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Study Point Spread Function for Eccentric Circular Aperture with Gaussian Illumination

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ABSTRACT:

In this research we studied the effect of Gaussian filter on point spread function(PSF) for an optical system consisted of circular aperture with non-central circular obscuration with Gaussian Illumination. The study was in case present different types of first and third order aberrations, and searched to get better combination of them to made optimum balance of third order aberration with first order error to get best image.

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دراسة دالة الانتشار النقطية الفتحة الدائرية الحلقية اللامركزية مع إضاءة كاوسية

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الكلمات المفتاحية:

دالة الانتشار النقطية
المرشح الكاوسي
الزيوغ
التوازن الامثل

الـــخُـــلاصـــة:

تم في هذا البحث دراسة تأثير المرشح الكاوسي على دالة الانتشار النقطية (point Spread Function) لمنظومة بصرية متكونة من فتحة دائرية معاقة بعائق دائري لا مركزي (فتحة حلقية دائرية لا مركزية) مع اضاءة كاوسية. وقد تمت هذه الدراسة في حالة وجود انواع مختلفة من زيوغ الرتبة الاولى والثالثة. وللحصول على أفضل صورة وتقليل تأثير بعض انواع الزيوغ تمت دراسة التوازن الأمثل لزيوغ الرتبة الثالثة مع ما يناسبها من زيوغ الرتبة الأولى.

1. INTRODUCTION

The properties of the optical system can be described by a point spread function. Point spread function is related to the phase at these exit pupil apertures through a Fourier transform [1]. The spread function depends on diffraction that produces by the lens aperture and the amount of the aberrations and its type in lens or in the optical system[2].Since the image of a point source in optical systems is not a point according to the theory of light, so the diffraction resulting from the aperture of the lens and the scattering makes the image of the point object extending to a spot distributed in a given distribution such as the Gaussian distribution [3]. Many researches have been doneoin improving the image using different filters, where in 2005 studied Virendra N. Mahajan Strehl ratio of a Gaussian beam where discuss the Strehl ratio of systems with a Gaussian pupil and determine the range of validity of its approximate expression based on the aberration variance. The results given are equally applicable to propagation of Gaussian beams[4]. In 2006Yangjian Cai and Lei Zhang studied Propagation of a decentered elliptical Gaussian beam through apertured aligned and misaligned paraxial optical systems[5]. In 2013 Fengqing Qin studied Blind Image Restoration Based on Signal to Noise Ratio and Point Spread Function with Gaussian filter. Using Technology SNR and PSF, Experimental results show that the quality and peak signal-to-noise of the restored image are better around the real value and justify the fact that the SNR an-d PSF estimation plays great important part in blind image restoration [6].

In 2016 Vidal F. Canales et.al. studied Analysis of Strehl ratio limit with superre solution binary phase filters[7]. In 2017Christi Jose and Pramod Panchal discuss wave front analysis of aberrated Laguerre-Gaussian beam using Shack-Hartmann wave front Sensor[8].

In 2007 Adnan Falih Hassan studied the effect of the Gaussian filter for an array of circular synthetic aperture and noted that the use of a Gaussian filter contributes to the reduction of the secondary peaks i.e. reducing the noise in the image [9]. In 2011 Azhar Abdul Zahra et.al. studied a line object optical system with Gaussian filter using different apertures. The results showed that the lower value of standard deviation for Gaussian filter leads to increase the intensity in the formed image, and the aberrations has small effect on the distribution of the intensity in the image plane [10]. In 2010 Raja Abdel Ameer Studied the effect of Gaussian Filter on image of point object for optical system using different apertures[11]. In the present study, the effect of the Gaussian filter on the eccentric circular aperture will be studied. derivation of the point spread function for eccentric circular aperture will presented , followed by calculations and results sections.

2. DERIVING THE EQUATION OF POINT SPREAD FUNCTION (PSF) FOR ECCENTRIC CIRCULAR APERTURE WITH GAUSSIAN FILTER

The pupil function for any point in exit pupil of any shape can be written as:

$$f(x, y) = \boldsymbol{\tau}(\mathbf{x}.\mathbf{y}) \cdot \mathbf{e}^{\mathbf{ikW}(\mathbf{x},\mathbf{y})} \qquad (1)$$

where $\tau(x.y)$ represents the real amplitude function distributed in exit pupil and it is called "pupil transparency" or " transmission function" and it is equal to1 if there is no apodization, and W(x,y) represents aberration function.

A pupil is referred to as a Gaussian pupil if the amplitude variation across it has the form of a Gaussian. It can be obtained in two different ways. In imaging applications, the wave incident on the enterance pupil has a uniform amplitude, but its transmission varies as a Gaussian. In applications of Gaussian beam propagation, the incident wave has a Gaussian amplitude, but its transmission is uniform. In this research Gaussian filter was used to write the transmission function as as [12].

$$\tau(x.y) = e^{-\gamma^2(x^2+y^2)}$$

where x and y are normalized at the edge of the circular pupil, which has a radius a and γ represents the Gaussian beam truncation ratio, which is given by $\gamma = a/\omega$, where ω is the $1/e^2$ of beam radius. The larger γ is, the less the truncation and the greater the transmitted energy, leading to stronger apodization.

The pupil function of eq. (1) becomes

$$f(x, y) = \mathbf{e}^{-\gamma^2(\mathbf{x}^2+\mathbf{y}^2)} \cdot \mathbf{e}^{\mathbf{ikW(x,y)}} \qquad (2)$$

The PSF for incoherent illumination is the square absolute value of Fourier transform of the pupil function[13], i.e.

$$\text{PSF}=|\boldsymbol{\mathcal{F}}\{\mathbf{f}(\mathbf{x},\mathbf{y})\}|^{\mathbf{2}} \qquad (3)$$

Figure (1) represents the Eccentric aperture that consists of circular aperture of radius1 unit with non-central obscuration of radius ε.

The equation of the outer circle is given by the relation:

$$\mathbf{x^2 + y^2 = 1} \qquad (4)$$

While the equation of the obscuration circle is:

$$(\mathbf{x} - \mathbf{x_0})^2 + (\mathbf{y} - \mathbf{y_0})^2 = \boldsymbol{\varepsilon}^2 \qquad (5)$$

Where (x_0,y_0) is the center of the obscuration circle.

Let $x' = x - x_0$ and $y' = y - y_0$ or

i.e. $\mathbf{x = x' + x_0}$ and $\mathbf{y = y' + y_0}$

(6)

Then eq.(5) becomes

$$\mathbf{x'^2 + y'^2 = \boldsymbol{\varepsilon}^2} \qquad (7)$$

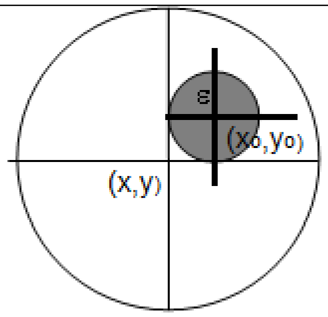
From eq.3 and using the limits of eq.s 3 and 6 for eccentric aperture the normalized PSF can be written as:

$$\text{PSF} = \text{n.f.}\left|\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)e^{i2\pi(ux+vy)}\,dx\,dy - \int_{-\varepsilon}^{\epsilon}\int_{-\sqrt{\varepsilon^2-y'^2}}^{\sqrt{\varepsilon^2-y'^2}} f(x,y)e^{i2\pi(u(x'+x_0)+v(y'+y_0))}\,dx'\,dy'\right|^2 \quad (8)$$

Where u and v are the spacial frequencies in the focal plane related to the angular distances with x and y axes, and n.f. is the normalizing factor which is for uniform illumination equal to reciprocal of the square of aperture area or the reciprocal of diffraction limited PSF (no aberration), i.e.

$$\text{n.f.} = \left|\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dx\,dy - \int_{-\varepsilon}^{\epsilon}\int_{-\sqrt{\varepsilon^2-y'^2}}^{\sqrt{\varepsilon^2-y'^2}} dx'\,dy'\right|^{-2}$$

Taking one coordinate of exit pupil (i.e. v=0)



Figure(1): Eccentric aperture

$$\text{PSF} = \text{n.f}\left|\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)e^{i2\pi(ux)}\,dx\,dy - \int_{-\varepsilon}^{\epsilon}\int_{-\sqrt{\varepsilon^2-y'^2}}^{\sqrt{\varepsilon^2-y'^2}} f(x'+x_0, y'+y_0)e^{i2\pi(u(x'+x_0))}\,dx'\,dy'\right|^2 \quad (9)$$

3. RESULTS AND DISCUSSION

The program(MathCAD) was used in this research to solve the equations used to study the effect of the Truncation ratio of the Gaussian filter with certain values, (γ = 0,1,2,3) on the Point Spread Function and the Strehl ratio and the optimum balance of aberrations for the eccentric circular aperture system, which is a circular aperture of unit area non-central obscuration of radius (ε =٠.٣) centered at (x=a=٠.٣,y=b=٠.٣) of the outer aperture: as follows.

Point Spread Function with Gaussian filter

The PSF was studied for Diffraction-Limited system with the presence of different types of 1st order aberration. (tilt and Focus Error) and of 3rd order aberration (spherical, coma, and Astigmatism) as follows

PSF with Tilt Aberration

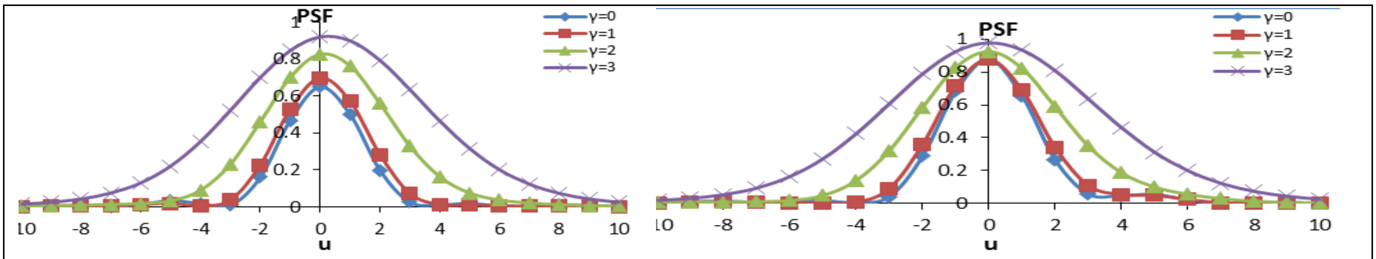
In this part, the effect of the Truncation ratio of the Gaussian filter was studied on the PSF for the eccentric circular aperture with Tilt Aberration of ($W_{11} = 0.2\lambda$) as shown in Figure (1-a), where we notice the central peak is equalt. (0.65) when ($\gamma = 0$) .This value increases with increasing (γ) to(0.699, 0.827, and 0.921), respectively. It must be known that the increase is in normalized PSF, and here the normalization constant does not equal the area of the aperture as in uniform Illumination but it is dependent on the value ofγ

PSF with Focus Error

The effect of the Truncation ratio of the Gaussian filter was calculated on the PSF with focus error of ($W_{20} = 0.2\lambda$) as shown in Figure (1-b) , where the central peak equalt. (0.878) when ($\gamma = 0$) ,this value increases with increasing (γ) to (0.878, 0.926 and 0.979), respectively.

PSF with Spherical Aberration

The effect of the Truncation ratio of the Gaussian filter on PSF of eccentric circular aperture with spherical aberration ($W_{40} = 0.2\lambda$) can be seen in figure (2-a), where the central peak is equal (0.867) when ($\gamma = 0$), This value increases with increasing (γ) to (0.884, 0.955 ,and 0.995) respectively.

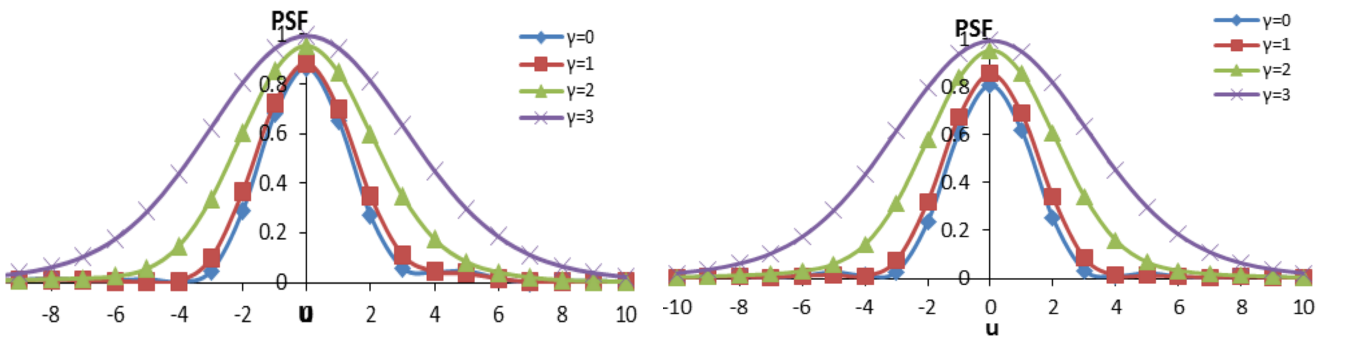


(a) (b)

Figure 1: The effect of the truncation ratio of the Gaussian filter on a PSF of eccentric circular aperture system (ε = 0.3, a = b = 0.3) with (a) Tilt ($W_{11} = 0.2\lambda$) (b) Focal error($W_{20} = 0.2\lambda$)

PSF with Coma Aberration

The effect of the truncation ratio of the Gaussian filter on PSF with Coma Aberration ($W_{31}=0.2\lambda$)is shown in Figure (2-b) where the central peak is equal (0.807) when ($\gamma = 0$).This value increases with increasing (γ) to (0.855, 0.953 ,and 0.993) , respectively.



(a) (b)

Figure 2: The effect of the truncation ratio of the Gaussian filter on a PSF of eccentric circular aperture system (ε = 0.3, a = b = 0.3) with (a) spherical aberration($W_{40} = 0.2\lambda$)(b) coma ($W_{31} = 0.2\lambda$)

3.2 Optimum Balance For Aberrations with Gaussian Illumination

The presence of aberration of all types affects the value of Point Spread Function and thus the Strehl ratio, which means reducing the quality of the image. To minimize this effect, the existence of a certain type of third order aberration with certain percentage of appropriate first order aberration can be led to optimum balance of aberrations and gives the best value of Strehl ratio, as follows:

3.2.1 Spherical Aberration with Focus Error Factor

Strehl ratio was calculated for each value of spherical aberration (W_{40}= 0λ,…. ,0.5λ) for a range of values of focus error ($W_{20} = -2\lambda \ldots, 2\lambda$), as shown in the left side of fig. (3), which shows that when W_{20} = W_{40} , the strehl ratio value is the best. This is illustrated in tables (1) that demonstrated the balanced values of spherical aberration and focus error with the values of strehl ratios before and after balancing. It can be seen also that the Strehl ratio value is improved after the balancing, where the acceptable value of spherical aberration which allows the Strehl ratio to begreaterthan 0.8 as Marchalcriteria [15]. $W_{40}$was ٠.٢ λ before balancing for (γ=1) and ٠.٤ λ for (γ=2), and after the balancing all the values taken have been acceptable for all Gaussian values.

Table 1: Balanced values of W40 and W20 with the values of S.R. before and after balancing, for eccentric annular aperture (=0.3,a=b=0.3) with Truncation ratio (=1,2,3)

W_{40}	W_{20}	S.R. before B.	S.R. after B.	S.R. before B.	S.R. after B.	S.R. before B.	S.R. after B.
		γ = 1		γ = 2		γ = 3	
0	0	1	1	1	1	1	1
0.1	0.1	0.97	0.998	0.988	0.997	0.999	0.999
0.2	0.2	0.884	0.991	0.955	0.99	0.995	0.999
0.3	0.3	0.757	0.98	0.904	0.988	0.989	0.997
0.4	0.4	0.609	0.964	0.843	0.98	0.981	0.996
0.5	0.5	0.462	0.944	0.778	0.966	0.971	0.994

3.2.2 Coma Aberration with Tilt Aberration

Strehl ratio was calculated for each value of coma Aberration (W_{31}= 0λ,…. ,0.5λ) with a range of values of tilt error ($W_{11} = -2\lambda \ldots, 2\lambda$) as illustrated in the right side of figure (3). Note that the appropriate values of W_{11} are not equal to the values of coma aberration as that of spherical but the percentage of increase increases with increasing value of comma aberration as illustrated in tables (2) according to the value of γ respectively. Also,the table shows how the strehl ratio improved after balancing when γ =1 and 2 but not when γ = 3. It can be noticed that the accepted value of W_{31} , when γ =1, before balancing is 0.2 λ, while after balancing W_{31}=0.4 λ, while when γ =2, the accepted value of W_{31} before and after balancing is 0.4λ.

Table 2: Balanced values of W_{31} and W_{11} with the values of S.R. before and after balancing, for eccentric annular aperture (ε=0.3,a=b=0.3) with Truncation ratio (γ=1,2,3).

W_{31}	W_{11}	S.R. before B.	S.R. after B.	W_{11}	S.R. before B.	S.R. after B.	W_{11}	S.R. before B.	S.R. after B.
	$\gamma = 1$			$\gamma = 2$			$\gamma = 3$		
0	0	1	1	0	1	1	0	1	1
0.1	0.1	0.961	0.983	0.2	0.988	0.988	0.3	0.998	0.993
0.2	0.3	0.855	0.955	0.3	0.953	0.956	0.6	0.993	0.973
0.3	0.4	0.708	0.904	0.5	0.901	0.907	0.9	0.985	0.942
0.4	0.6	0.551	0.83	0.7	0.837	0.842	1.3	0.974	0.902
0.5	0.7	0.41	0.755	0.9	0.769	0.765	1.7	0.961	0.855

3.2.3 Astigmatism aberration with Focus Error Factor

The addition of focus error does not helps in reducing astigmatism aberration because the sign of astigmatism aberration on the x-axis and the y- axis are different while the sign at the two axes of focus error are similar, so when adding focus error to astigmatism aberration be either a reduction astigmatism aberration on the x-axis and increase on the y- axis or vice versa. So, it was added x- or y- focus error alone, as in the right side of figure (4). Table (3) shows that the value that satisfies the Marshall condition has changed from W_{22} = 0.2λ to 0.3λ in the case of γ= 1, and from W_{22}= 0.3 λ to 0.4 λ in the case of γ = 2. The last balance can be improved if it is added x-focus and y-focus separately and made $W_{22} = W_{20y} = -W_{20x}$ as in right side of figure (4), Which shows clearly how the strel ratios changed after balancing to unity for all values of W_{22} and all values of γ.

Table 3: Balanced values of W_{22} with W_{20x} with the values of S.R. before and after balancing for eccentric annular aperture (ε=0.3,a=b=0.3) with truncation ratio (γ=1,2,3).

W_{22}	W_{20x}	S.R. before B.	S.R. after B. withW_{20x}	W_{20x}	S.R. before B.	S.R. after B. withW_{20x}	W_{20x}	S.R. before B.	S.R. after B. withW_{20x}
	γ = 1			γ = 2			γ = 3		
0	0	1	1	0	1	1	0	1	1
0.1	0.1	0.946	0.978	0.1	0.978	0.99	0.1	0.995	0.997
0.2	0.3	0.8	0.917	0.2	0.917	0.96	0.2	0.979	0.99
0.3	0.4	0.605	0.832	0.3	0.826	0.914	0.3	0.955	0.977
0.4	0.5	0.41	0.724	0.4	0.719	0.856	0.4	0.923	0.96
0.5	0.6	0.251	0.606	0.5	0.609	0.791	0.5	0.884	0.94

4. CONCLUSIONS

1. The normalized Point Spread Function with the presence of a certain types of aberration increases with increasing the truncation ratio of the Gaussian filter . It must be known here that the increasing is normalized PSF and the normalization constant does not equal to the area of the aperture as that in uniform Illumination but it is dependent on the value of truncation ratio γ.
2. The effect of 3rd order aberration (spherical, coma, astigmatism) can be reduced by adding suitable value of 1st order aberration (tilt and Focus Error)

3. 3.The values of focal error $W_{20}$and spherical aberration $W_{40}$for optimum balance are equal.
4. The values the tilt aberrationW_{11} and coma aberration W_{31} that suitable for optimum balance were not equal, but the ratio between them increases with increasing coma aberration
5. 5.The addition of focus error does not helps in reducing astigmatism aberration because the sign of astigmatism aberration on the x-axis and the y- axis are different while the sign at the two axes of focus error are similar, so when adding focus error to astigmatism aberration be either a reduction astigmatism aberration on the x-axis and increase on the y- axis or vice versa.
6. Astigmatisn can be balanced by adding x-axis and y-axis focus error separately by making$W_{22} = W_{20y} = -W_{20x}$.

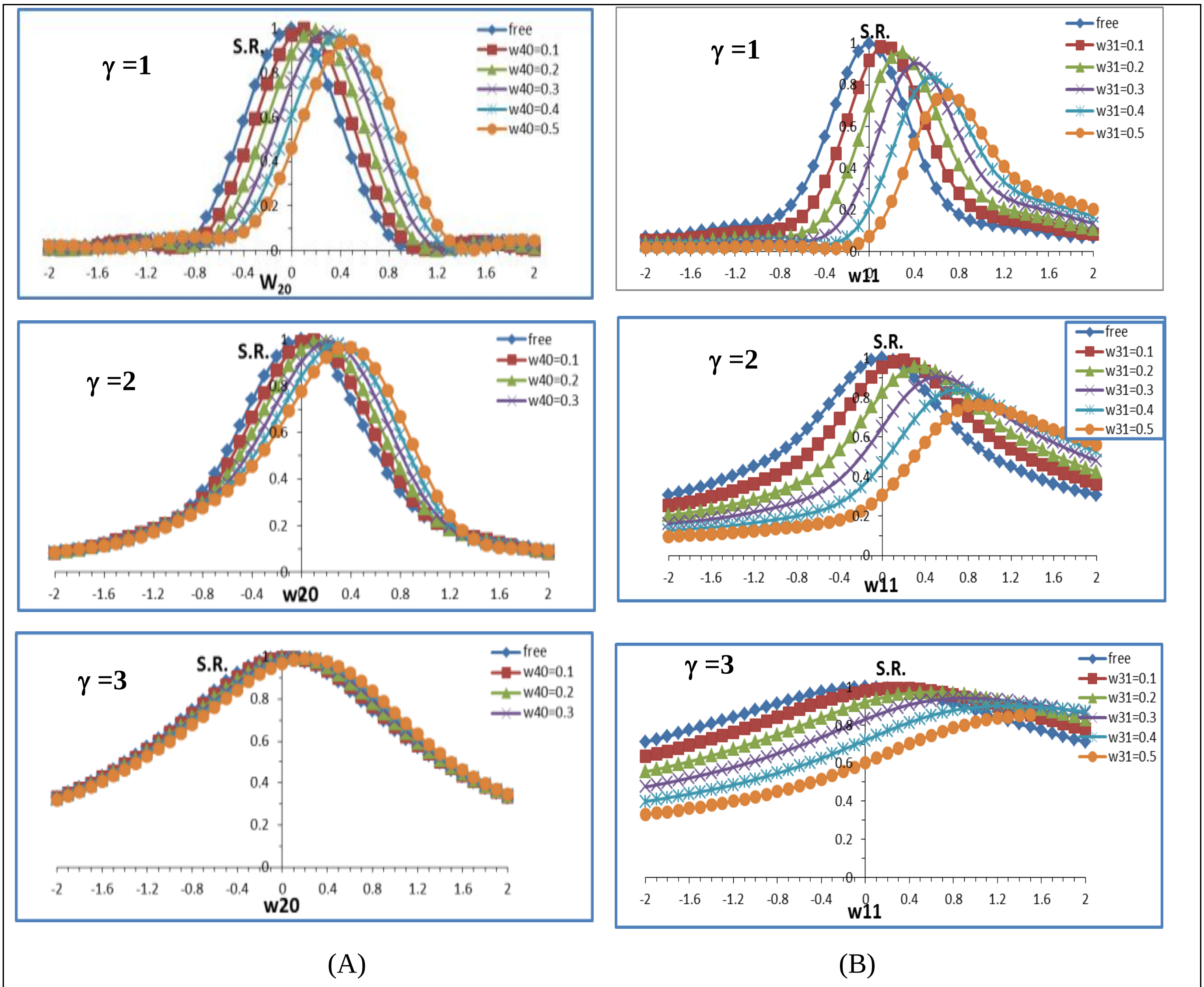


Figure 3: (A) Strehl ratio with spherical aberration and focal error. (B) Strehl ratio with coma aberration and tilt error of an eccentric Annular aperture with Gaussian Illumination

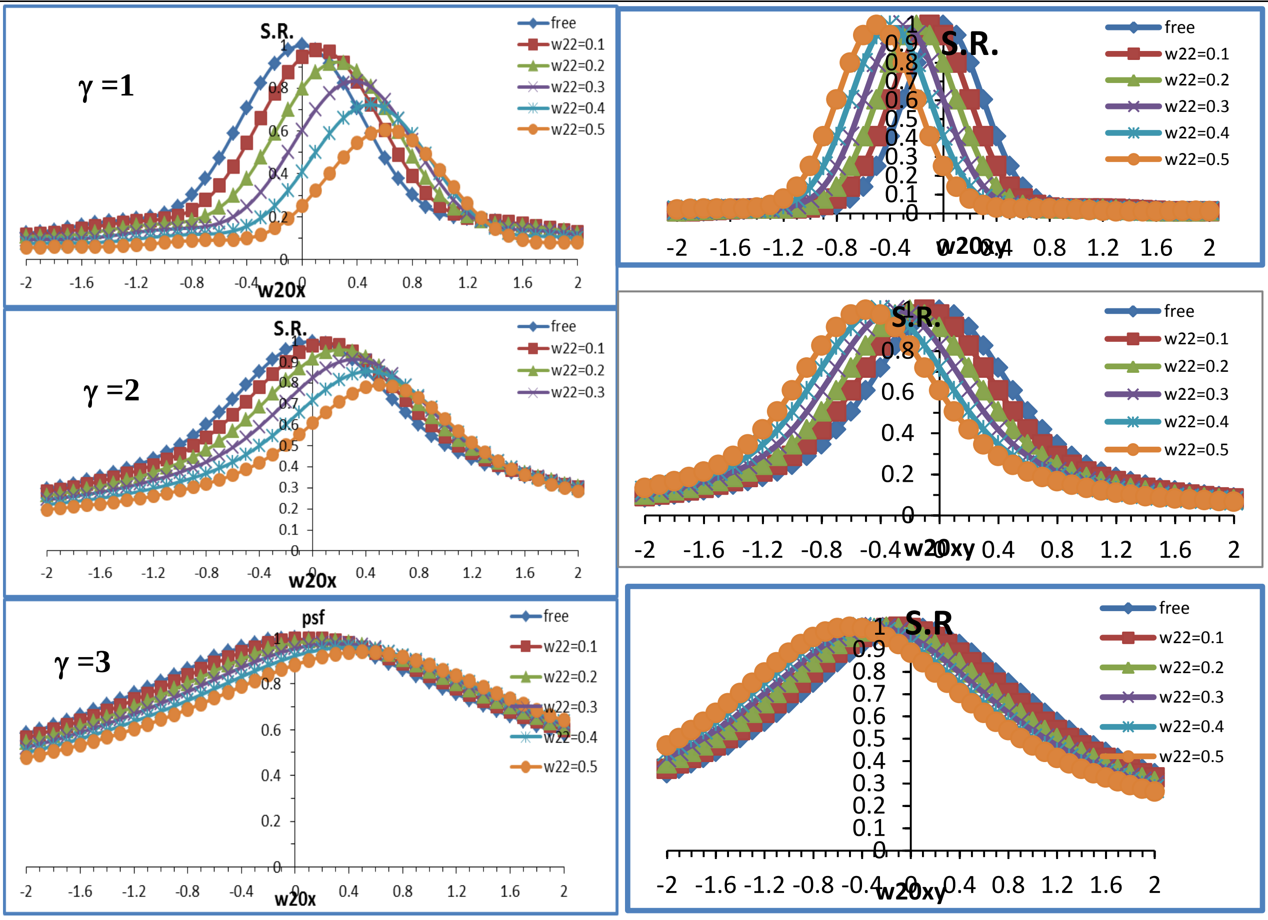


Figure 4: Left side: Strehl ratios with Astigmatism and focal error-x axis. Right side: Strehl ratios with Astigmatism and focal error-x axis and y-axis and $W_{22} = W_{20y} = -W_{20x}$ of an eccentric Annular aperture with Gaussian Illumination.

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