

On

Certain Types of Minimal Proper function

Dr.Sattar Hameed

Hamza

Department of Mathematics,
College of Education
Al-Qadisiyh university

E-mail:

sattar_math@yahoo.com

Fieras Joad Obead Al-

Ysary

Department of Mathematics,
College of Education
Al-Qadisiyh university

E-mail:

fieras_math@yahoo.com

Abstract:

The main aim of this work is to create a new types of proper function namely m -proper, m_* -proper, $*m$ -proper and $*m_*$ -proper function. Also, we gave the relation among the certain types of minimal proper function.

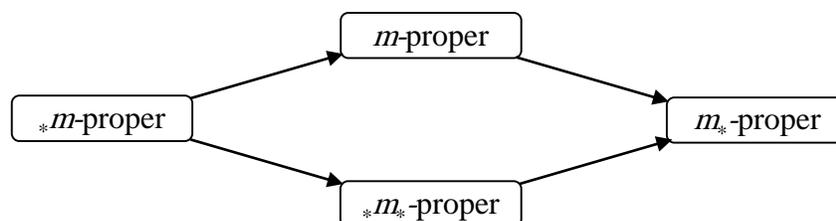
Introduction:

In 1950 Maki H., Umehara J. and Noiri T. introduced the notions of minimal structure and minimal space. They achieved many important results compatible by the general topology case. We recall the basic definitions and facts concerning minimal structures and minimal spaces.

Alimohammady M. and Roohi M. in [1] give the definition of minimal closed set (m -closed set) and give the definition of minimal continuous function (m -continuous function) and study the properties of it. And Ravi O., Ganesan S., Tharmar S. and Balamukugan in [6] give the definition of minimal closed function (m -closed function) and study some properties of it.

In this work we give the definitions of certain types of minimal continuous functions and minimal closed and used it to construct a definition of minimal proper function and certain types of it (m_* -proper, $*m$ -proper and $*m_*$ -proper functions). And prove that the composition of m -proper ($*m$ -proper, $*m_*$ -proper) is m -proper ($*m$ -proper, $*m_*$ -proper) respectively (2.1, 2.2 and 2.3)but the composition of m_* -proper function is not necessarily m_* -proper function. The

restriction of m -proper (m_* -proper) function from um -space into m -space on an m -closed subset is m -proper (m_* -proper) function respectively (3.2, 3.3). Also the restriction of $*m$ -proper ($*m_*$ -proper) function on a closed subset is $*m$ -proper ($*m_*$ -proper) function respectively (3.4, 3.5). we give the relation among these types and from (4.1, 4.2, 4.3, 4.4 and 4.5) we have the following diagram.



1. Basic Definitions and Notations

1.1 Definition [1], [4]

Let X be a non-empty set and $P(X)$ the power set of X . A subfamily M_X of $P(X)$ is called a minimal structure (briefly m -structure) on X if $\phi, X \in M_X$. In this case (X, M_X) is said to be minimal space (briefly m -space). A set $A \in P(X)$ is said to be an m -open set if $A \in M_X$. $B \in P(X)$ is an m -closed set if $B^c \in M_X$.

1.2 Example

Let $X = \{a, b, c, d\}$ and $M_X = \{\phi, X, \{a\}, \{b\}, \{c, d\}\}$. Then M_X is an m -structure on X , and (X, M_X) is an m -space.

1.3 Remark

Every topological space is m -space but the converse is not necessarily true as the following example shows. Let $X = \{a, b, c\}$ then $M_X = \{\phi, X, \{a\}, \{b\}\}$ is m -space but not topological space. Since $\{a\} \cup \{b\} = \{a, b\} \notin M_X$.

1.4 Remark

If (X, M_X) is m -space then there is always a subfamilies T_{M_X} of M_X satisfies the conditions of topological spaces (at least the family $\{\phi, X\}$) and the intersection of these

families represent the indiscrete topology on X . T_{M_x} called induced topology from minimal structure.

Note: in this work if A is open set in X is mean $A \in T_{M_x}$. Also if B is closed set in X mean that $B^c \in T_{M_x}$.

1.5 Definition [7]

Let X be a non-empty set and M_x an m -structure on X . For a subset A of X , the minimal closure of A (briefly \bar{A}^m) and the minimal interior of A (briefly $A^{\circ m}$), are defined as follows:

$$\bar{A}^m = \bigcap \{F : A \subseteq F, F^c \in M_x\}$$

$$A^{\circ m} = \bigcup \{V : V \subseteq A, V \in M_x\}$$

1.6 Proposition [1],[4],[5]

Let X be a non-empty set and M_x an m -structure on X . For $A, B \subseteq X$ the following properties hold:

- i. $A \subseteq \bar{A}^m$ and $A^{\circ m} \subseteq A$;
- ii. if $A^c \in M_x$, then $\bar{A}^m = A$ and if $A \in M_x$, then $A^{\circ m} = A$;
- iii. $\bar{\phi}^m = \phi$, $\bar{X}^m = X$, $\phi^{\circ m} = \phi$ and $X^{\circ m} = X$;
- iv. $\overline{(\bar{A}^m)^m} = \bar{A}^m$ and $(A^{\circ m})^{\circ m} = A^{\circ m}$.
- v. $(A^c)^{\circ m} = (\bar{A}^m)^c$ and $\overline{(A^c)^m} = (A^{\circ m})^c$;
- vi. if $A \subseteq B$, then $\bar{A}^m \subseteq \bar{B}^m$ and $A^{\circ m} \subseteq B^{\circ m}$;
- vii. $(A \cap B)^{\circ m} = A^{\circ m} \cap B^{\circ m}$ and $A^{\circ m} \cup B^{\circ m} \subseteq (A \cup B)^{\circ m}$;
- viii. $\overline{(A \cup B)^m} = \bar{A}^m \cup \bar{B}^m$ and $\overline{(A \cap B)^m} \subseteq \bar{A}^m \cap \bar{B}^m$.

1.7 Remark

Let (X, M_X) be an m -space, if A, B are m -open sets then $A \cap B, A \cup B$ not necessarily m -open set as the following example shows. Let $X = \{a, b, c, d\}$, $M_X = \{\emptyset, X, \{a\}, \{b\}, \{a, b, c\}, \{a, b, d\}\}$ be an m -structure on X then $\{a\}, \{b\}, \{a, b, c\}, \{a, b, d\} \in M_X$ but $\{a\} \cup \{b\} = \{a, b\} \notin M_X$ and $\{a, b, c\} \cap \{a, b, d\} = \{a, b\} \notin M_X$. So, we introduce the following definition.

1.8 Definition

An m -space (X, M_X) is called an

- (i) um -space if the arbitrary union of m -open sets is an m -open set.
- (ii) im -space if the any finite intersection of m -open sets is an m -open set.

1.9 Proposition

Let (X, M_X) be a um -space, and A be a subset of X then:

- i. $A \in M_X$ if and only if $A^{\circ m} = A$;
- ii. A is an m -closed if and only if $\overline{A}^m = A$.
- iii. $A^{\circ m} \in M_X$ and $(\overline{A}^m)^c \in M_X$.

Proof:

(i) Let A be an m -open set, then $A^{\circ m} = A$ by using proposition (1.6-ii).

Conversely: Let $A^{\circ m} = A$, since $A^{\circ m} = \bigcup \{U : U \subseteq A, U \in M_X\}$ is an m -open set (X is a um -space), hence $A \in M_X$.

(ii) Let A be an m -closed set, therefore $\overline{A}^m = A$ by using proposition (1.6-ii).

Conversely: Let $\overline{A}^m = A$, then $(\overline{A}^m)^c = A^c$ from propositions (1.6-v) and (1.6-ii) we have $(\overline{A}^m)^c = (A^c)^{\circ m}$ then $(A^c)^{\circ m} = A^c$, therefore A^c is an m -open set, hence A is an m -closed set in X .

(iii) Obvious.

1.10 Remark

If X be a um -space and A, B be m -closed set in X then $A \cap B$ is m -closed set in X .

Note that, if (X, M_X) is an m -space and $A \subseteq X$ then $M_A = \{W \cap A : W \in M_X\}$ is a minimal structure on A . [3]

1.11 Definition [3]

Let (X, M_X) be an m -space and $A \subseteq X$ then the pair (A, M_A) is called the minimal subspace (briefly m -subspace) of (X, M_X) .

1.12 Proposition

Let A be an m -subspace of a um -space X such that A be an m -closed set in X , and let $B \subseteq A$, then B is m -closed set in A if and only if B is m -closed set in X .

Proof:

Let B be an m -closed set in X then $(X - B) \cap A \in M_A$, and then $A - ((X - B) \cap A)$ is m -closed set in A . Since $A \subseteq X$ thus $A - ((X - B) \cap A) = A - ((A - B) \cap A) = A - (A - B) = B$ is an m -closed set in A .

Conversely: Let B be an m -closed set in A , then there is an m -closed set V in X such that $B = A \cap V$. Since $X - V, X - A \in M_X$ then by definition (1.8-i) we have $(X - A) \cup (X - V) \in M_X$, therefore $X - (A \cap V) = X - B \in M_X$. Hence B is an m -closed set in X .

1.13 Theorem [8]

Let (X, M_X) and (Y, M_Y) be two m -spaces, then

$$M_{X \times Y} = \{U \times V : U \in M_X \text{ and } V \in M_Y\}$$

is an m -structure on $X \times Y$.

Now, we can introduce the following definition.

1.14 Definition

Let (X, M_X) and (Y, M_Y) be two m -space then the pair $(X \times Y, M_{X \times Y})$ is called minimal product space (briefly m -product space).

1.15 Proposition

Let (X, M_X) and (Y, M_Y) be two *im*-spaces, then the *m*-product space $(X \times Y, M_{X \times Y})$ is an *im*-space.

Proof:

Let W_1 and W_2 are *m*-open set in $X \times Y$. To prove that $W_1 \cap W_2 \in M_{X \times Y}$.

Then $W_i = U_i \times V_i$ such that U_i, V_i are *m*-open set in X and Y respectively

$i = 1, 2$. Since X and Y are *im*-spaces then $U_1 \cap U_2, V_1 \cap V_2$ are *m*-open set in X and Y respectively. Thus by theorem (1.13) we have

$(U_1 \cap U_2) \times (V_1 \cap V_2) = (U_1 \times V_1) \cap (U_2 \times V_2) = W_1 \cap W_2$ is an *m*-open set in $X \times Y$. Hence $(X \times Y, M_{X \times Y})$ is an *im*-space.

1.16 Definition [7]

Let $f : (X, M_X) \rightarrow (Y, M_Y)$ be a function from *m*-space X into *m*-space Y then f is called a minimal continuous (briefly *m*-continuous) if $f^{-1}(B) \in M_X$, for every $B \in M_Y$.

1.17 Remark

In general if $f : (X, M_X) \rightarrow (Y, M_Y)$ be a function from *m*-space X into *m*-space Y , $B \in T_{M_Y}$ then it is not necessarily $f^{-1}(B) \in M_X$ for all non-indiscrete topology T_{M_Y} induced from M_Y . As the following example shows.

1.18 Example

Let $X = \{a, b, c\}$ and $Y = \{1, 2, 3\}$ such that $M_X = \{\phi, X, \{a\}, \{c\}\}$, $M_Y = \{\phi, Y, \{3\}, \{2, 3\}\}$ are *m*-structure on X and Y respectively and let $f : (X, M_X) \rightarrow (Y, M_Y)$ be a function defined as $f(a) = 2, f(b) = 3, f(c) = 1$, then non-indiscrete topologies T_{M_Y} is $T_{1M_Y} = \{\phi, Y, \{3\}\}$. Then $\{3\} \in T_{1M_Y}$, $f^{-1}(\{3\}) = \{b\} \notin M_X$.

So we introduce the following definition.

1.19 Definition

Let (X, M_X) and (Y, M_Y) be two m -spaces and $f : (X, M_X) \rightarrow (Y, M_Y)$ be a function, then f is called:

- i. m_* -continuous if there is non-indiscrete topology T_{M_Y} such that $f^{-1}(B) \in M_X, \forall B \in T_{M_Y}$.
- ii. $*m$ -continuous if there is non-indiscrete topology T_{M_X} such that $f^{-1}(B) \in T_{M_X}, \forall B \in M_Y$.
- iii. $*m_*$ -continuous if there are non-indiscrete topologies T_{M_X} and T_{M_Y} such that $f^{-1}(B) \in T_{M_X}, \forall B \in T_{M_Y}$.

1.20 Example

- i. Let $X = \{a, b, c\}$ and $Y = \{1, 2, 3\}$ such that $M_X = \{\phi, X, \{b\}, \{c\}\}$,
 $M_Y = \{\phi, Y, \{1\}, \{2\}, \{2, 3\}\}$ are
 m -structure on X and Y respectively and let :

A. $f : (X, M_X) \rightarrow (Y, M_Y)$ be a function defined as $f(a) = 2, f(b) = 3, f(c) = 1$. Then f is:

- a. m_* -continuous since there is non-indiscrete topology $T_{M_Y} = \{\phi, Y, \{1\}\}$ which satisfies the conditions of definition (1.19-i).
- b. $*m_*$ -continuous since there are non-indiscrete topologies $T_{M_X} = \{\phi, X, \{c\}\}$ and $T_{M_Y} = \{\phi, Y, \{1\}\}$ which satisfies the conditions of definition (1.19-iii).
- c. not $*m$ -continuous since all non-indiscrete topologies T_{M_X} are $T_{1M_X} = \{\phi, X, \{b\}\}$ and $T_{2M_X} = \{\phi, X, \{c\}\}$ which are not satisfies the conditions of definition (1.19-ii).

B. $g : (X, M_X) \rightarrow (Y, M_Y)$ be a constant function defined as $f(a) = f(b) = f(c) = 1$. Then g is

$*m$ -continuous since there is non-indiscrete topology $T_{M_X} = \{\phi, X, \{c\}\}$ which satisfies the conditions of definition (1.19-ii).

ii. Let $X = \{a, b, c\}$ and $Y = \{1, 2, 3\}$ such that $M_x = \{\phi, X, \{a\}, \{b\}, \{c\}\}$, $M_y = \{\phi, Y, \{2\}\}$ are m -structure on X and Y respectively and let $h: (X, M_x) \rightarrow (Y, M_y)$ be a function defined as $h(a) = h(b) = 2, h(c) = 3$, then h is neither m_* -continuous nor $*m_*$ -continuous since only non-indiscrete topologies T_{M_y} is $T_{M_y} = \{\phi, Y, \{2\}\}$ which is not satisfies the conditions of definition (1.19-i) and definition (1.19-iii).

1.21 Definition [6]

A function $f: (X, M_x) \rightarrow (Y, M_y)$ is said to be m -closed if for each m -closed set B of X , $f(B)$ is m -closed set in Y .

1.22 Remark

In general if $f: (X, M_x) \rightarrow (Y, M_y)$ be a function from m -space X into m -space Y and $B \in T_{M_x}$ then it is not necessarily $f(B) \in M_y$ for all non-indiscrete topology T_{M_x} induced from M_x . As the following example shows.

1.23 Example

Let $X = \{a, b, c\}$ and $Y = \{1, 2, 3\}$ such that $M_x = \{\phi, X, \{c\}, \{b, c\}\}$, $M_y = \{\phi, Y, \{1\}, \{3\}\}$ are m -structure on X and Y respectively and let $f: (X, M_x) \rightarrow (Y, M_y)$ be a function defined as $f(a) = 3, f(b) = 1, f(c) = 2$, then all non-indiscrete topologies T_{M_x} is $T_{1M_x} = \{\phi, X, \{c\}\}$, $T_{2M_x} = \{\phi, X, \{b, c\}\}$, $T_{3M_x} = \{\phi, X, \{c\}, \{b, c\}\}$. Then $\{a, b\}^c \in T_{1M_x}$, $f(\{a, b\}) = \{1, 3\}$ which is not m -closed set in Y , $\{a\}^c \in T_{2M_x}$, $f(\{a\}) = \{3\}$ which is not m -closed set in Y , and $\{a, b\}^c \in T_{3M_x}$, $f(\{a, b\}) = \{1, 3\}$ which is not m -closed set in Y .

So we introduce the following definitions.

1.24 Definition

Let (X, M_X) and (Y, M_Y) be two m -spaces and $f : (X, M_X) \rightarrow (Y, M_Y)$ be a function, then f is called:

- i. m_* -closed if there is non-indiscrete topology T_{M_X} such that for each m -closed set B of X , $f(B)$ is closed in Y .
- ii. $*m$ -closed if there is non-indiscrete topology T_{M_Y} such that for each closed set B in X , $f(B)$ is m -closed set in Y .
- iii. $*m_*$ -continuous if there are non-indiscrete topologies T_{M_X} and T_{M_Y} such that for each closed set B of X , $f(B)$ is a closed set in Y .

1.25 Example

- i. Let $X = \{a, b, c\}$ and $Y = \{1, 2, 3\}$ such that $M_X = \{\phi, X, \{a\}, \{b\}, \{b, c\}\}$, $M_Y = \{\phi, Y, \{2\}, \{3\}\}$ are m -structure on X and Y respectively and let :

A. $f : (X, M_X) \rightarrow (Y, M_Y)$ be a function defined as $f(a) = 2, f(b) = 3, f(c) = 1$. Then f is:

- a. m_* -closed since there is non-indiscrete topology $T_{M_X} = \{\phi, X, \{a\}\}$ which satisfies the conditions of definition (1.24-i).
- b. $*m$ -closed since there are non-indiscrete topologies $T_{M_X} = \{\phi, X, \{a\}\}$ and $T_{M_Y} = \{\phi, Y, \{2\}\}$ which satisfies the conditions of definition (1.24-iii).
- c. not $*m$ -closed since all non-indiscrete topologies T_{M_Y} is $T_{1M_Y} = \{\phi, Y, \{2\}\}$ and $T_{2M_X} = \{\phi, X, \{c\}\}$ which are not satisfies the conditions of definition (1.24-ii).

- B. Let $M'_X = \{\phi, X, \{a\}, \{a, c\}\}$ and $M'_Y = \{\phi, Y, \{1, 2\}\}$ are m -structure on X and Y respectively and let $g : (X, M'_X) \rightarrow (Y, M'_Y)$ be a constant function defined as $f(a) = f(b) = f(c) = 1$. Then g is $*m$ -closed since there is non-indiscrete topology $T_{M_Y} = \{\phi, Y, \{1, 2\}\}$ which satisfies the conditions of definition (1.24-ii).

- ii. Let $X = \{a, b, c\}$ and $Y = \{1, 2, 3\}$ such that $M_x = \{\phi, X, \{a, c\}\}$, $M_y = \{\phi, Y, \{1\}, \{2\}, \{3\}\}$ are m -structure on X and Y respectively and let $h : (X, M_x) \rightarrow (Y, M_y)$ be a function defined as $h(a) = h(b) = 2, h(c) = 3$, then h is not m_* -closed or $*m_*$ -closed since all non-indiscrete topologies T_{M_x} is $T_{1M_x} = \{\phi, X, \{a, c\}\}$ which is not satisfies the conditions of definition (1.24-i) and definition (1.24-iii).

1.26 The following definition is given in [2]

Let f be a function of a topological space X into a topological space Y then f is called proper function if and only if f is continuous f and the function $f \times I_Z : X \times Z \rightarrow Y \times Z$ is closed for every topological space Z .

1.27 The following result is given in [2]

Every proper function is closed function (take a topological space Z to consist of single point in definition (1.14)).

Now we introduce the following definitions.

1.28 Definition

Let $f : (X, M_x) \rightarrow (Y, M_y)$ be a function from m -space X into m -space Y . Then f is said to be an minimal proper (briefly m -proper) if:

- i. f is an m -continuous.
- ii. The function $f \times id_Z : X \times Z \longrightarrow Y \times Z$ is an m -closed for every m -space Z .

1.29 Example

- i. The identity function $id_X : (X, M_x) \longrightarrow (X, M_x)$ on X is an m -proper function.

- ii. Let $id_Z : (Z, M_Z) \longrightarrow (Z, M'_Z)$ be identity function such that $M_Z = \{\phi, Z, Z_e, Z_o\}$ be an m -structure pace on Z and M'_Z be indiscrete m -structure on Z then id_Z is not m -proper, Since id_Z is not m -closed function.

1.30 Definition

Let $f : (X, M_X) \longrightarrow (Y, M_Y)$ be a function from m -space X into m -space Y .

Then f is called:

- i. m_* -proper function if and only if f is an m_* -continuous and the function $f \times I_Z : X \times Z \longrightarrow Y \times Z$ is an m_* -closed for every m -space Z .
- ii. $*m$ -proper function if and only if f is an $*m$ -continuous and the function $f \times I_Z : X \times Z \longrightarrow Y \times Z$ is a $*m$ -closed for every m -space Z .
- iii. $*m_*$ -proper function if and only if f is a $*m_*$ -continuous and the function $f \times I_Z : X \times Z \longrightarrow Y \times Z$ is a $*m_*$ -closed for every m -space Z .

1.31 Example

- i. Let $X = \{a, b\}$ and $Y = \{1, 2\}$ such that $M_X = \{\phi, X, \{a\}\}$, $M_Y = \{\phi, Y, \{1\}\}$ are m -structure on X and Y respectively and let:
 - A. The identity function $id_X : (X, M_X) \longrightarrow (X, M_X)$ is an m_* -proper and $*m_*$ -proper.
 - B. The constant function $f : (X, M_X) \longrightarrow (Y, M_Y)$ defined as $f(a) = f(b) = 1$ is an $*m$ -proper.
- ii- The function $f : (X, M_X) \longrightarrow (Y, M_Y)$ in example (1.18) is not m_* -proper.
- iii. The function $h : (X, M_X) \longrightarrow (Y, M_Y)$ in example (1.20-ii) is neither $*m$ -proper nor $*m_*$ -proper.

1.32 Remark

As a consequence of definition (1.28) and definition (1.30) we have every m -proper

(m_* -proper, ${}_m$ -proper, ${}_m m_*$ -proper) function is m -closed (m_* -closed, ${}_m m$ -closed, ${}_m m_*$ -closed) function respectively if we take in these definitions an m -space Z to consist of a single point.

2. Composition of Certain Type of m -proper function

2.1 Proposition

Let $f : X \rightarrow Y$ and $g : Y \rightarrow W$ be two functions, then:

- i. If f and g are m -proper then $g \circ f$ is an m -proper.
- ii. If $g \circ f$ is an m -proper and f is surjective then g is an m -proper.
- iii. If $g \circ f$ is an m -proper and g is injective then f is an m -proper.

Proof:

(i) (a) To prove $g \circ f : X \rightarrow W$ is an m -continuous.

Let $B \in M_W$ then $g^{-1}(B) \in M_Y$ (g is an m -continuous) and $f^{-1}(g^{-1}(B)) \in M_X$ (f is an m -continuous), hence $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B)) \in M_X$.

(b) Let Z be an m -space to prove that the function $(g \circ f) \times I_Z : X \times Z \rightarrow Y \times Z$ is an m -closed.

Since $(g \circ f) \times I_Z = (g \times I_Z) \circ (f \times I_Z)$, also since $g \times I_Z$ and $f \times I_Z$ are m -closed function (f and g are m -proper), then easy to show that the composition of two m -closed function is an m -closed function, hence $(g \circ f) \times I_Z$ is an m -closed function. From (a) and (b) we have $g \circ f$ is an m -proper.

(ii) (a) To prove $g : Y \rightarrow W$ is an m -continuous.

Let B be an m -closed in W then $(g \circ f)^{-1}(B)$ is an m -closed in X ($g \circ f$ is an m -continuous), hence $f((g \circ f)^{-1}(B)) = f(f^{-1}(g^{-1}(B)))$ is an m -closed in Y (f is surjective m -closed (1.24)).

(b) Let Z be an m -space to prove that the function $g \times I_Z$ is an m -closed. Let B be an m -closed in $Y \times Z$. Then $(f \times I_Z)^{-1}(B)$ is an m -closed in $X \times Z$ and then $((g \circ f) \times I_Z)((f \times I_Z)^{-1}(B)) = ((g \times I_Z) \circ (f \times I_Z))((f \times I_Z)^{-1}(B)) = (g \times I_Z)(B)$ is an m -closed in $W \times Z$. From (a) and (b) we have g is an m -proper function.

(iii) (a) To prove f is an m -continuous.

Let B be an m -closed set in Y then $g(B)$ is an m -closed set in W (g is an m -closed), hence $(g \circ f)^{-1}(g(B))$ is an m -closed in X ($g \circ f$ is an m -continuous) and then $f^{-1}(g^{-1}(g(B))) = f^{-1}(B)$ (g is injective) is an m -closed in X .

(b) Let Z be an m -space to prove that the function $f \times I_Z$ is an m -closed.

Let F be an m -closed in $X \times Z$ then $(g \circ f) \times I_Z(F)$ is an m -closed in $W \times Z$. Since $(g \circ f) \times I_Z$ is an m -closed function and then $(g \circ f) \times I_Z(F)$ is an m -closed in $W \times Z$, hence $(g \times I_Z)^{-1}((g \circ f) \times I_Z(F)) = (f \times I_Z)(F)$ is an m -closed in $Y \times Z$.

2.2 Proposition

Let $f : X \rightarrow Y$ and $g : Y \rightarrow W$ be two functions, then:

- i. If f and g are $*m$ -proper then $g \circ f$ is a $*m$ -proper.
- ii. If $g \circ f$ is a $*m$ -proper and f is surjective then g is a $*m$ -proper.
- iii. If $g \circ f$ is a $*m$ -proper and g is injective then f is a $*m$ -proper.

Proof:

(i) (a) To prove $g \circ f : X \rightarrow W$ is a $*m$ -continuous.

Let $B \in M_W$ then $g^{-1}(B) \in T_{M_Y}$ (g is an $*m$ -continuous) and by remark (1.4) then

$g^{-1}(B) \in M_Y$, also $f^{-1}(g^{-1}(B)) \in T_{M_X}$ (f is $*m$ -continuous), hence $(g \circ f)^{-1}(B) \in T_{M_X}$.

(b) Let Z be an m -space to prove that the function $(g \circ f) \times I_Z : X \times Z \rightarrow W \times Z$ is a $*m$ -closed. Since $(g \circ f) \times I_Z = (g \times I_Z) \circ (f \times I_Z)$, also since $f \times I_Z$ and $g \times I_Z$ are $*m$ -closed function (f and g are $*m$ -proper), to prove the composition of $f \times I_Z$ and $g \times I_Z$ is a $*m$ -closed.

Let F be an m -closed set in $X \times Z$ then $(f \times I_Z)(F)$ is a closed set in $Y \times Z$ ($f \times I_Z$ is

a

$*m$ -closed), by remark (1.4) we have $(f \times I_Z)(F)$ is an m -closed set in $Y \times Z$, thus

$(g \times I_Z)((f \times I_Z)(F))$ is a closed set in $W \times Z$ ($g \times I_Z$ is a $*m$ -closed). Therefore

$((g \times I_Z) \circ (f \times I_Z))(F)$ is a closed set in $W \times Z$, hence $(g \circ f) \times I_Z$ is a $*m$ -closed

function. From (a) and (b) we have $g \circ f$ is a $*m$ -proper function.

(ii) (a) To prove $g : Y \rightarrow W$ is a $*m$ -continuous.

Let B be an m -closed set in W then $(g \circ f)^{-1}(B)$ is a closed set in X ($g \circ f$ is a $*m$ -continuous), then by remark (1.4) $(g \circ f)^{-1}(B)$ is an m -closed set in X , then $f((g \circ f)^{-1}(B)) = f(f^{-1}(g^{-1}(B))) = g^{-1}(B)$ is an m -closed set in Y (f is surjective $*m$ -closed (1.24)).

(b) Let Z be an m -space to prove that the function $g \times I_Z$ is a $*m$ -closed. Let B be an m -closed set in $Y \times Z$. Then $(f \times I_Z)^{-1}(B)$ is a closed set in $X \times Z$ and then by remark (1.4) we have $(f \times I_Z)^{-1}(B)$ is an m -closed set in $X \times Z$ therefore $((g \circ f) \times I_Z)((f \times I_Z)^{-1}(B)) = ((g \times I_Z) \circ (f \times I_Z))((f \times I_Z)^{-1}(B)) = (g \times I_Z)(B)$ is a closed set in $W \times Z$. From (a) and (b) we have g is $*m$ -proper.

(iii) (a) To prove f is an $*m$ -continuous.

Let B be an m -closed set in Y then $g(B)$ is a closed set in W (g is a $*m$ -closed), then by remark (1.4) we have $g(B)$ is an m -closed set in W , hence $(g \circ f)^{-1}(g(B))$ is a closed set in X ($g \circ f$ is a $*m$ -continuous), also by remark (1.4) we have $(g \circ f)^{-1}(g(B))$ is an m -closed set in X . And then $f^{-1}(g^{-1}(g(B))) = f^{-1}(B)$ (g is injective) is a closed set in X .

(b) Let Z be an m -space to prove that the function $f \times I_Z$ is an $*m$ -closed.

Let F be an m -closed in $X \times Z$ then $(g \circ f) \times I_Z(F)$ is a closed in $W \times Z$. Since $(g \circ f) \times I_Z$ is an $*m$ -closed function then by remark (1.4) we have $(g \circ f) \times I_Z(F)$ is an m -closed in $W \times Z$, hence $(g \times I_Z)^{-1}((g \circ f) \times I_Z(F)) = (f \times I_Z)(F)$ is a closed in $Y \times Z$.

2.3 Proposition

Let $f : X \rightarrow Y$ and $g : Y \rightarrow W$ be two functions, then:

i. If f and g are $*m*$ -proper then $g \circ f$ is a $*m*$ -proper.

ii. If $g \circ f$ is a $*m_*$ -proper and f is surjective then g is a $*m_*$ -proper.

iii. If $g \circ f$ is a $*m_*$ -proper and g is injective then f is a $*m_*$ -proper.

Proof:

(i) (a) To prove $g \circ f : X \rightarrow W$ is a $*m_*$ -continuous. Let $B \in \mathcal{T}_{M_W}$ then $g^{-1}(B) \in \mathcal{T}_{M_Y}$ (g is a $*m_*$ -continuous) and $f^{-1}(g^{-1}(B)) \in \mathcal{T}_{M_X}$ (f is a $*m_*$ -continuous), hence $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B)) \in \mathcal{T}_{M_X}$.

(b) Let Z be an m -space to prove that the function $(g \circ f) \times I_Z : X \times Z \rightarrow Y \times Z$ is a $*m_*$ -closed. Since $(g \circ f) \times I_Z = (g \times I_Z) \circ (f \times I_Z)$, also since $g \times I_Z$ and $f \times I_Z$ are $*m_*$ -closed function (f and g are $*m_*$ -proper), then easy to show that the composition of two $*m_*$ -closed functions is a $*m_*$ -closed function, hence $(g \circ f) \times I_Z$ is a $*m_*$ -closed function. From (a) and (b) we have $g \circ f$ is a $*m_*$ -proper.

(ii) (a) To prove $g : Y \rightarrow W$ is a $*m_*$ -continuous. Let B be a closed in W then $(g \circ f)^{-1}(B)$ is a closed in X ($g \circ f$ is a $*m_*$ -continuous), hence $f((g \circ f)^{-1}(B)) = f(f^{-1}(g^{-1}(B)))$ is a closed in Y (f is surjective $*m_*$ -closed (1.24)).

(b) Let Z be an m -space to prove that the function $g \times I_Z$ is a $*m_*$ -closed.

Let B be a closed in $Y \times Z$. Then $(f \times I_Z)^{-1}(B)$ is a closed set in $X \times Z$ and then $((g \circ f) \times I_Z)((f \times I_Z)^{-1}(B))$ is a closed in $W \times Z$. From (a) and (b) we have g is a $*m_*$ -proper function.

(iii) (a) To prove f is a $*m_*$ -continuous. Let B be a closed set in Y then $g(B)$ is a closed set in W (g is a $*m_*$ -closed), then $g(B)$ is a closed set in W , hence $(g \circ f)^{-1}(g(B))$ is a closed in X ($g \circ f$ is a $*m_*$ -continuous) and then $f^{-1}(g^{-1}(g(B))) = f^{-1}(B)$ (g is injective) is a closed in X .

(b) Let Z be an m -space to prove that the function $f \times I_Z$ is a $*m_*$ -closed. Let F be a closed in $X \times Z$ then $(g \circ f) \times I_Z(F)$ is a closed in $W \times Z$. Since $(g \circ f) \times I_Z$ is a $*m_*$ -closed function and then $(g \circ f) \times I_Z(F)$ is a closed in $W \times Z$, hence $(g \times I_Z)^{-1}((g \circ f) \times I_Z(F)) = (f \times I_Z)(F)$ is a closed in $Y \times Z$.

2.4 Remark

Let (X, M_X) , (Y, M_Y) and (W, M_W) are m -structure on X , Y and W respectively. And let $f : (X, M_X) \longrightarrow (Y, M_Y)$, $g : (Y, M_Y) \longrightarrow (W, M_W)$ be m_* -continuous functions then not necessarily the function $g \circ f$ is an m_* -continuous. As the following example shows.

2.5 Example

Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3\}$ and $W = \{w_1, w_2, w_3\}$ such that $M_X = \{\phi, X, \{x_1\}\}$, $M_Y = \{\phi, Y, \{y_2\}, \{y_3\}\}$ and $M_W = \{\phi, W, \{w_1\}, \{w_2\}, \{w_1, w_3\}\}$ are m -structure on X , Y and W respectively then $f : (X, M_X) \longrightarrow (Y, M_Y)$ defined as $f(x_1) = y_2, f(x_2) = y_3, f(x_3) = y_1$ is m_* -continuous since there is non-indiscrete topology $T_{M_Y} = \{\phi, Y, \{y_2\}\}$ which satisfies the conditions of definition, and $g : (Y, M_Y) \longrightarrow (W, M_W)$ defined as $g(y_1) = w_2, g(y_2) = w_3, g(y_3) = w_1$ is m_* -continuous since there is non-indiscrete topology $T_{M_W} = \{\phi, W, \{w_1\}\}$ which satisfies the conditions of definition. But $g \circ f$ is not m_* -continuous function since all non-indiscrete topologies T_{M_W} is $T_{1M_W} = \{\phi, W, \{w_1\}\}$, $T_{2M_W} = \{\phi, W, \{w_2\}\}$ and $T_{3M_W} = \{\phi, W, \{w_1, w_3\}\}$ which are not satisfies the conditions of definition (1.19-i).

2.6 Remark

The composition of two m_* -proper functions is not necessarily m_* -proper function. And we can show that from remark (2.4) and example (2.5).

3. Restriction of Certain Type of m -proper function

3.1 Remark

Let $f : X \rightarrow Y$ be m -proper (m_* -proper, $*m$ -proper, $*m_*$ -proper) function and let $A \subseteq X$ the restriction function $f|_A$ is not necessarily m -proper (m_* -proper, $*m$ -proper, $*m_*$ -proper).

3.2 Proposition

Let $f : X \rightarrow Y$ be an m -proper function from um -space X into m -space Y and let A be an m -closed subset of X then $f|_A : A \rightarrow Y$ is m -proper function.

Proof: Let $g = f|_A$

(i) To prove $g : A \rightarrow Y$ is an m -continuous.

Let B be an m -closed set in Y then $f^{-1}(B)$ is an m -closed set in X (f is an m -continuous function) and then by definition (1.11) $A \cap f^{-1}(B)$ is an m -closed set in X , hence $A \cap f^{-1}(B)$ is an m -closed set in A by using Proposition (1.12). Therefore $g^{-1}(B) = A \cap f^{-1}(B)$ is an m -closed set in A .

(ii) Let Z be an m -space to prove that the function $g \times I_Z : A \times Z \rightarrow Y \times Z$ is an m -closed. Since $g \times I_Z = (f \times I_Z)|_{A \times Z}$ and $A \times Z$ is an m -closed set in $X \times Z$ by using Theorem (1.13). Let F be an m -closed set $A \times Z$ then by Proposition (1.12) F is an m -closed set in $X \times Z$. Hence $(f \times I_Z)(F)$ is an m -closed in $Y \times Z$ (f is an m -proper function). Thus $(g \times I_Z)(F) = (f \times I_Z)|_{A \times Z}(F)$ is an m -closed in $Y \times Z$. From (i) and (ii) we have $g = f|_A$ is an m -proper function.

3.3 Proposition

Let $f : X \rightarrow Y$ be an m_* -proper function from um -space X into m -space Y and let A be an m -closed subset of X then $f|_A : A \rightarrow Y$ is m_* -proper function.

Proof: Let $g = f|_A$

(i) To prove $g : A \rightarrow Y$ is an m_* -continuous.

Let B be a closed set in Y then $f^{-1}(B)$ is an m -closed set in X (f is an m_* -continuous function) and then by Remark (1.10) $A \cap f^{-1}(B)$ is an m -closed set in X , hence by Proposition (1.12) we have $A \cap f^{-1}(B)$ is an m -closed set in A . Therefore $g^{-1}(B) = A \cap f^{-1}(B)$ is an m -closed set in A .

(ii) Let Z be an m -space to prove that the function $g \times I_Z : A \times Z \rightarrow Y \times Z$ is an m_* -closed. Since $g \times I_Z = (f \times I_Z)|_{A \times Z}$ and $A \times Z$ is an m -closed set in $X \times Z$ by Theorem (1.13). Let F be an m -closed set $A \times Z$ then by Proposition (1.12) F is an m -closed set in $X \times Z$. Hence $(f \times I_Z)(F)$ is a closed in $Y \times Z$ (f is an m_* -proper function). Thus $(g \times I_Z)(F) = (f \times I_Z)|_{A \times Z}(F)$ is a closed in $Y \times Z$. From (i) and (ii) we have $g = f|_A$ is an m_* -proper function.

3.4 Proposition

Let $f : X \rightarrow Y$ be an $*m$ -proper function from m -space X into m -space Y and let A be a closed subset of X then $f|_A : A \rightarrow Y$ is $*m$ -proper function.

Proof: Let $g = f|_A$

(i) To prove $g : A \rightarrow Y$ is an $*m$ -continuous.

Let B be an m -closed set in Y then $f^{-1}(B)$ is a closed set in X (f is a $*m$ -continuous function) and then $A \cap f^{-1}(B)$ is a closed set in A . Hence $g^{-1}(B) = A \cap f^{-1}(B)$ is a closed set in A .

(ii) Let Z be an m -space to prove that the function $g \times I_Z : A \times Z \rightarrow Y \times Z$ is an $*m$ -closed. Since $g \times I_Z = (f \times I_Z)|_{A \times Z}$ and $A \times Z$ is a closed set in $X \times Z$ (A is a closed set). Let F be an m -closed set in $A \times Z$ then F is an m -closed set in $X \times Z$ ($A \times Z$ is an m -closed set, Proposition (1.12)). Hence $(f \times I_Z)(F)$ is a closed in $Y \times Z$ (f is an $*m$ -proper function). Thus $(g \times I_Z)(F) = (f \times I_Z)|_{A \times Z}(F)$ is a closed in $Y \times Z$. From (i) and (ii) we have $g = f|_A$ is an $*m$ -proper function.

3.5 Proposition

Let $f : X \rightarrow Y$ be a $*m_*$ -proper function from m -space X into m -space Y and let A be a closed subset of X then $f|_A : A \rightarrow Y$ is $*m_*$ -proper function.

Proof: Let $g = f|_A$

(i) To prove $g : A \rightarrow Y$ is a $*m_*$ -continuous.

Let B be a closed set in Y then $f^{-1}(B)$ is a closed set in X (f is a $*m_*$ -continuous function) and then $A \cap f^{-1}(B)$ is a closed set in A . Hence $g^{-1}(B) = A \cap f^{-1}(B)$ is a closed set in A .

(ii) Let Z be an m -space to prove that the function $g \times I_Z : A \times Z \rightarrow Y \times Z$ is a $*m_*$ -closed. Since $g \times I_Z = (f \times I_Z)|_{A \times Z}$ and $A \times Z$ is a closed set in $X \times Z$ (A is closed set). Let F be a closed set $A \times Z$ then F is a closed set in $X \times Z$. Hence $(f \times I_Z)(F)$ is

a closed in $Y \times Z$ (f is a $*m_*$ -proper function). Thus $(g \times I_Z)(F) = (f \times I_Z)|_{A \times Z}(F)$ is a closed in $Y \times Z$. From (i) and (ii) we have $g = f|_A$ is a $*m_*$ -proper function.

4. Relation Among Types of m -proper function

4.1 Proposition

Every m -proper function is m_* -proper function.

Proof: Let $f : X \rightarrow Y$ be an m -proper function then f is m -continuous and the function $f \times I_Z : X \times Z \rightarrow Y \times Z$ is m -closed for every m -space Z (1.28).

(i) To prove f is m_* -continuous function.

Let $B \in \mathcal{T}_{M_Y}$ then $B \in M_Y$ thus $f^{-1}(B) \in M_X$ (f is m -continuous function).

(ii) Let Z be an m -space. To prove that the function $f \times I_Z : X \times Z \rightarrow Y \times Z$ is m_* -closed. Let F be a closed set in $X \times Z$ then F is an m -closed set in $X \times Z$, thus $(f \times I_Z)(F)$ is an m -closed set in $Y \times Z$ (f is an m -proper function). From (i) and (ii) we have f is m_* -proper function.

4.2 Proposition

Every $*m$ -proper function is m -proper function.

Proof: Let $f : X \rightarrow Y$ be an $*m$ -proper function then f is $*m$ -continuous and the function $f \times I_Z : X \times Z \rightarrow Y \times Z$ is $*m$ -closed for every m -space Z (1.30-ii).

(i) To prove f is m -continuous function.

Let $B \in M_Y$ then $f^{-1}(B) \in \mathcal{T}_{M_X}$ (f is $*m$ -continuous function) and then $f^{-1}(B) \in M_X$.

(ii) Let Z be an m -space. To prove that the function $f \times I_Z : X \times Z \rightarrow Y \times Z$ is m -closed. Let F be an m -closed set in $X \times Z$ then $(f \times I_Z)(F)$ is a closed set in $Y \times Z$ (f is an $*m$ -proper function). Thus $(f \times I_Z)(F)$ is an m -closed set in $Y \times Z$. From (i) and (ii) we have f is m -proper function.

4.3 Proposition

Every $*m$ -proper function is m_* -proper function.

Proof: Let $f : X \rightarrow Y$ be an $*m$ -proper function then f is $*m$ -continuous and the function $f \times I_Z : X \times Z \rightarrow Y \times Z$ is $*m$ -closed for every m -space Z (1.30-ii).

(i) To prove f is m_* -continuous function.

Let $B \in \mathcal{T}_{M_Y}$ then $B \in M_Y$ therefore $f^{-1}(B) \in \mathcal{T}_{M_X}$ (f is $*m$ -continuous function) and then $f^{-1}(B) \in M_X$.

(ii) Let Z be an m -space. To prove that the function $f \times I_Z : X \times Z \rightarrow Y \times Z$ is m_* -closed. Let F be a closed set in $X \times Z$ then $(f \times I_Z)(F)$ is a closed set in $Y \times Z$ (f is an $*m$ -proper function). Thus $(f \times I_Z)(F)$ is an m -closed set in $Y \times Z$. From (i) and (ii) we have f is m_* -proper function.

4.4 Proposition

Every $*m$ -proper function is $*m_*$ -proper function.

Proof: Let $f : X \rightarrow Y$ be an $*m$ -proper function then f is $*m$ -continuous and the function $f \times I_Z : X \times Z \rightarrow Y \times Z$ is $*m$ -closed for every m -space Z (1.30-ii).

(i) To prove f is $*m_*$ -continuous function.

Let $B \in \mathcal{T}_{M_Y}$ then $B \in M_Y$ therefore $f^{-1}(B) \in \mathcal{T}_{M_X}$ (f is $*m$ -continuous function).

(ii) Let Z be an m -space. To prove that the function $f \times I_Z : X \times Z \rightarrow Y \times Z$ is $*m_*$ -closed. Let F be a closed set in $X \times Z$ then F be an m -closed set in $X \times Z$ therefore $(f \times I_Z)(F)$ is a closed set in $Y \times Z$ (f is an $*m$ -proper function). From (i) and (ii) we have f is m -proper function.

4.5 Proposition

Every $*m_*$ -proper function is m_* -proper function.

Proof: Let $f : X \rightarrow Y$ be an $*m_*$ -proper function then f is $*m_*$ -continuous and the function $f \times I_Z : X \times Z \rightarrow Y \times Z$ is $*m_*$ -closed for every m -space Z (1.30-iii).

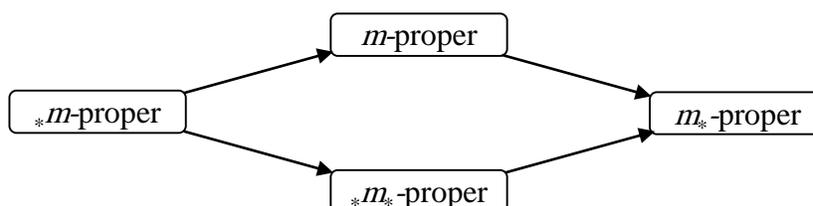
(i) To prove f is m_* -continuous function.

Let $B \in \mathcal{T}_{M_Y}$ then $f^{-1}(B) \in \mathcal{T}_{M_X}$ (f is $*m_*$ -continuous function). Thus $f^{-1}(B) \in M_X$.

(ii) Let Z be an m -space. To prove that the function $f \times I_Z : X \times Z \rightarrow Y \times Z$ is m_* -closed. Let F be a closed set in $X \times Z$ then $(f \times I_Z)(F)$ is a closed set in $Y \times Z$ (f is an $*m_*$ -proper function). From (i) and (ii) we have f is m_* -proper function. a

$*m_*$ -proper function) and then $(f \times I_Z)(F)$ is m -closed set in $Y \times Z$. From (i) and (ii) we have f is m_* -proper function.

The following diagram shows the relation among types of minimal proper functions.



References

- [1] Alimohammady M. and Roohi M., (2005), "Fixed Point in Minimal Spaces", Nonlinear Analysis. Modelling and control, vol.10, no.4, 305-314.
- [2] Bourbaki N., Elements of Mathematics, (1989), "General Topology", chapter 1-4, springer-verlog, Berlin, Heidelberg, New-York, London, Paris, Tokyond Edition.
- [3] Parimelazhagan R., Nagaveni N. and Sundara Krishuan Sai, (2009), "On mg-continuous function in minimal structure", Proceedings of the International Multico-nference of Engineers and Computer Scientists, Hongkong, vol.I, march 18-20.
- [4] Popa V. and Noiri T., "Minimal Structures, Punctually m-open function in the sense of kuratowski and bitopological spaces", (2007), Mathematical communications, vol.12, 247-253.

- [5] Ravi O., Tharmar S. and GAnesan S., “Some locally closed sets in ideal minimal spaces”, (2011), Issuel, vol.1, 89-105.
- [6] Ravi O., Ganesan S., Tharmar S. and Balamurhgan R. G., “Minimal g -closed sets with respect to an Ideal”, (2011), Issue 2, vol.1, 1-12.
- [7] Sunisa Buadong, Chokchai Viriyapong and Chawalit Boonpok, “On Generalized Topology and Minimal Structure Spaces”, (2011), Int. Journal of Math. Analysis, vol.5, no.31, 1507-1516.
- [8] Won Keun Min¹ and Yong Key Kim², “M*-continuity and Product Minimal Structure on Minimal Structure”, (2011), vol.69, no.3, 329-339.

أنماط معينه من الدوال السديدة الأصغرية

فراس جواد عبید

قسم الرياضيات / كلية التربية

جامعة القادسية

البريد الإلكتروني:

feras_math@yahoo.com

د. ستار حمید حمزة

قسم الرياضيات / كلية التربية

جامعة القادسية

البريد الإلكتروني:

sattar_math@yahoo.com

المستخلص:

الهدف الرئيسي من هذا العمل هو تقديم نوع جديد من الدوال السديدة وبالتحديد الدالة السديدة الأصغرية m -proper ، m_* -proper ، m -proper و m_* -proper. أيضاً أوضحنا العلاقة فيما بين أنواع الدوال السديدة الأصغرية.

المقدمة:

في عام 1950 ،Maki H. ،Umehara J. و Noiri T. قدموا ملاحظات للبنية الأصغرية والفضاءات الأصغرية. حيث أنجزوا حقائق هامة منسجمة في حالة التبولوجيا العامة.

Alimohammady M. و Roohi M. في [1] أعطوا تعريفاً للمجموعة المغلقة الاصغرية (m -closed set) وأعطوا تعريف للدالة المستمرة الاصغرية (m -continuous function) ودرسوا خصائصها. و Tharmar S. ،Ganesan S. ،Ravi O. و Balamukugan في [6] أعطوا التعريف للدالة المغلقة الأصغرية (m -closed function) ودرسوا بعض خصائصها.

في هذا البحث أعطينا تعاريف للدوال المستمرة الأصغرية وكذلك الدوال المغلقة الأصغرية واستخدمناها لتعريف الدالة السديدة الأصغرية وأنواعها (m_* -proper و m -proper). وبرهنا التركيب للدوال m -proper (m_* -proper, m_* -proper) هو m -proper, m_* -proper (m -proper, m_* -proper) على التوالي في الخواص (2.1، 2.2 و 2.3)، لكن تركيب دالتي m_* -proper ليس بالضرورة دالة m_* -proper. كذلك القصر للدوال m -proper (m_* -proper) من um -space الى الفضاء الاصغري على المجموعة الجزئية المغلقة الأصغرية هي دالة m -proper (m_* -proper) على التوالي (3.2 و 3.3). أيضاً القصر لدالة m -proper (m_* -proper) على المجموعة الجزئية المغلقة هي دالة m -proper (m_* -proper) على التوالي (3.4 و 3.5). ومن خلال (4.1، 4.2، 4.3 و 4.4) أوضحنا العلاقة فيما بين أنواع الدوال السديدة الأصغرية كما في المخطط التالي:

