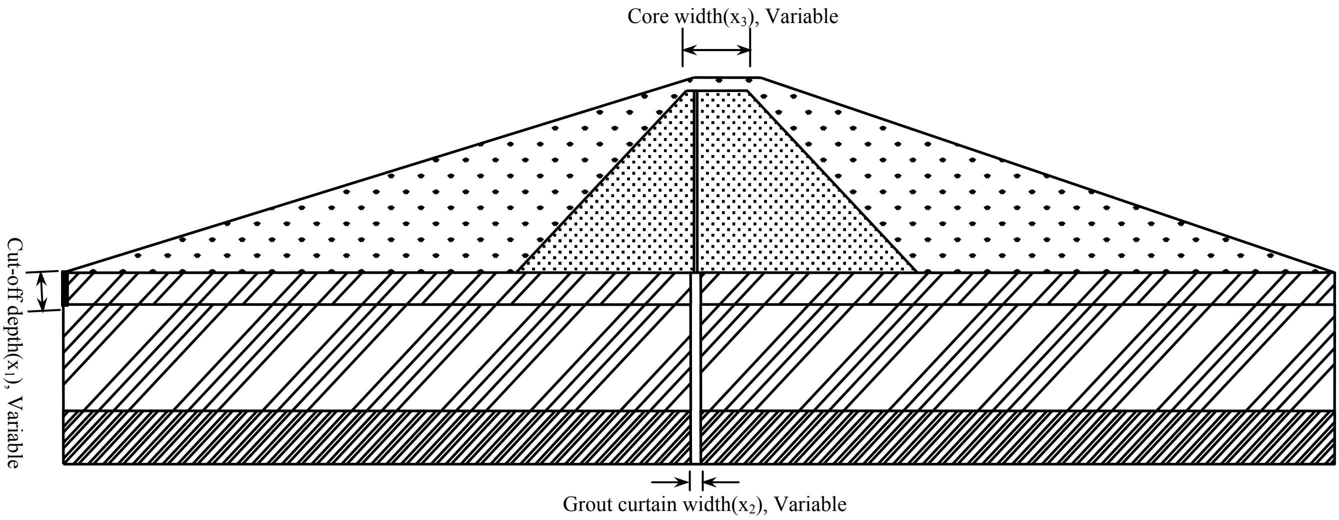
The project comprises mainly of an earth dam, 9 km long. Because of its considerable length and diversity of its topography and geological conditions, the design of the dam embankment varies from section to section but in general it preserves the features of the basic type which cover most of the dam length. [**Irzooki,** (1998)].

The body of the dam consists of a central dolomite core and shells made of sand/gravel material and/or a rock muck (random rock material). [**Salih,** (2000)]. An asphaltic concrete diaphragm through the core was provided as an antiseepage measure through the body of the dam. A grout curtain was constructed to provide treatment for the foundation against seepage. Shcematic cross-section of Haditha dam is shown in Figure 1.

By using the GMS-Seep2D software, the solution region is discretized with the upstream and downstream water levels being 147 m.a.s.l and 109.15 m.a.s.l respectively. The mesh is modified many times in order to calibrate the model and the final finite element mesh, is as shown in Figure 2.



A: The Actual Dam



B: Hypothetical Dam Examined in the Study

Figure 1: Shcematic cross-section of Haditha Dam

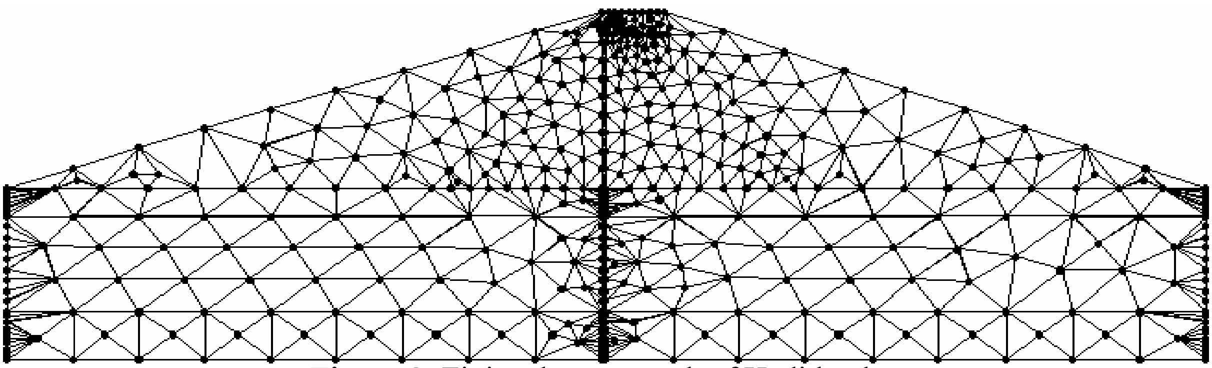


Figure 2: Finite element mesh of Haditha dam

We first consider the case of only two of the three measures of seepage control are provided. It is assumed now that a cut-of wall is provided at the upstream face of the dam, the top width of the dolomite core (x_3) is varied from 1 to 24 m in steps of 5.75 m for a cut-off depth (x_1) of 0,3,6,9 and 12 m and the quantity of seepage is determined from GMS-Seep2D program. Hence, a core top width versus seepage relationship is established by regression for each of these cut-off depth as shown in Figures from 3 to 7. From these curves the relationship between the core width and the cut-off depth for allowable seepage of 1.487 l/s/m is obtained and given as Figure 8.

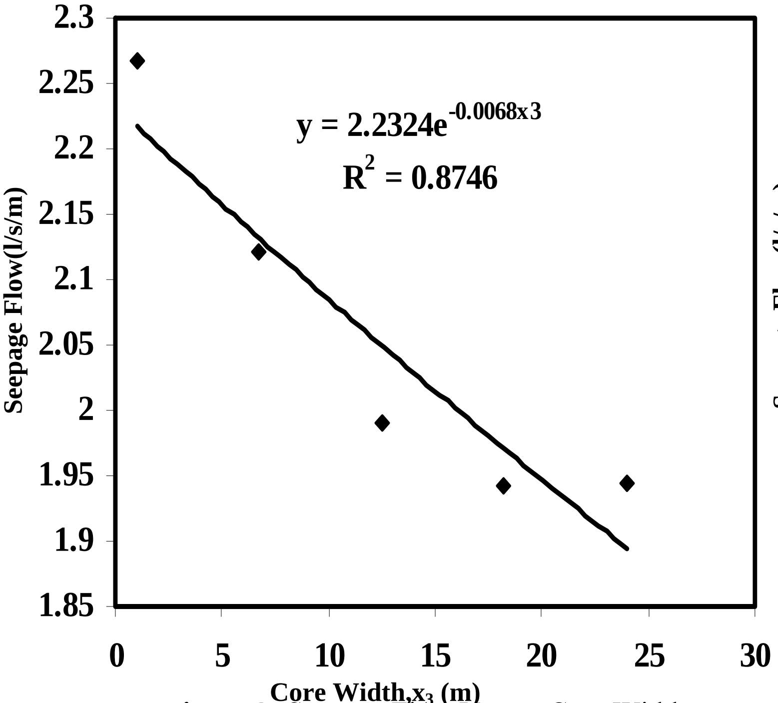


Figure 3: Seepage Flow Versus Core Width Relationship (no Cut-off wall or Grout Curtain)

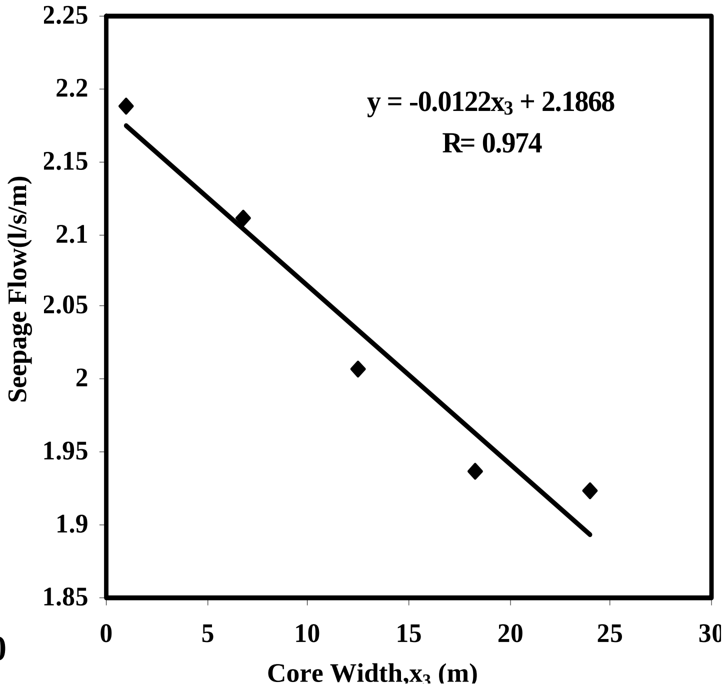


Figure 4: Seepage Flow Versus Core Width Relationship for Cut-off Depth = 3 m and no Grout Curtain

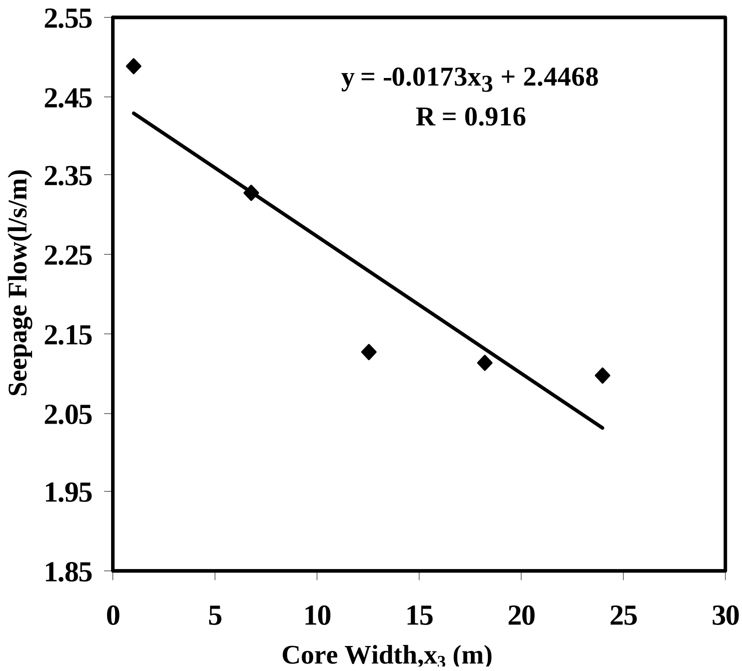


Figure 5: Seepage Flow Versus Core Width Relationship for Cut-off Depth = 6 m and no Grout Curtain

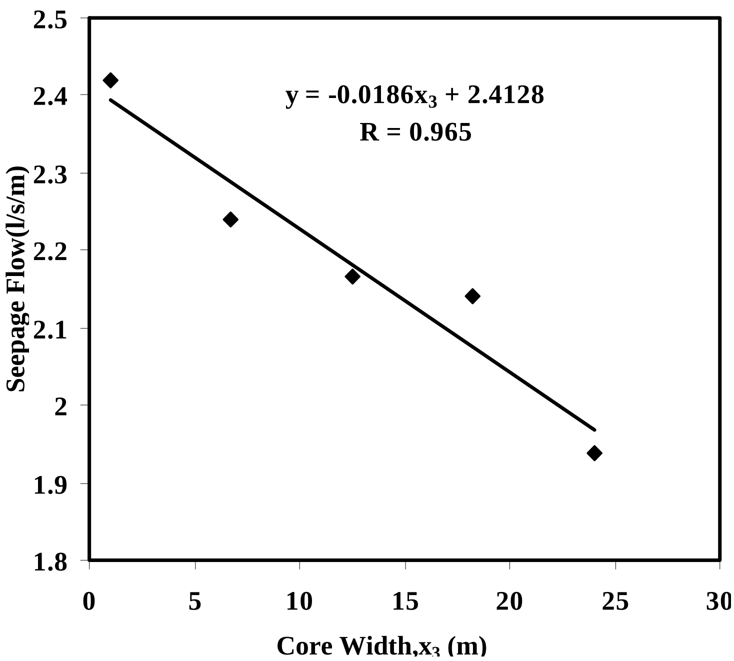


Figure 6: Seepage Flow Versus Core Width Relationship for Cut-off Depth = 9 m and no Grout Curtain

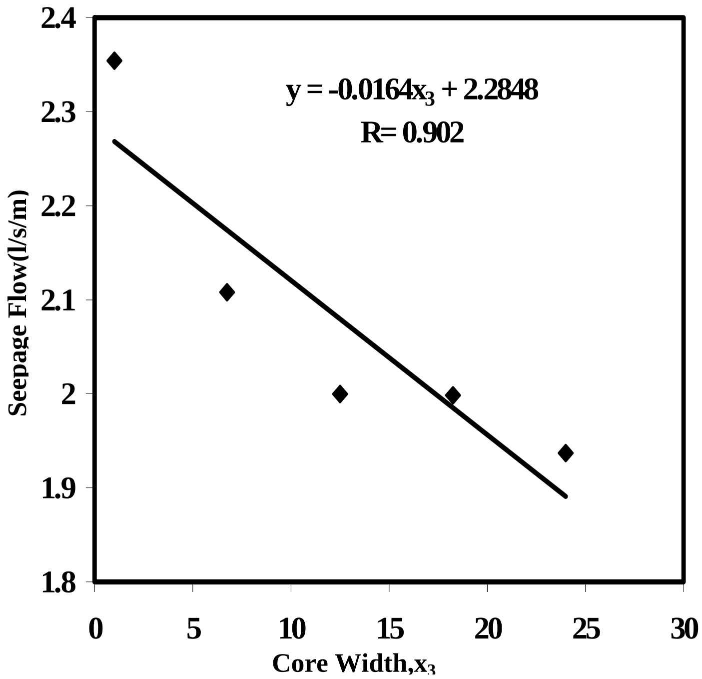


Figure 7: Seepage Flow Versus Core Width Relationship for Cut-off Depth = 12 m and no Grout Curtain

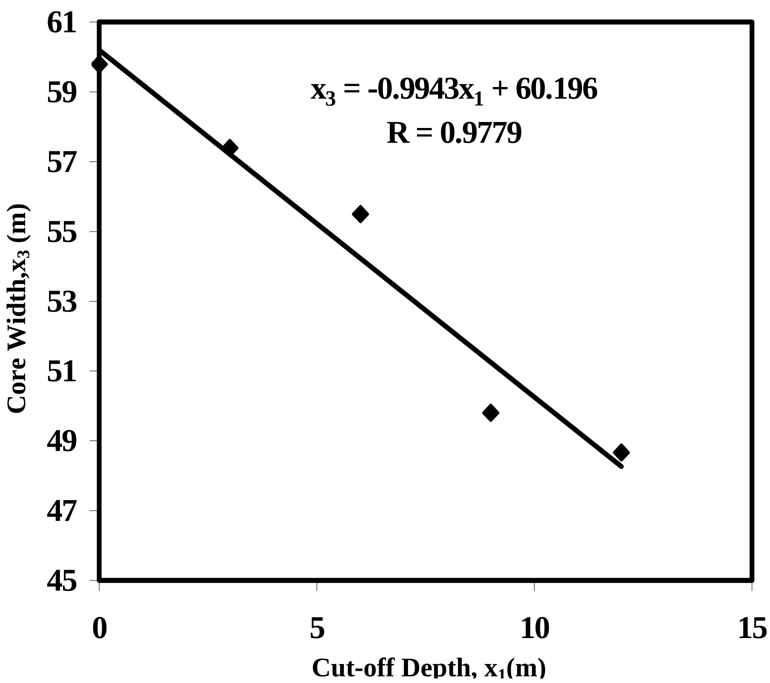


Figure 8: Core Width Versus Cut-off Depth Relationship for Seepage Flow=1.487 l/s/m

This procedure is repeated to establish the core top width against cut-off depth relationship corresponding to a safe pore pressure value of 34.135 m H_2O as shown in Figure 9.

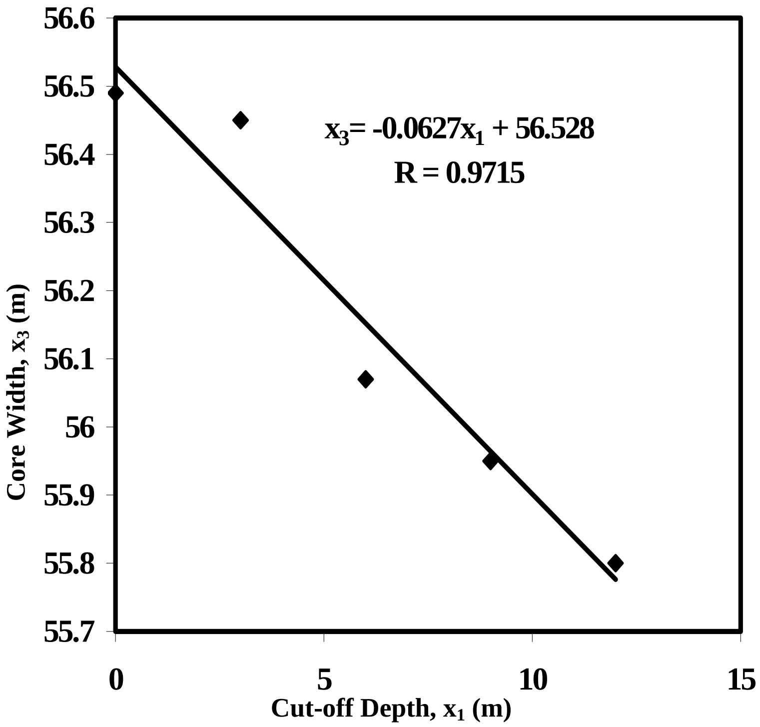


Figure 9: Core Width Versus Cut-off Depth Relationship for Pore Pressure = 34.135 m H_2O

In a similar manner, the core top width versus grout curtain width relationship corresponding to the allowable seepage and the same relationship corresponding to the safe pore pressure are obtained and presented as Figures 10 and 11 respectively.

Next, we assume that the cut-off wall, grout curtain, and the core are all used. The top width of the dolomite core (x_3) is varied from 1 to 24 m in steps of 5.75 m for a grout curtain width (x_2) of 0, 1.5, 3, 4.5 and 6 m with different cut-off depths and the relationship corresponding to the allowable seepage flow of 1.487 l/s/m is established using modflow model coupled with regression technique. Similarly, other equation corresponding to the safe pore pressure value of 34.135 m H_2O is also obtained. Due to space limitation only two of the many figures used in this analysis are presented here as Figure 12 and 13 respectively.[**Saleh**,(2006)]

The equations that correlate all the variables using multiple regression are:

$x_3 = 44.705+0.05593x_1-6.711x_2$ (for seepage = 1.487 l/s/m) … (6)

$x_3 = 43.027-0.0893x_1-7.93x_2$ (for pore pressure = 34.135 m H_2O) … (7)

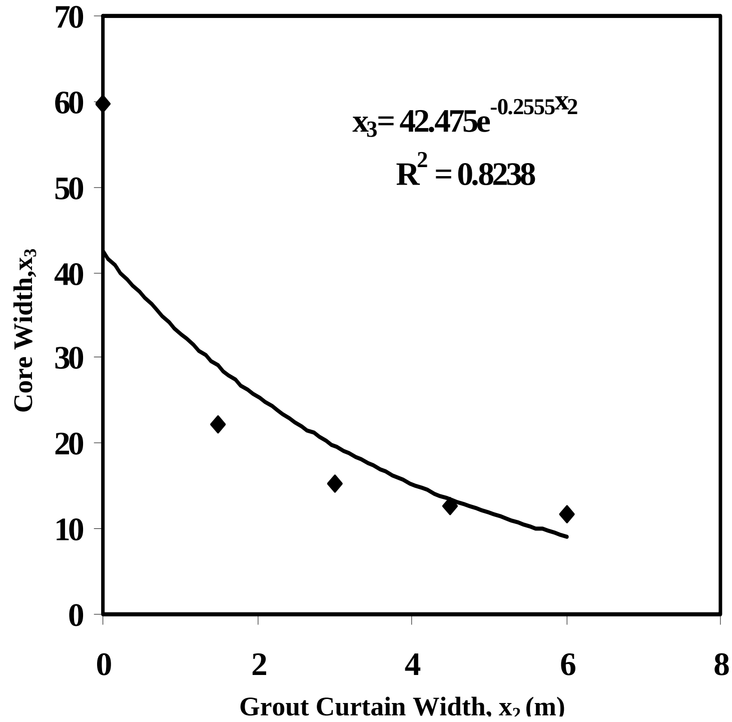


Figure 10: Core Width Versus Grout Curtain Width Relationship for Seepage Flow=1.487 l/s/m

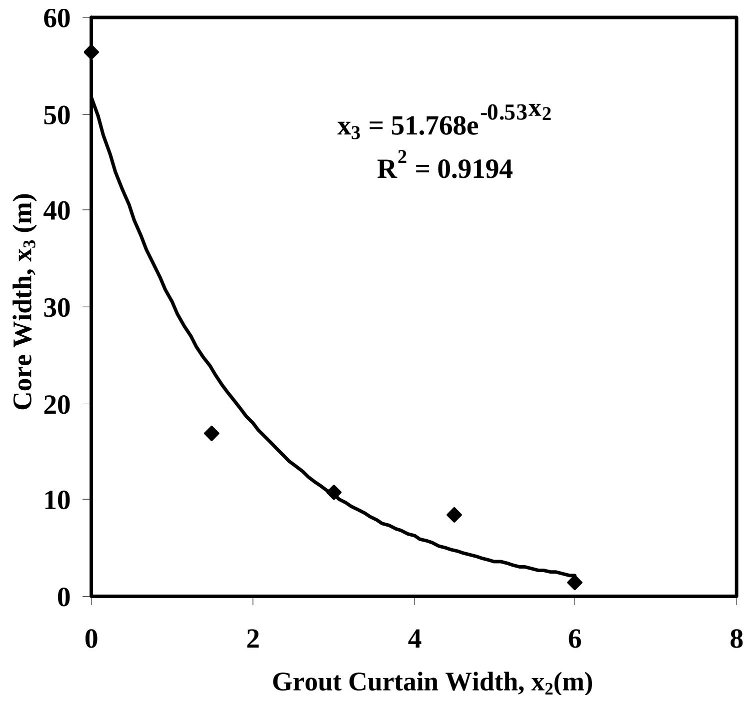


Figure 11: Core Width Versus Grout Curtain Width Relationship for Pore Pressure = 34.135 m H_2O

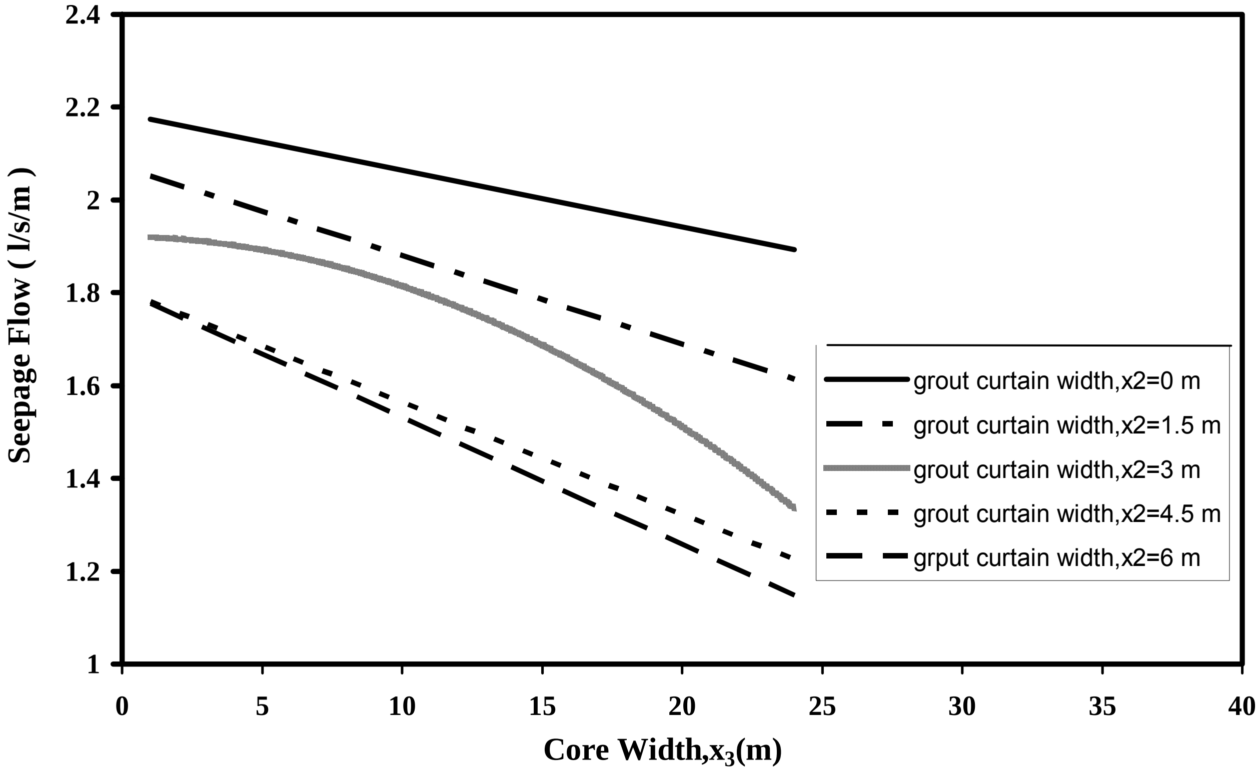


Figure 12: Seepage Flow Versus Core Width Relationship for Different Grout Curtain Width and Cut-off Depth=3 m

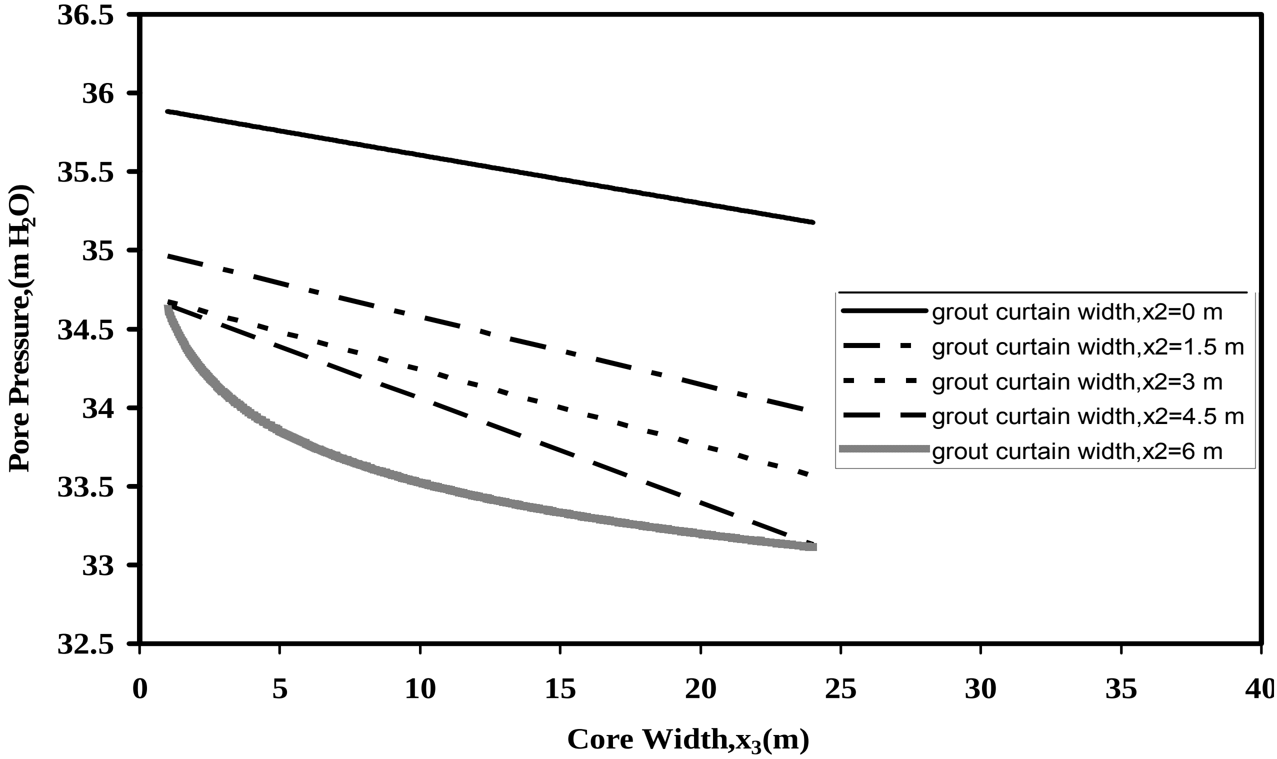


Figure 13: Pore Pressure Versus Core Width Relationship for Different Grout Curtain Width and Cut-off Depth=3 m

3. Optimum Solution

The variables are the depth of the upstream cut-off, x_1, the width of grout curtain, x_2, and the width of core, x_3.

There are three cost objective functions:

1-The cost objective function (Z_1) deals with two control devices, concrete cut-off 1m in width and dolomite core. Such function is formulated as follows:

$Z_1=c_1x_1+c_3x_3$, …. (8)

where:

c_1=cost of one cubic meter of cut-off,

x_1=depth of upstream cut-off (m),

c_3=cost of one cubic meter of dolomite core,

x_3=width of core (m).

2-The cost objective function (Z_2) dealing with another two control devices, grout curtain and dolomite core, that is:

$Z_2=c_2x_2+c_3x_3$, ….. (9)

where:

c_2=cost of one cubic meter of grout curtain,

x_2=width of grout curtain (m).

3-The cost objective function (Z_3) dealing with all control devices that used in this research together, that is:

$Z_3=c_1x_1+c_2x_2+c_3x_3$ ….(10)

The objective function (Z_1) is minimum and subject to the following constraints:

1. The relationship between core width and cut-off depth for safe pore pressure (34.135 m H_2O), is given by:

$x_3= - 0.0627x_1 + 56.528$ … (11)

2. The relationship between core width and cut-off depth for safe seepage flow (1.4866 l/s/m), is given by:

$x_3= - 0.9943x_1 + 60.196$ …. (12)

3. The cut-off depth should be equal or less than (12 m).

The objective function (Z_2) is minimum and subject to the following constraints:

1. The relationship between core width and grout curtain width for safe pore pressure (34.135 m H_2O), is given by :

$x_3= 51.768\, e^{-0.53x_2}$ …(13)

2. The relationship between core width and grout curtain width for safe seepage flow (1.4866 l/s/m), is given by:

$x_3 =42.475\, e^{-0.2555x_2}$ … (14)

The objective function (Z_3) is minimum too and subject to the following constraints:

1-The relationship between core width, cut-off depth and grout curtain width for safe seepage flow (1.4866 l/s/m), is given by:

$$x_3= 44.705+0.05593x_1-6.711x_2 \quad \dots (6)$$

2-The relationship between core width, cut-off depth and grout curtain width for safe pore pressure (34.135 m H_2O), is given by:

$$x_3 = 43.027-0.0893x_1-7.93x_2 \quad \dots (7)$$

Both equation (6) and (7) are obtained by using a software entitled " SPSS" with applying multiple linear regression.

3. The cut-off depth should be equal or less than (12 m).

The methods of optimization employed in this study are the simplex and the Lagrange multiplier methods.

For the first case the objective function (Z_1), as in equation (8) and all the constraints form linear relationships with the design variables, therefore the simplex method is used to solve this problem. [**Phillips** *et al.*, (1976)] and [**Wu** and **Coppins**, (1981)].

Minimize: $Z_1=c_1x_1 +c_3x_3$ …(8)

Minimize: $Z_1= 250x_1 + 1026x_3$

Subject to: $0.0627x_1 + x_3 = 56.528$

$0.9943x_1 + x_3 = 60.196$

$x_1 \leq 12$

$x_1, x_2 \geq 0$

For the second case the objective function (Z_2) is linear relationship as in equation (9) but all the constraints form non-linear relationship with the design variables, therefore, the Lagrange multiplier method is used to solve this problem. [**Dimitri**, (1982)].

Minimize: $Z_2=c_2x_2 +c_3x_3$ …(9)

Minimize: $Z_2= 18000x_2 + 1026x_3$

Subject to: $x_3 = 51.768e^{-0.53x_2}$

$x_3 = 42.475e^{-0.2555x_2}$

For the third case the objective function (Z_3), as in equation (10) and all the constraints form linear relationship with the design variables, therefore, the simplex method is used to solve this problem.

Minimize: $Z_3=c_1x_1 + c_2x_2 + c_3x_3$ …(10)

Minimize: $Z_3= 250x_1 + 18000x_2 + 1026x_3$

Subject to: $0.0893x_1 + 7.93x_2 + x_3 = 43.027$

$-0.05593x_1 + 6.711x_2 + x_3 = 44.705$

$x_1 \leq 12$

$x_1, x_2, x_3 \geq 0$

Table (1): The summary results of optimum design of control devices

case No.	Cut-off depth, x_1(m)	Grout curtain width,x_2(m)	Core width,x_3(m)
1	3.94	___	56.28
2	___	0.72	35.33
3	Infeasible problem		

4. Conclusions

A simple method to obtain optimum solution for seepage through earth dam is presented and applied. In this method, the GMS-Seep2D model is used together with regression technique to establish the constraints of optimization.

References

Dimitri, P.B., (1982), "**Constrained optimization and Lagrange multiplier methods**", Academic Press, New York.

Hydroprojekt, (1988), "**Al-Qadisiya project on the Euphrates River, operation and maintenance manual for earth-fill dam**", Book1, No.950-10-76, Moscow.

Irzooki, R.H., (1998), "**Investigation and analysis of seepage problems on the left side of Al-Qadisiya Dam",** M.Sc. Thesis, University of Technology.

Phillips, D.T., Ravindran, A. and James J., (1976), "**Operations research: Principles and Practice**", John Wiley and Sons, Canada.

Saleh, Z.A., (2006), **"Optimum design of control devices for safe seepage through earth dam",** M.Sc. Thesis, Department of Civil Engineering, University of Babylon.

Salih, R.A., (2000), "**performance of the right side of Al-Qadisiya Dam",** M.Sc. Thesis, Department of Civil Engineering, College of Engineering, University of Tikrit.

Wu, N. and Coppins, R., (1981), "**Linear programming and extensions**", McGraw-Hill, Inc., New York.

Zienkiewicz, O.C., Mayer, P. and Cheung, Y.K., (1966), "**Solution of anisotropic seepage by finite elements**", Journal of the Engineering Mechanics Division, ASCE, Vol.92, No.EM1, pp.111-120.