Effect of Width/Depth (B_f/D) Section Ratio and Curvature on the Behavior of Horizontally Curved Steel Beam

Raad Kamil Hashim
Babylon University, College of Engineering

Abstract

The aim of this research is to study the effect of the width to depth ratio (b_f/d) and the curvature of the steel I-section horizontally curve beams on their ultimate strength and the statical response. The research adopts three – dimensional nonlinear finite element analysis of steel I-section horizontally curved beams under static load. The twenty-node isoparametric brick element has been used to represent the steel element; also, the Von-Mises yield criterion is used to compute the stress level of plastic deformation. Elastic-perfect plastic model is used to represent the behavior of steel under tension and compression stresses. Thirty beams have been analyzed with the same length, mesh, material properties and boundary conditions. The effect of b_f/d ratio is considered by taken different values (0.5-2.5) with the same area of cross section. Also, the effect of curvature was studied by considering different values from beam with half circle to straight beam. It is obtained that the change of (b_f/d) ratio reveals that the load-carrying capacity increasing about 30-50% when the ratio changed from (1) to (2) for the same area. So, it is found that the change of (b_f/d) ratio is more effectively when the curvature is high. The results appear that the (b_f/d) ratio equal to 1.5-2 is optimum for different values of curvature.

الخلاصة

ان الغرض من هذا البحث هو دراسة تأثير نسبة عرض مقطع الى ارتفاعه وكذلك الانحناء لعتب منحني افقيا على تحمله الاقصى وتصرفه الساكن. اعتمد هذا البحث في تحليل العتب المنحني افقيا والمحمل باحمال ساكنة، على النموذج اللاخطي ثلاثي الابعاد للعناصر المحددة. تم استخدام العنصر الطابوقي ذو العشرين عقدة لتمثيل العناصر الفولاذية، وكذلك تم استعمال معيار الخضوع لـ (Von-Mises) لمراقبة مستوى الاجهاد اللازم لحدوث التشوهات اللدنة. تم استعمال نموذج Elastic-perfect plastic لتمثيل سلوك الفولاذ تحت تأثير اجهادات الشد والضغط. تم تحليل ثلاثين عتب لنفس الطول ، التقسيم، خصائص الماد، والشروط الحدودية. تأثير نسبة b_f/d تم اخذها بنظر الاعتبار بأخذ مختلف القيم لها والتي تتروح ما بين (0.5-2.5) لنفس مساحة المقطع. كذلك تم اخذ تأثير الانحناء بنظر الاعتبار باخذ عدة قيم للانحناء تتروح ما بين كون العتب نصف دائرى الى عتب مستقيم. النتائج المستحصلة بينت ان لنسبة العرض الى العمق اهمية كبيرة وجيدة ، حيث ان تحمل العتب قد ازداد بنسبة تتروح من 30 الى 50 % عندما غيرت نسبة العرض/ العمق من 1 الى 2 لنفس مساحة المقطع، كذلك وجد ان تغيير نسبة العرض الى العمق تكون اكثر فعالية عندما تكون قيمة الانحناء عالية. اخيرا النتائج المستحصلة بينت ان نسبة العرض الى العمق المثالثة هي (1.5-2) لكل انواع الانحناء.

1. Introduction

1.1 Background and Previous Research

Steel I-Section horizontally curve beams are used in building construction especially in bridges, where the need to augment traffic capacity in urban highways is vital.

The plastic analysis for the determination of collapse loads for horizontally circular curve beams was done by Jordian, *et. al*., 1974. The analysis was for the case of two concentrated loads placed at any point on the arc of the beam. In 1985, Kaoro, *et. al.* investigate the effects of curvature and the effect of cross section dimensions on the effective width of horizontally simply supported curve beam for box and channel cross-section under uniformly distributed loads. In 1990, Hsu, Fu and Schelling, developed a more exact horizontally curve beam finite element in which the true warping degree of freedom conforms the warping. Shanmugham, *et. al.* and Tan, 1995, tested a ten steel horizontally curved beam. The results obtained from experiments on two sets of I-beams. The object of this study is to determine the ultimate load capacity of steel I-beams, with intermediate lateral restraint and to examine the effect of curvature on the behavior of these beams under bending loads. Khalid and John, 1999, summarized the results from an extensive parametric study, using finite element method in which simply supported curved composite multi-cell bridge prototypes are analyzed to evaluate the moment and deflections between girders due to truck loading and dead load. In 2001, Young-Lin and Bradford investigate the nonlinear behavior of steel I-section beams curved in plan under vertical loading and proposed straight forward formulas for their design against combined bending and torsion actions. Recently, in 2008, Al-Mutairee investigates the analysis of three dimensional horizontal beam (steel, concrete and composite) subjected to static and dynamic loads. For steel beam, the effect of tapering ratios, curvature and many other parameters, were taken into account on the behavior and dynamic response of curved beams.

1.2 Object and Scope of Research

As shown all previous researchers did not study the effect of width to depth (b_f/d) ratio on the statical response of steel I-section horizontally curved beam. The main object of this research is to study the effect of change (b_f/d) ratio of I-section (which is taken into account for the same area of section). This is important in manufacturing of the section, and one can gain best performance of section for the same area. In addition, the curvature of the beam was investigated to take their effect of b_f/d ratio.

2. Finite Element Idealization

2.1 Steel Idealization

Twenty-node isoparametric quadratic brick element was employed for analysis of in-plane curved beam. This element has been successfully used by many three dimension nonlinear studies (Cervera, *el. al*.,1988), (Al-Sherrawi, 2001) and (Al-Mutairee, 2008). The element has its own local coordinate system ***r,s,t,*** as shown in Figure(1), with origin at the center of the element such that each local coordinate ranges from (-1) to (+1).

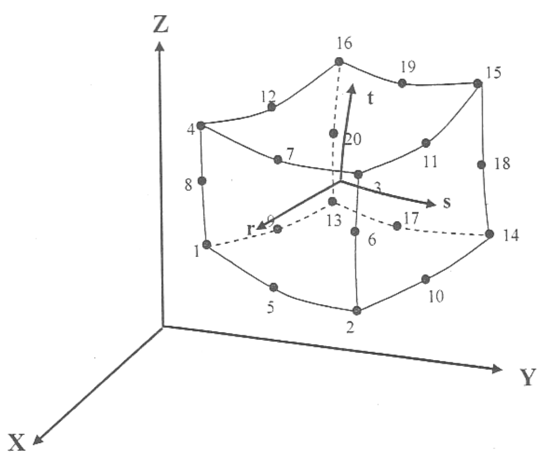


Figure (1): The Twenty-Node Isoparametric Quadratic Brick Element in Cartesian Coordinates.

2.1.1 Shape Functions of the 20-Node Brick Element

The displacement components at a particular point p (r,s,t) within the element are defined using the nodal values at each of the twenty nodes and the quadratic shape functions such that: (Cook, 1974)

$$u(r,s,t)=\sum_{i=1}^{20} N_i(r,s,t)u_i$$
$$v(r,s,t)=\sum_{i=1}^{20} N_i(r,s,t)v_i \qquad \ldots(1)$$
$$w(r,s,t)=\sum_{i=1}^{20} N_i(r,s,t)w_i$$

Where ***N_i(r,s,t)*** is the shape function at node ***i***. The shape functions for the twenty-nodes have the following forms:

$$N_i=\frac{1}{8}(1+rr_i)(1+ss_i)(1+tt_i)(rr_i+ss_i+tt_i-2), \quad i=1-4,13-16$$
$$N_i=\frac{1}{4}(1+r^2)(1+ss_i)(1+tt_i), \qquad i=6,8,18,20$$
$$N_i=\frac{1}{4}(1+rr_i)(1-s^2)(1+tt_i), \qquad i=5,17,19,7 \qquad \ldots(2)$$
$$N_i=\frac{1}{4}(1+rr_i)(1+ss_i)(1+t^2), \qquad i=9,10,11,12$$

The Cartesian coordinate values of any point ***p(r,s,t)*** within the element may be defined as:

$$x(r,s,t)=\sum_{i=1}^{20} N_i(r,s,t)x_i$$
$$y(r,s,t)=\sum_{i=1}^{20} N_i(r,s,t)y_i \qquad \ldots(3)$$
$$z(r,s,t)=\sum_{i=1}^{20} N_i(r,s,t)z_i$$

where x_i, y_i and z_i are the Cartesian coordinates of node ***i***

2.1.2 Strain-Displacement Representation

For three-dimensional finite element analysis, the strain components can be evaluated as: (Zienkiewicz, 1977)

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \sum_{i=1}^{20} \begin{bmatrix} \partial N_i/\partial x & 0 & 0 \\ 0 & \partial N_i/\partial N_i & 0 \\ 0 & 0 & \partial N_i/\partial z \\ \partial N_i/\partial y & \partial N_i/\partial x & 0 \\ 0 & \partial N_i/\partial z & \partial N_i/\partial y \\ \partial N_i/\partial z & 0 & \partial N_i/\partial x \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} \qquad \ldots(4)$$

The shape function N_i is a function of both local and Cartesian coordinates. The derivative of the shape function can be given by the usual chain rule as:

$$\begin{Bmatrix} \partial N_i/\partial r \\ \partial N_i/\partial s \\ \partial N_i/\partial t \end{Bmatrix} = \begin{bmatrix} \partial x/\partial r & \partial y/\partial r & \partial z/\partial r \\ \partial x/\partial s & \partial y/\partial s & \partial z/\partial s \\ \partial x/\partial t & \partial y/\partial t & \partial z/\partial t \end{bmatrix} \begin{Bmatrix} \partial N_i/\partial x \\ \partial N_i/\partial y \\ \partial N_i/\partial z \end{Bmatrix} = [J] \begin{Bmatrix} \partial N_i/\partial x \\ \partial N_i/\partial y \\ \partial N_i/\partial z \end{Bmatrix} \quad \ldots(5)$$

where, [J] is known as the Jacobin matrix.

Then the shape function derivatives with respect to ***x, y*** and ***z*** axes can be obtained as:

$$\begin{Bmatrix} \partial N_i/\partial x \\ \partial N_i/\partial y \\ \partial N_i/\partial z \end{Bmatrix} == [J]^{-1} \begin{Bmatrix} \partial N_i/\partial r \\ \partial N_i/\partial s \\ \partial N_i/\partial t \end{Bmatrix} \quad \ldots(6)$$

2.1.3 Stiffness Matrix for Twenty-Noded Brick Element

For an element of volume V, the stiffness matrix may be expressed as: (Dawe, 1984)

$$[k]_e = \int_{v_e} [B]^T.[D].[B]dv_e \quad \ldots(7)$$

In three dimensional elements, the differential volume, dv_e, may be written as;:

$$dv_e = dx.dy.dz \quad \ldots(8)$$

Equation (8) can be transformed into natural coordinates as:

$$dv_e = |J|\ dr.ds.dt \quad \ldots(9)$$

where |**J**| is the determinant of the Jacobin matrix. The limits of integration in the natural coordinates become -1 to +1 and the element stiffness matrix can be expressed as:

$$[k]_e = \int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1} [B]^T.[D].[B].|J|dr.ds.dt \quad \ldots(10)$$

2.2 Numerical Integration

In the present study, the Gaussian-Legender quadratic numerical integration technique has been used to evaluate the stiffness matrix [Zienkiweixz, 1977]. In this technique the element stiffness matrix for the brick element may be written as:

$$[k]_e = \int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1} f(r,s,t)dr.ds.dt \cong \sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p} W_i.W_j.W_k.\ f(r_i, s_{ji}, t_k) \quad \ldots(11)$$

where ***P*** are the number of Gaussian points in the ***r,s*** and ***t*** direction, and the expression ***f(r,s,t)*** represents the matrix multiplication $\left([B]^T.[D].[B].\det|J|\right)$.

For brick element representing steel, the (3x3x3) Gauss quadratic integration rule has been used in this study.

3. Modeling of Steel Material

3.1 Elastic-plastic stress-strain relation for steel

The elastic – perfect plastic relationship based on von-Mises criterion is used for in-plane curved I-steel beams, study as shown in Figure (2)

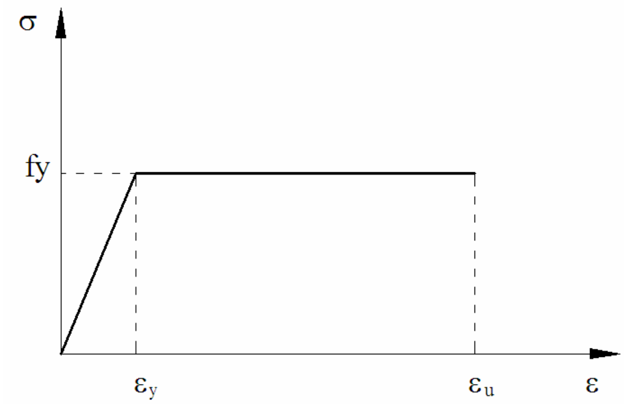


Figure (2): Idealization of Stress-Strain Curve for Steel Beams

3.2 Yield Criterion Used for Steel

The von-Mises criterion is used as a yield criterion in this research, where it is monitor the stress level at the onset plastic deformation for steel material. It can be expressed as (Chen, 1982), (see Figure (3)):

$$f(\sigma)=\sqrt{3J_2}=\sigma_o \qquad \ldots(12)$$

Where J_2 is equal to:

$$J_2=\frac{1}{3}\left[\left(\sigma_1^2+\sigma_2^2+\sigma_3^2\right)-\left(\sigma_1.\sigma_2+\sigma_2.\sigma_3+\sigma_3.\sigma 1\right)\right] \qquad \ldots(13)$$

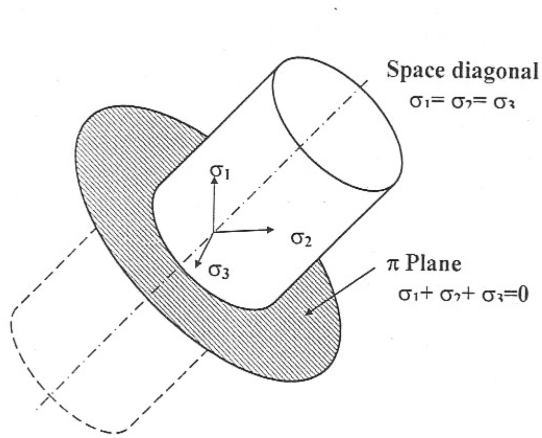


Figure (3): Three – Dimensional Representation of von-Mises Yield Surfaces in Principal Stress Space.

4. Nonlinear Solution Technique

In the present study, the incremental-iterative method is adopted to consider the material nonlinearity problem, where the load applied as a series of increments and at each increment iterative solution is carried out to find the true response, see Figure(4).

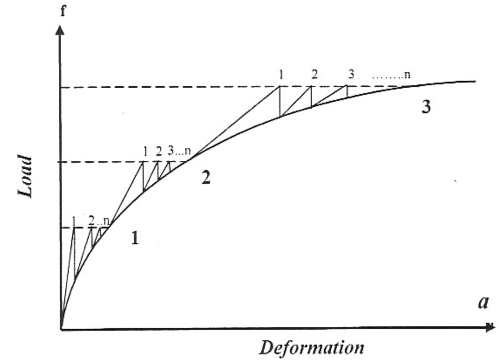


Figure (4): Incremental-Iterative Technique for the Solution of Nonlinear Equations

5. Convergence Criterion

A termination criterion for any iterative process should be used to stop iteration. At certain iteration within the increment of loading, if the difference between the external and the internal forces becomes negligibly small, the convergence is assumed to be occurred. The force convergence criterion which considered in this study, can be expressed in the form:

$$\frac{\sqrt{\{r(a)\}^T.\{r(a)\}}}{\sqrt{\{f\}^T.\{f\}}} \leq \textit{Specified tolerance} \qquad \ldots(14)$$

<u>6. Application and Discussions</u>

6.1 Comparison with other studies

In order to evacuate the program, the curve beam which tested by Shanmugam in 1995, is analyzed. Figure (5) shows the geometry, properties and loading for curved beam.

ν=222.GPa
$\sigma_{y)web}$=395 MPa
E_{flange}=205.1 GPa
$\sigma_{y)flange}$=337 MPa

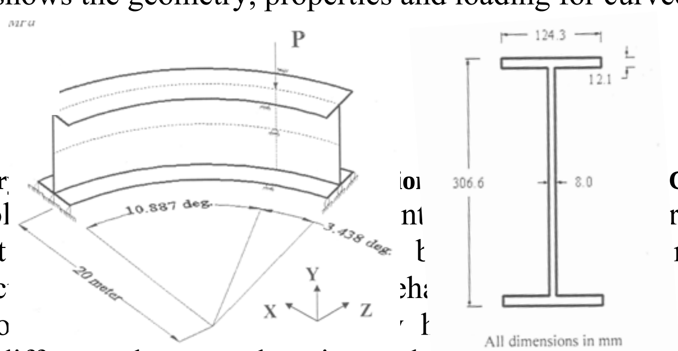


Figure (5): Geometr ... ion ... Curved Beam

The curve beam is sol ... nt ... re (6). It is clear from Figure (7) that ... t ... mental and the predicted load deflec ... ha ... specimen. The computer ultimate lo ... h ... perimental one (192 kN), hence, the difference between them is equal t...

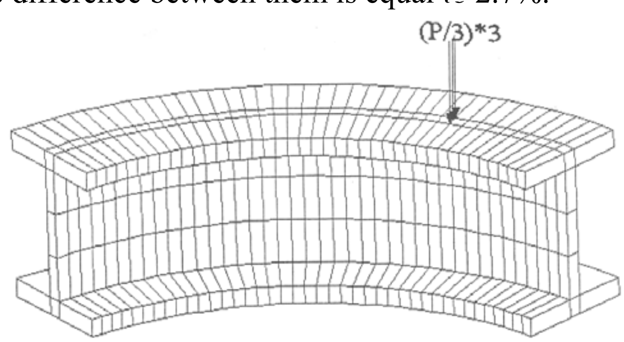


Figure (6): Finite Element Mesh of 360 Brick Elements of Curve Beam

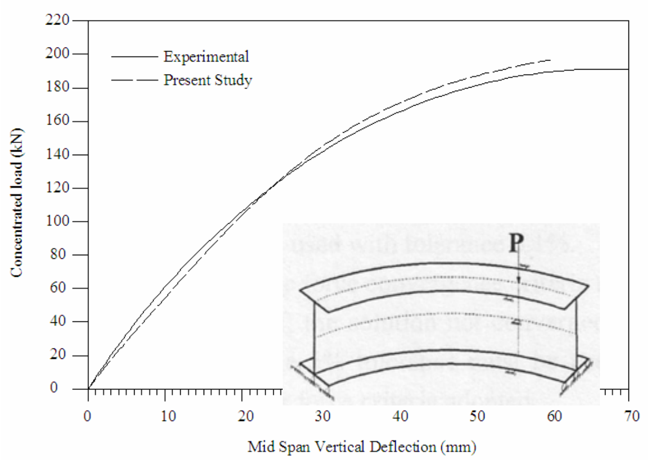
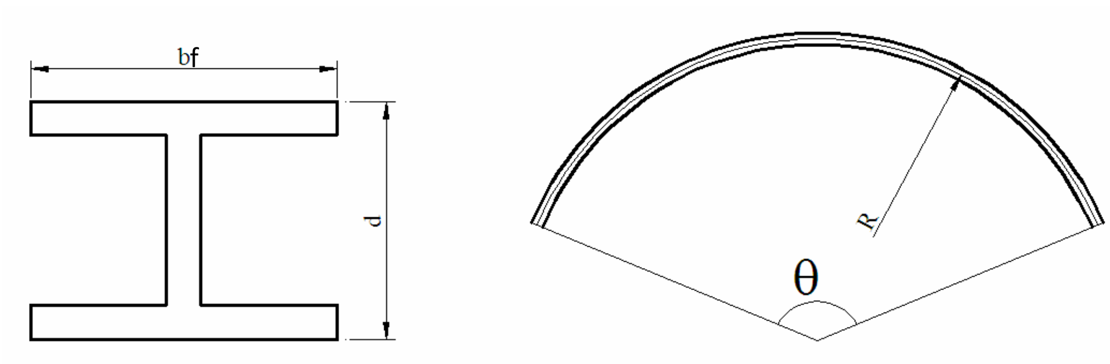


Figure (7): Load Deflection Curve for Shanmugam Curve Beam

6.2 Parametric Study

In order to study the effect of (b_f/d) ratio on the behavior of steel curve beam, thirty beams with constant length, cross-sectional area and boundary condition are studied. The values of (b_f/d) ratio (0.5, 1.0, 1.5, 2.0, and 2.5) are studied for (θ=180^o, 135^o, 90^o, 45^o and zero degree), as shown in Figure(8), then studying the above values with the other curvatures (θ=135^o,90^o,45^o and straight beam). The constant area of the section is taken equal to 2640 mm^2 and the length of the member is 3000 mm, also the thickness of web is assumed 8 mm and the one of top and bottom flange is 10 mm. The cases studied information, can be summarized in the Table (1) and Table (2). The boundary conditions of the beam are assumed to be fixed at both ends.



A= 2640 mm^2 , E=210 MPa, σ_y=350 MPa, t_w=8 mm, t_f=10 mm , L=3000 mm

(a) (b)

Figure (8): Configuration of in-plane curve beam (a) Section in beam; (b) Top view of beam

Table (1): Values of (b_f/d) considered in the present study

b_f/d	b (mm)	d (mm)
2.5	175	70
2.0	155.55	77.77
1.5	175	87.5
1.0	100	100
0.75	80.77	107.70
0.50	58.33	116.66

Table (2): Values of (θ) and R in the present study

θ(^{o})	R (mm)
180	954.9
135	1273.26
90	1909.86
45	3819.72
0 (Straight Beam) ∞	

The present in-plane curved beam was modeled by (240) elements mesh with 20-node isoperimetric element, as shown in Figure (9). Finally, the program NFHCBSL is adopted to analyze the above curve beams [Al-Mutairee, 2008].

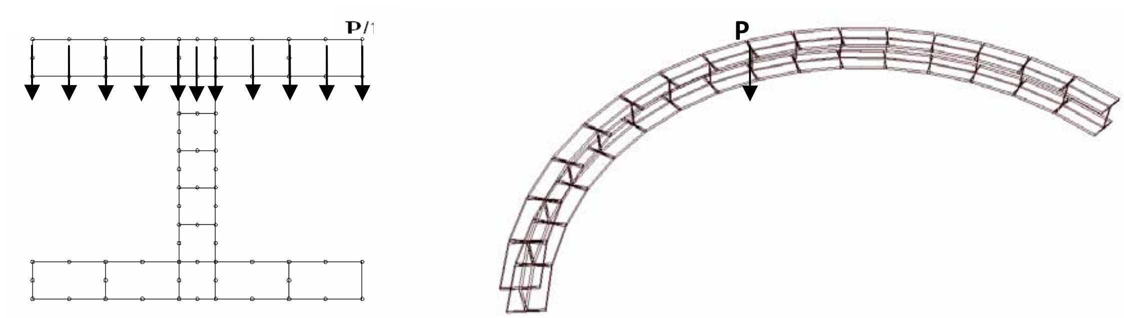


Figure (9): Mesh Configuration of in-plane Curved Beam Under Concentrated Load,
a) Mesh of End Section of Brick Element and load distribution, b) Mesh Along the Beam

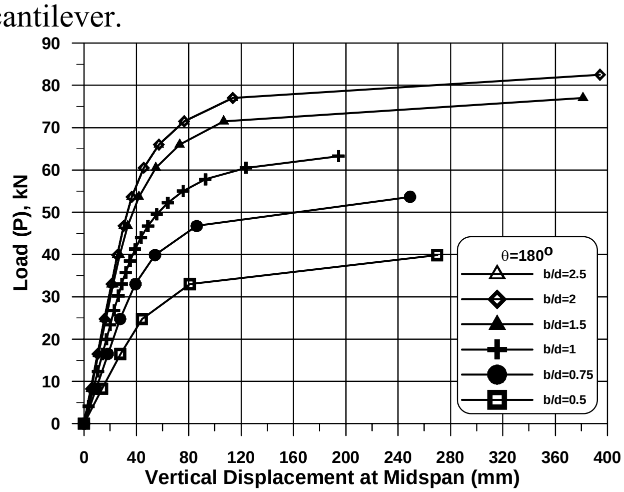
7. Finite Element Results

7.1 Effect of (b_f/d) ratio

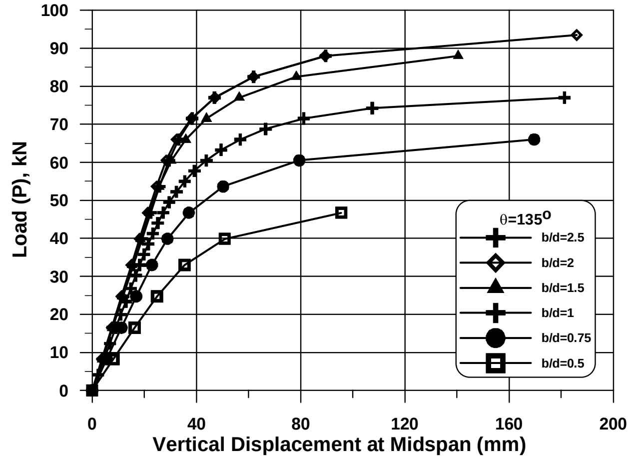
It is clear from Figure (10), the changing of (b_f/d) ratio has important role in the response of the steel horizontally curve beam and on the load-carrying capacity. It can be seen that, the load-carrying capacity of the beam will increase as soon as the (b_f/d) ratio is increased. For a range of (b_f/d) ratio (0.5-2.0, the load carrying capacity will increase about 50% to 105%, also, the (b_f/d) equal to 2 gives increasing of load-carrying capacity of about 30% from the ratio 1, which are widely used in practice.

7.2 Effect of curvature

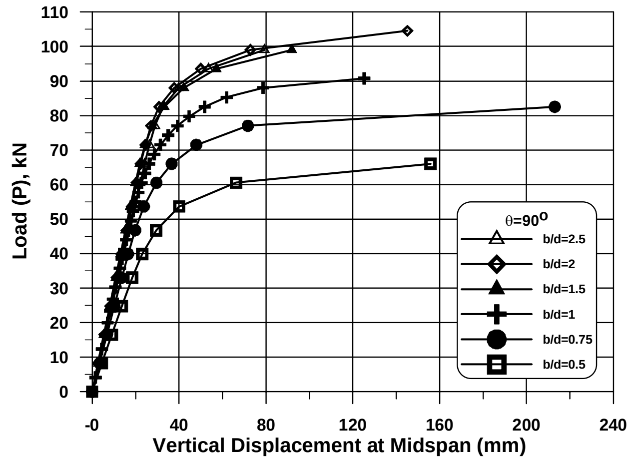
Figure(10), show that for the same (b_f/d) ratio, beam capacity is different according to the curvature, where the capacity of straight beam is increased about 30%-50% for (θ=180^o), i.e. half circle. This is due to the ability of straight beam to support additional load after plastic zone appear, i.e. if the plastic zone appear at support the beam will be work as simply supported beam, or when the plastic zone appear at midspan, support can resisting additional stress as cantilever.



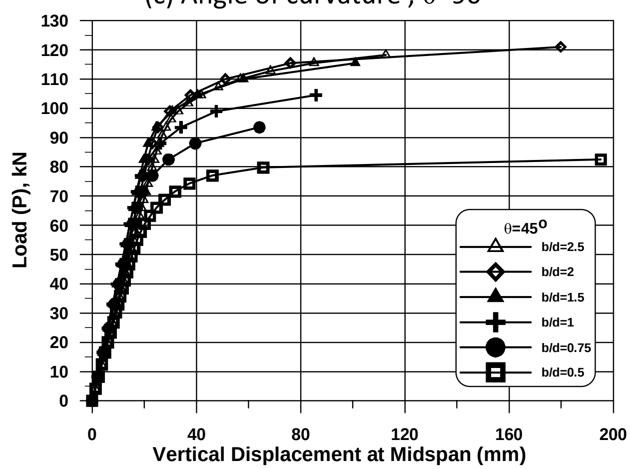
(a) Angle of curvature , θ=180^o



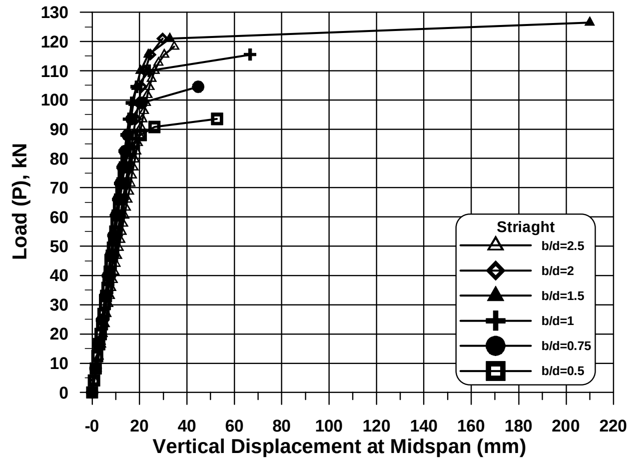
(b) Angle of curvature , θ=135°



(c) Angle of curvature , θ=90°



(d) Angle of curvature , θ=45°



(e) Angle of curvature , $\theta=0^o$ [Straight beam]

Figure (10): Load – Deflection curve at midspan of beam
a) $\theta=180^o$, b) $\theta=135^o$, a) $\theta=90^o$, a) $\theta=45^o$, e) Straight

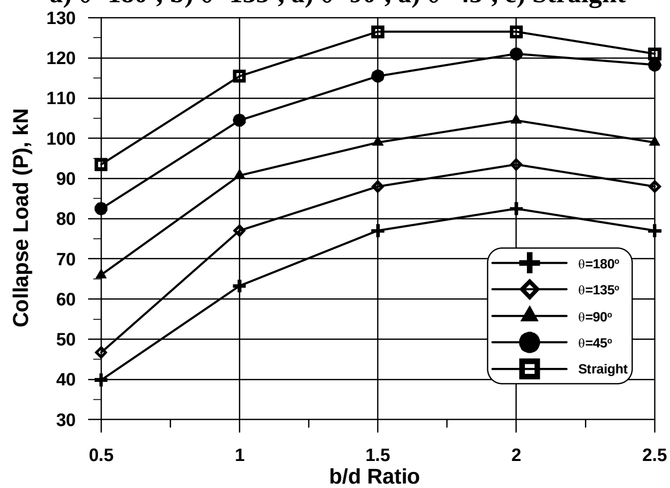


Figure (11): Collapse Load with b_f/d

However, it can be seen from Figure (11) that the optimum value of (b_f/d) approximately lying between (1.5 to 2). Also, it can be noted that this ratio depend on curvature of in-plane curved steel beam, where if the curvature of beam is high, the (b_f/d) ratio will be changed more effectively change noticed.

8. Conclusions

From the obtained results, it can be concluded that:

1- The selection of b_f/d ratio in the analysis of steel curve beam is so important, where it is improve the performance of section for the same area. The load-carrying capacity increase about 30-50% when the (b_f/d) ratio changed from (1) to (2) for the same area.

2- Selection of (b_f/d) ratio depends on the curvature of in-plane curvature, where (b_f/d) ratio is more effective if the curved beams be straight. The load – carrying capacity of straight beam is increased about 30%-50% from for ($\theta=180^o$), i.e. half circle.

3- It is found from results that the optimal value of (b_f/d) ratio lying between (1.5) to (2.0), where the benefit reached to about 121%.

9. References

Al-Mutairee, H.M.K., “Nonlinear Static and Dynamic Analysis of Horizontally Curved Beams”, Ph.D Thesis, University of Babylon, 2008.

Al-Sherrawi, M.H.M. “Shear and Moment Behavior of Composite Concrete Beams”, Ph.D Thesis, University of Baghdad, Iraq, 2001.

Cervera, M., Hinton, E., Bonet, J., and Bicanic, N., “Nonlinear Transient Dynamic Analysis of Three Dimensional Structure. A finite Element Program for Steel and Reinforced Concrete Materials”, pp.320-550. In Hinton E. Book’s “Numerical Methods and Software for Dynamic Analysis of Plates and Shell”, Pinerdge Press, 550 pages, 1988, ISBN, 0-906674-48-4.

Chen, W.F., “Plasticity in Reinforced Concrete”, Third Edition, McGraw-Hill Book Company, 1982.

Cook, R.D., “Concept and Application of Finite Element Analysis”, John Wiley and Sons, Inc., New York, 1974.

Dawe, D.J., “Matrix and Finite Element Displacement Analysis of Structures”, Clarendon Press, Oxford, U.K., 1984.

Hsu, Y. T., Fu,C.C., and Schelling, D.R., “An Improved Horizontally Curved Beam Element”, Journal of Computer and Structure, Vol.34, No.2, pp.313-318.

Jordaan, I.J., Khalifa, M.M.A., and McMullen, A.E., “Collapse of Curved Reinforced Concrete Beams”, Journal of ASCE, Vol.100, No.ST11, Nov.,1974.

Kaoru, H., Seizo, U., and Yasushi, H., “Shear Lag Analysis and Effective Width of Curved Girder Bridges”, Journal of Engineering Mechanics, Vol.111, No.1, pp.87-92, January, 1985.

Khalid, S., and John, B.K. “Simply Supported Curved Cellular Bridges: Simplified Design Method”, Journal of Bridge Engineering, Vol.4, No.2, pp.85-94, May, 1999.

Shanmugam, N.E., Thevendran, V., Richard Liew, J.Y., and Tan, L.O., “Experimental Study on Steel Beams Curved in Plan”, Journal of Structural Engineering, Vol.121, No.2, pp.249-259, February, 1995.

Yong-Lin, P., and Bradford, M.A., “Strength Design of Steel I-Section Beams Curved in Plan”, Journal of Structural Engineering, Vol.127, No.6, pp.639-646, June, 2001.

Zienkiweixz, O.C., “The Finite Element Method”, Third Edition, McGraw Hill, London, 1977.