ON SEMI OPEN FUNCTION

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Abstract

In this work we study inductively open functions, and we introduce a new concept of Inductively semi open functions, multifunction and prove the now result about inductively S-open function also we gives a remarks and examples to show this result.

1- Introduction

The concept of function is considered as on of the important concept in mathematics, especially in general topology. There are several types of functions such as inductively open functions…

In this work, in section one we study Inductively semi open we are symbol by S-open functions [Single valued and multi-valued].

In section two we study some theories about inductively semi open functions which are valid for Inductively open function will be also true for inductively S-open function (sometime with extra conditions).

1- Definition and the concept of inductively

1.1 Definition

Let f: $X \to Y$ be a function, we say that f is S-open iff the image of every open set in X is S-open in Y.

1.2 Definition

A multifunction F: $X \to Y$ is a set valued function, that as sings empty subset of Y for each $x \in Y$.

1.3 Definition

Let $f : X \to Y$ be a single valued function [$F : X \to Y$ be a multi function] we say that f[F] is inductively S-open function [multi function] iff there exists a subset X^* $\subseteq X$, such that $f(X^*) = f(X)$ [$F(X^*) = F(X)$ and $f|_{X^*}: X^* \to f(X)$ is S-open function [$F|_{X^*} : X^* \to F(X)$ is S-open multifunction].

1.4 Remarks

i- Every single valued function is a multifunction, but the converse is not true in general.

ii- Every Inductively S-open onto function is Inductively S-open multifunction.

2- Inductively semi open functions

2.1 Definition

Let F: $X \to Y$ be a multifunction, Let $\phi \neq A \subseteq X$, and $\phi \neq B \subseteq Y$, then

i- $F(A) = \{F(x): x \in A\}$

ii- $F^+(B) = \{x \in X : F(x) \subseteq B\}$

iii- $F^-(B) = \{x \in X : F(x) \cap B \neq \phi\}$ and

iv- $F^*(B) = \{x \in X : F(x) \cap B = \phi\}$ and

2.2 Theorem

If f: $X \to Y$ is inductively S-open function and let $\phi \neq T \subseteq Y$, then f_T: $f^{-1}{}_{(T)} \to T$ be also inductively S-open function .

Proof

Since f: $X \to Y$ inductively S-open function, then there exists a subset $X_1 \subseteq X$, such as that $f(X_1) = Y$ and $f|X_1 : X_1 \to Y$ is S-open function.

Now, to prove $f_T : f^{-1}{}_{(T)} \to T$ is inductively S-open function.

Let $X^*{}_1$ be a subset of $f^{-1}{}_{(T)}$, such that $X^*{}_1 = X_1 \cap f^{-1}{}_{(T)}$ we needed to show that $f_T(X^*{}_1)$ T and $f_T|X^*{}_1 : X^*{}_1 \to T$ is open $F_T(X^*{}_1) = f(X^*{}_1) = f(X_1 \cap f^{-1}{}_{(T)}) = f(X_1) \cap T = Y \cap T = T$

Now, let w^* be open set in $X^*{}_1$

Hence, there exists an open set w in x_1, such that $w^* = w \cap x^*{}_1$

$f(w^*) = f(w \cap x^*{}_1) = f[w \cap x^1 \cap f^{-1}{}_{(T)}] = f[w \cap f^{-1}{}_{(T)}]$

$= f(w) \cap T$

Since w is open in x1 and $f_T | X_1 : X_1 \to Y$ is S-open, therefore f(w) is S-open in Y so $f(w) \cap T$ is S-open in T. Hence $f_T : f^{-1}{}_{(T)} \to T$ is inductively S-open function

2.3 Theorem

Let f: $X \to Y$ be a function, $X = W_1 \cup W_2$ with $f(w_1)$ and $f(w_2)$ are open in f(X), if $f|W_1 : w_1 \to Y$ and $f|W_2 : w_2 \to Y$, are inductively S-open functions, then f: $X \to Y$ inductively S-open function.

Proof

Since $f|W_1 : w_1 \to Y$ inductively S-open function. Then, there exists a subset $X_1 \subseteq w_1 \ni f(X_1) = f(w_1)$ and $f|X_1 : X_1 \to f(w_1)$ S-open function also $f|_{W_1} : w_1 \to Y$ inductively S-open function, then, there exists a subset $X_2 \subseteq w_2 \ni f(x_2) = f(w_2)$ and $f|X_2 : X_2 \to f(w_2)$ S-open function.

Now, To show $f : X \to Y$ inductively S-open function

Let $X^* = X_1 \cup X_2 \subseteq X$, $f(X^*) = f(X_1 \cup X_2) = f(X_1) \cup f(X_2)$

$= f(w_1) \cup f(w_2) = f(w_1 \cup w_2) = f(X)$

So $f(X^*) = f(X)$ and to show $f|_{X^*} : x^* \to f(x)$ S-open function let T open in X^* so $T = T \cap X^* = T \cap (X_1 \cup X_2) = (T \cap X_1) \cup (T \cap X_2)$

So $f(T) = f[(T \cap X_1) \cup (T \cap X_2)] = f(T \cap X_1) \cup f(T \cap X_2)$

Since T is open in X^*, so $T \cap X_1$ is open in X1 and $f|X_1 : X_1 \to f(w_1)$ S-open, then $f(T \cap X_1)$ S-open in f(X). Similarly $f(T \cap X_2)$ S-open in f(X).

$f(T) = f(T \cap X_1) \cup f(T \cap X_2)$ S-open in f(X) $f|X^*{}_1 : X^*{}_1 \to f(X)$ is S-open function. Therefore f: $X \to Y$ inductively S-open function.

2.4 Remarks and examples

If F: Y is an inductively S-Open multifunction and if $T \subseteq Y$ then

$F : F^-{}_{(T)} \to T$ is not necessary inductively S-open multifunction. But if we assume that T has the following extra property

$F^-{}_{(T)} = F^+{}_{(T)}$ (in this case we write $F^x{}_{(T)} = F^-{}_{(T)} = F^+{}_{(T)}$ and we can say in this case (that T is a good subset of Y) Now we can show this case by the following example:-

Let X = {a, b, c} and T_x= {ϕ, X , {a}, {b,c}} be a topology defined on X.

Let Y = {d, e, f, g} and T_Y= {ϕ, Y , {d}, {e}, {d,e}} be a topology defined on Y. A multifunction F: $x \to Y$ defined by:

F(a) = {f} F(b) = {f,g} , F(c) = {d,e}. Now, let B = {f,g}

$F^+(B) = F^+(\{f,g\}) = \{a,b\}$ $F^-(B) = F^-(\{f,g\}) = \{a,b\}$

So $F^+(B) = F^-(B) = \{a,b\}$

Now, we can prove the following theorem for a multifunction.

..1 Theorem

Let $F : X \to Y$ be inductively S-open onto multifunction and T be a good subset of Y, with $F^*_{(T)} \subseteq X$, then a multifunction $F_T : F^*_{(T)} \to T$ is inductively S-open multifunction.

Proof

Since $F: X \to Y$ inductively S-open onto multifunction.

So, $\exists$ a subset $X_1 \subseteq X$, $\ni F(x_1) = F(x) = Y$ and $f|_{X_1} : X_1{:}Y$ is S-open.

To show $F_T : F^*_{(T)} \to T$ inductively S-open multifunction. Consider $X^*_1 = X_1 \cap F^*_{(T)}$ be a subset of $F^*_{(T)}$ To check $F_T(X^*_1) = T$,

$F_T(X^*_1) = F(X^*_1) = F(X_1 \cap F^*_{(T)})$

To show $F(x_1 \cap F^*_{(T)}) = F(x_1) \cap T$ ......(1) We needed to show first $F(x_1 \cap F^*_{(T)}) \subseteq F(x_1) \cap T$. Let $y \in F(x_1 \cap F^*_{(T)})$ So $\exists$ at least one element $x \in X_1 \cap F^*_{(T)} \ni y \in F(x)$

$\therefore x \in X_1$ and $x \in F^*_{(T)}$, Since $F(x) \subseteq F(x1)$ and $F(x) \subseteq T$ So by definition $F^*(B) = F+(B)$, $F(x) \subseteq F(x) \cap T$. $Y \in F(x)$

Then $y \in F(x_1) \cap T$. Hence $F(x_1 \cap F^*_{(T)}) \subseteq F(x_1) \cap T$ .... (2)

Now to show $F(x_1) \cap T \subseteq F(x_1 \cap F^*_{(T)})$ Let $y \in F(X_1) \cap T$.

Then $y \in F(x_1)$ and $y \in \exists$ at least one element $x \in X_1$, such that $y \in F(x)$, Then $y \in F(x) \cap T$. So $F(x) \cap T \neq \phi$ $\therefore x \in F^-_{(T)} = F^*_{(T)}$ and $x \in y_1$

Therefore $x \in X_1 \cap F^*_{(T)} F(x) \subseteq F(x_1 \cap F^*_{(T)})$ and $y \in F(x)$

Then $y \in F(x_1 \cap F^*_{(T)})$ So $F(x_1) \cap T \subset F(x_1 \cap F^*_{(T)})$ .... (3)

And by (2) and (3) we get the relation (1)

So $F(x_1 \cap F^*_{(T)}) = F(x_1) \cap T = Y \cap T = T$

Now, to show $F_T|_{x_1^*} : X_1^* \to T$ is S- open Let W open set in X^*_1

So, $\exists$ w^* open in x_1^*, $\ni w = w^* \cap x_1^*$ So $F(w) = F(w^* \cap x_1^*)$

$= F[w^* \cap x_1^* \cap F^*_{(T)}] = F[w^* \cap F^*_{(T)}] = F[(w_1^*) \cap T]$ is S-open in T

So $F_T : = F^*_{(T)} \to T$ is inductively S-open multifunction

3.2 Theorem

Let $F : X \to Y$ be a multifunction, let $X = w_1 \cup w_2$ and $F|_{w_1}: w_1 \to Y$, $F|_{w_2}: w_2 \to Y$ are inductively S-open , $\ni f(w_1)$ open in $F(x)$ and $F(w_2)$ open in $F(x)$, then $F: X \to Y$ be inductively S-open multifunction

Proof

Since $F|_{w_1}: w_1 \to Y$ inductively S-open multifunction.

So, $\exists$ $x_1 \subseteq w_1 \ni F(x_1= F(w_1)$ and $F|_{X_1}: X_1 \to F(w_1)$ is S-open and $F|_{w_2}: w_2 \to Y$, inductively S-open multifunction.

So, $\exists$ $x_2 \subseteq w_2 \ni F(x_2= F(w_2)$ and $F|_{X_2}: X_2 \to F(w_2)$ is S-open .

Now, to show $F: x \to Y$ inductively S-open multifunction.

Let $X^* = X_1 \cup X_2$ be a subset of X Now , to show $F(x^*) = F(x)$ and $F|_{X^*}: x^* \to F(x)$ is S-open . Hence $F(x^*) = F(x_1 \cup x_2) = F(x_1) \cup F(x_2)$

$= F(w_1) \cup F(w_2) = F(w_1 \cup w_2) = F(x)$ Now, Let T open set in X^*

$T = T \cap X^* = T \cap (x_1 \cup x_2) = (T \cap x_1) \cup (T \cap x_2)$ So $T \cap x_1$ open in x_1 and $T \cap x_2$ open in x_2 .Therefore $F_{(T)} = F[(T \cap x_1) \cup (T \cap x_2)]$

$= F(T \cap x_1) \cup F(T \cap x_2)$ $F(T \cap x_1)$ S-open in $F(w_1)$ and $F(w_1)$ S-open in $F(x)$, So $F(T \cap x_1)$ S-open in $F(x)$

Similarly F $(T \cap x_2)$ S-open in F(x), So $F_{(T)} = F(T \cap x_1)\ F(T \cap x_2)$ S-open in F(x). Therefore $F|_{X^*}: X^* \rightarrow F(x)$ is S-open So F: X→ Y inductively S-open multifunction.

References

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الدوال الشبه مفتوحة

الخلاصة

في هذا البحث درسنا الدوال المفتوحة استقرائياً وقدمنا مفهوماً جديداً وهو مفهوم الدوال شبه المفتوحة استقرائياً، وكذلك برهنا نتائج جديدة حول الدوال شبه المفتوحة استقرائياً واعطاء بعض الملاحظات الخاصة بها وامثلة توضح هذه النتيجة.