Journal of Thi-Qar University No.2 Vol.5 Septemper/2009

Basic Concepts on L_n −connectedness

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2009

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Abstract:

In this paper, the concepts of L_n −*connectedness* are defined depending on the definition of L −spaces which was presented by (Kelley, J.C. : 1963 in [3]) and the concept of *connectedness*. Also we used this concept to study some theorms which are related to the concept of *connectedness*.

الخلاصة:

في هذا البحث ًعُرف مفهوم L_n – *conneectedness* بالاعتمـــاد على مفهـوم spaces – L الذي قدمـــــه (العالم *Kelley, J.C.* ســــــنة ١٩٦٣ في المصدر [3]) وعلى مفهـــــوم *conneectedness* . كذلك أســـتخدم هذا المفهوم لدراســــة بعض المبرهنات التي تتعلق بمفهـوم *conneectedness*.

§1 Introduction

***1 Introduction*:**

Kelley, J.C. at 1963 in [3] presented a new concept, namely, Bitopological spaces which theirs members are sets depend upon two topologies in theirs definition on the same nonempty set.

Bitopological space which their members are $L-$open sets are called $L-$topological spaces or $L-$spaces.

The concept of $L-$spaces are used in many Principle topological concepts, for example Compactness, Connectedness, Separation axioms, Convergence and others.

In this paper, the concepts of L_n-*connectedness* are defined depending on the definition of $L-$spaces which was presented by (Kelley, J.C. : 1963 in [3]) and the concept of *connectedness*. Also we used this concept to study some theorms which are related to the concept of *connectedness*.

§2 Basic Concepts

2.1 Definitions:

1. A subset *A* of a topological space *X* is *disconneced* if there exist open sets *G* and *H* of *X* such that $A\bigcap G$ and $A\bigcap H$ are disjoint non-empty sets whose union is *A*. In this case $G\bigcup H$ is called disconnection of A. Equivalently, A is disconneced in *X* if there exist open sets *G* and *H* of *X* such that $A\bigcap G\neq\phi$, $A\bigcap H\neq\phi$, $A\subseteq G\bigcup H$, and $G\bigcap H\subseteq A^c$.

A set is *connected* if it is not disconnected [1], [2], [4], [5].

2. Two subsets *A* and *B* of a topological space *X* are said to be *separated* if $A\bigcap\overline{B}=\phi$ and $\overline{A}\bigcap B=\phi$, where $\overline{A}$, $\overline{B}$ is the closure set of *A, B* respectively [1], [2], [4], [5].

2.2 Theorem: **A subset *A* of the real line $\Re$ containing at least two points is connected if and only if *A* is an interval [1], [2], [4], [5].**

2.3 Proposition: **If *A* and *B* are connected sets which are not separated, then $A\bigcup B$ is connected [1], [4], [5].**

2.4 Proposition: **If A is a connected subset of a space X such that $A \subseteq B \subseteq \overline{A}$, then B is a connected set, in particular $\overline{A}$ is connected [1], [4], [5].**

2.5 Definitions:

***1.* Let $\tau_1, \tau_2, \ldots\ldots, \tau_n$ $(n \geq 2)$ be topologies on a nonempty set X and $A, B \subseteq X$. Then**

***i.* A is called an L_n-open set in X if there is a τ_1-open set U in X such that $U \subseteq A \subseteq \bigcup_{i=2}^{n} \overline{U}^{\,i}$, where $\overline{U}^{\,i}$ is the *i-th* closure set of U in X $w.r.t.$ τ_i $(2 \leq i \leq n)$.**

The collection of all L_n-open sets in X is denoted by $L_n - O(X)$, and $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ is called an L_n-topological space or an L_n-space [for easiness written X is called an L_n-space], and the open sets in $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ are L_n-open sets in X . If *n*=2, then for easiness (X, τ_1, τ_2) is called an $L-$topological space or an $L-$space (for easiness written X is called an $L-$space) [3].

***ii.* B is called an L_n-closed set in X if and only if B^c is an L_n-open set in X .**

The collection of all L_n-closed sets in X is denoted by $L_n - C(X)$.

***2.* The L_n-subspace A of an L_n-space $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ is the L_n-space ($A, \tau_{1A}, \tau_{2A}, \ldots\ldots, \tau_{nA}$), where $\tau_{1A}, \tau_{2A}, \ldots\ldots, \tau_{nA}$ are the relative topologies of $\tau_1, \tau_2, \ldots\ldots, \tau_n$ respectively on A in X .**

***3.* Let $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ & $(Y, \tau'_1, \tau'_2, \ldots\ldots, \tau'_n)$ be L_n-spaces, and let $f:(X, \tau_1, \tau_2, \ldots\ldots, \tau_n) \to (Y, \tau'_1, \tau'_2, \ldots\ldots, \tau'_n)$ be a function. Then f is called L_n-continuous if $f^{-1}(A)$ is an L_n-open set in X for all $A \in \tau'_1$.**

2.6 Remarks:

***1.* Let $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ be an L_n-space. Then**

i. τ_1 **is a sub collection of** $L_n - O(X)$**, (since** $U \subseteq U \subseteq \bigcup_{i=2}^{n} \overline{U}^{\,i}$ **for all** $U \in \tau_1$**) .**

ii. **If** ***n1*** **<** ***n2*** **≤*n*, then** $L_{n1} - O(X) \subseteq L_{n2} - O(X)$**, since if** $A \in L_{n1} - O(X) \Rightarrow \exists U \in \tau_1 \ni U \subseteq A \subseteq \bigcup_{i=2}^{n1} \overline{U}^{\,i} \;\&\; \bigcup_{i=2}^{n1} \overline{U}^{\,i} \subseteq \bigcup_{i=2}^{n2} \overline{U}^{\,i} \Rightarrow U \subseteq A \subseteq \bigcup_{i=2}^{n2} \overline{U}^{\,i} \Rightarrow A \in L_{n2} - O(X) \Rightarrow L_{n1} - O(X) \subseteq L_{n2} - O(X)$**. Moreover** $L_2 - O(X) \subseteq L_3 - O(X) \subseteq \ldots\ldots L_n - O(X)$ **.**

iii. **If** $\overline{A}^{L_n} = \bigcap\{F \in L_n - C(X) / A \subseteq F\}$**, then** $\overline{A}^{L_n}$ **is called the** $L_n -$**closure of** A ***w.r.t.*** **the** $L_n -$**space** $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$**. Notice that** $\overline{A}^{L_n} \subseteq \overline{A}^{1}$ **by using (*i*).**

2. **Let** $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ **&** $(Y, \tau'_1, \tau'_2, \ldots\ldots, \tau'_n)$ **be** $L_n -$**spaces, and let** $f : (X, \tau_1, \tau_2, \ldots\ldots, \tau_n) \to (Y, \tau'_1, \tau'_2, \ldots\ldots, \tau'_n)$ **be a function. Then if** $f : (X, \tau_1) \to (Y, \tau'_1)$ **is a continuous function, then** $f : (X, \tau_1, \tau_2, \ldots\ldots, \tau_n) \to (Y, \tau'_1, \tau'_2, \ldots\ldots, \tau'_n)$ **is an** $L_n -$**continuous function. The converse is not true, e. g., if** u **is the usual topology on** $\Re$**, and** $f : \Re \to \Re$ **is a function defined as follows** $f(x) = n$ **for every** $x \in \Re, n \in Z$ **such that** $n \le x < n+1$**, then** $f : (\Re, u) \to (\Re, u)$ **is not continuous function, since** $(0,2) \in u$**, but** $f^{-1}((0,2)) = f^{-1}(0,1) \cup f^{-1}(\{1\}) \cup f^{-1}((1,2)) = \phi \cup [1,2) \cup \phi = [1,2) \notin u$**; while** $f : (\Re, u, u, \ldots\ldots, u) \to (\Re, u, u, \ldots\ldots, u)$ **(*n*-times,** $n \ge 2$**) is an** $L_n -$**continuous function .**

§3 L_n-connectedness

3.1 Definitions:

1. A subset *A* of an L_n −space *X* is called L_n −*disconnected* if there exist L_n −open sets *G* and *H* of *X* such that $A \cap G$ and $B \cap H$ are disjoint non-empty sets whose union is A. In this case $G \cup H$ is called L_n −disconnection of A. *Equivalently*, *A* is disconneced in *X* if there exist L_n −open sets *G* and *H* of *X* such that $A \cap G \neq \phi$, *A* $\cap H \neq \phi$, $A \subseteq G \cup H$, and $G \cap H \subseteq A^c$. A set is L_n −*connected* if it is not disconnected.

2. Two subsets *A* and *B* of an L_n −space $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ are said to be L_n − *separated* if $A \cap \overline{B}^{L_n} = \phi$ and $\overline{A}^{L_n} \cap B = \phi$.

3.2 Proposition: **Let $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ be an L_n −space such that *A* is disconnected with respect to (X, τ_1). Then *A* is L_n −disconnected with respect to $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ for every topologies τ_i , $(2 \leq i \leq n)$ on *X*.**

Proof:

By using *2.1* and *2.6.*

3.3 Examples:

***i.* Let *U* be the usual topology on $\Re^2$ and *A*={*(x,y): x²-y²* ≥ 4} [4].**

For the two open half-planes *G*={*(x,y): x < -1*} and *H*={*(x,y): x >1*} form a disconnection of *A* with respect to the space $(\Re^2, U)$, therefore G and *H* *A* form an L_n −disconnection of *A* with respect to the L_n −space $(\Re^2, U, \ldots, U)$ (*n*-times) by using *3.2* as in indicated *Fig.1* bellow:

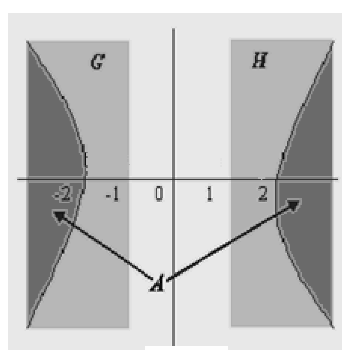

Fig.1

ii. Let u be the usual topology on $\Re$ and a,b$\in\Re$, and let A=[a,b). Then A is an connected with respect to $(\Re,u)$ by using 2.2 but A is L_n-disconnected with respect to $(\Re,u,\ldots,u)$ (n-times) since if G=[a,(a+b)/2), H=[(a+b)/2),b), then G,H$\in L_n-O(\Re)$, A$\cap$G$\neq\phi$, A$\cap$H $\neq\phi$, $A\subseteq G\cup H$, and $G\cap H\subseteq\Re-A$.

<u>3.4 Proposition:</u> Let $(X,\tau_1,\tau_2,\ldots\ldots,\tau_n)$ be an L_n-space such that A and B are separated with respect to (X,τ_1). Then A and B are L_n-separated with respect to $(X,\tau_1,\tau_2,\ldots\ldots,\tau_n)$ for every topologies τ_i, $(2\leq i\leq n)$ on X.

<u>Proof:</u>

By using 2.1 and 2.6.

<u>3.5 Examples:</u>

i. Let u be the usual topology on $\Re$, and let A=(0,1), B=(1,2) [4]. Then A and B are separated with respect to the space $(\Re,u)$ since $\overline{A}^1$=[0,1], $\overline{B}^1$=[1,2], A$\cap\overline{B}^1=\phi$, $\overline{A}^1\cap$B $=\phi$, therefore A and B are L_n-separated with respect to the L_n-space $(\Re,u,\ldots,u)$ (n-times) by using 3.4.

ii. Let u be the usual topology on $\Re$ and a,b,c$\in\Re$ such that a<b<c, and let A=(a,b), B=[b,c). Then A and B are not separated with respect to the space $(\Re,u)$ since $\overline{A}^1$=[a,b] and $\overline{B}^1$=[b,c], and $\overline{A}^1\cap$B ={b}$\neq\phi$, but A and B are L_n-separated with respect to the L_n-space $(\Re,u,\ldots,u)$ (n-times) since $\overline{A}^{L_n}$ = A, $\overline{B}^{L_n}$ =B, and A$\cap\overline{B}^{L_n}$ = $\overline{A}^{L_n}\cap$B = A$\cap$ B $=\phi$.

<u>3.6 Proposition:</u> Let $(X,\tau_1,\tau_2,\ldots\ldots,\tau_n)$ be an L_n-space and A$\subseteq$X such that A is L_n-disconnected with respect to the $(A,\tau_{1A},\tau_{2A},\ldots\ldots,\tau_{nA})$. Then A contains a subset S which is disconnected with respect to (X,τ_1).

<u>Proof:</u>

Since A is L_n-disconnected $w.r.t.$ $(A,\tau_{1A},\tau_{2A},\ldots\ldots,\tau_{nA})$. $\Rightarrow$ $\exists\, G,H\in L_n-O(A)$ such that A$\cap$G$\neq\phi$, A$\cap$H $\neq\phi$, $A\subseteq G\cup H$, and $G\cap H\subseteq A^c$. $\Rightarrow\exists\, u,v\in\tau_1$ such

that $u\bigcap$ **A** $\subseteq$ **G** $\subseteq \bigcup_{i=2}^{n}\overline{u\bigcap A}^{\,i}$, $v \bigcap$ **A** $\subseteq$ **H** $\subseteq \bigcup_{i=2}^{n}\overline{v\bigcap A}^{\,i}$ **. If S =A** $\bigcap (u\bigcup v)$**, then A contains S . Since A** $\bigcap$ **G** $\neq \phi$**, A** $\bigcap$ **H** $\neq \phi$**, and** $G\bigcap H \subseteq X-A \Rightarrow$ **A** $\bigcap$ **u** $\neq \phi$**, A** $\bigcap$ **v** $\neq \phi$**,** $S\subseteq u\bigcup v$**, and** $u\bigcap v \subseteq G\bigcap H \subseteq X-A \Rightarrow$ **S is disconnected** *w.r.t.* **the space** (X,τ_1)**.**

<u>3.7 Corollary:</u> Let $(X,\tau_1,\tau_2,.....,\tau_n)$ **be an** L_n-**space and A** $\subseteq$ **X such that A is** L_n- **disconnected with respect to** $(A,\tau_{1A},\tau_{2A},.....,\tau_{nA})$**. Then A contains a subset S which is** L_n-**disconnected with respect to** $(X,\tau_1,\tau_2,.....,\tau_n)$**.**

<u>Proof:</u>

By using 3.6 and 3.2.

<u>3.8 Proposition:</u> If $f:(X,\tau_1,\tau_2,.....,\tau_n)\rightarrow(Y,\tau'_1,\tau'_2,.....,\tau'_n)$ **is an** L_n-**continuous function such that A is** L_n-**connected with respect to** $(X,\tau_1,\tau_2,.....,\tau_n)$**, then f(A) is connected with respect to** (Y,τ'_1)**.**

<u>Proof:</u>

We show that f(A) is connected *w.r.t.* (Y,τ'_1)**. Suppose that f(A) is disconnected** *w.r.t.* (Y,τ'_1)**.** $\Rightarrow \exists\, G,H\in\tau'_1$ **such that f(A)** $\bigcap$ **G** $\neq\phi$**, f(A)** $\bigcap$ **H** $\neq\phi$**, f(A)** $\subseteq G\bigcup H$ **&** $G\bigcap H\subseteq Y-f(A)$**.** $\Rightarrow f^{-1}(G)\bigcap A\neq\phi$**,** $f^{-1}(H)\bigcap A\neq\phi$**, A** $\subseteq f^{-1}(G)\bigcup f^{-1}(H)$ **&** $f^{-1}(G)\bigcap f^{-1}(H)\subseteq X-A$**. Since** $f:(X,\tau_1,...,\tau_n)\rightarrow(Y,\tau'_1,...,\tau'_n)$ **is** L_n-**continuous.** $\Rightarrow f^{-1}(G),f^{-1}(H)\in L_n-O(X)$ **and hence A is** L_n-**connected with respect to** $(X,\tau_1,\tau_2,.....,\tau_n)$**, which is impossible.** $\Rightarrow$ **f(A) is connected** *w.r.t.* (Y,τ'_1)**.**

<u>3.9 Proposition:</u> If A and B are L_n-**connected sets which are not** L_n-**separated with respect to the** L_n-**space** $(X,\tau_1,\tau_2,.....,\tau_n)$**, then A** $\bigcup$ **B is** L_n-**connected.**

<u>Proof:</u>

Suppose A $\bigcup$ **B is** L_n-**disconnected and suppose G** $\bigcup$ **H is an** L_n-**disconnection of A** $\bigcup$ **B . Since**

A is an L_n-**connected subset of A** $\bigcup$ **B, either** $A\subseteq G$ **or** $A\subseteq H$**.Similarly, either** $B\subseteq G$ **or** $B\subseteq H$**. Now if** $A\subseteq G$ **and** $B\subseteq H$ **(or** $B\subseteq G$ **and** $A\subseteq H$**)** $\Rightarrow$ **A & B**

are L_n −separated sets. But this contradicts the hypothesis; hence either A$\bigcup$B$\subseteq G$ or A$\bigcup$B$\subseteq H$, and so G$\bigcup$H is not an L_n −disconnection of A$\bigcup$B. In the other word, A$\bigcup$B is L_n −connected.

<u>3.10 Example:</u> Let U be the usual topology on $\Re^2$, and let A={(0,y): $\frac{1}{2} \leq y \leq 1$}, B={(x,y): sin(1/x), $0 < x \leq 1$} [4]. Now A and B are not separated with respect to the space $(\Re^2, U)$, since each point in A is an a commulation point of B and therefore A $\bigcup$B is connected with respect to the space $(\Re^2, U)$ by using 2.3, moreover A and B are L_n −separated with respect to the L_n −space $(\Re^2, U, \ldots, U)$ (n-times), since $\overline{A}^{L_n}$ = A, $\overline{B}^{L_n}$ =B, and A$\bigcap \overline{B}^{L_n}$ = $\overline{A}^{L_n} \bigcap$B = A$\bigcap$ B $=\phi$. Therefore A$\bigcup$B is L_n − disconnected as indicated in Fig.2 bellow:

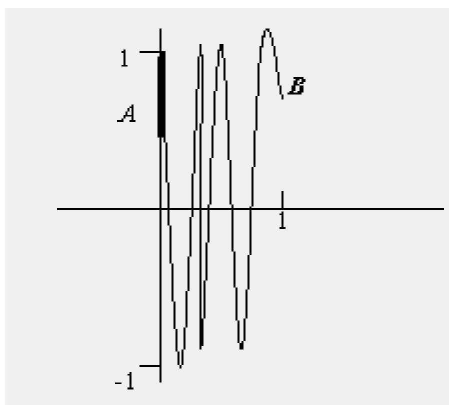

Fig.2

<u>3.11 Proposition:</u> Let $(X, \tau_1, \tau_2, \ldots, \tau_n)$ be an L_n −space and A,B$\subseteq$X such that A is L_n −connected such that $A \subset B \subseteq \overline{A}^{L_n}$, then B is L_n −connected, in particular $\overline{A}^{L_n}$ is L_n −connected.

<u>Proof:</u>

Suppose B is L_n −disconnected and suppose G$\bigcup$H is an L_n −disconnection of B . Now A is L_n −connected subset of B, either $A \bigcap G = \phi$ or $A \bigcap H = \phi$; say, $A \bigcap G = \phi$. Then $X - G$ is a L_n −closed set and therefore $A \subset B \subseteq \overline{A}^{L_n} \subseteq X - G$.

Consequently, $B \bigcap G = \phi$. But this contradicts the fact that G$\bigcup$H is an $L_n -$ disconnection of B; B is $L_n -$disconnected.

<u>3.12 Example:</u> Let u be the usual topology on $\Re$ and a,b$\in \Re$, and let A=(0,2), B=[0,2). Then $A \subseteq B \subseteq \overline{A}^{1}$, therefore A & B are connected sets with respect to $(\Re, u)$ by using 2.2 and 2.4 respectively, but B is $L_n -$disconnected with respect to $(\Re, u, \ldots, u)$ (n-times) since if G=[0,1), H=[1,2), then G,H$\in L_n - O(\Re)$, B$\bigcap$G$\neq \phi$, B $\bigcap$H $\neq \phi$, $B \subseteq G \bigcup H$, and $G \bigcap H \subseteq \Re - B$.

<u>3.13 Conclusions:</u>

1. If A is disconnected with respect to the space (X, τ_1), then A is $L_n -$ disconnected with respect to the $L_n -$space $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ for every topologies τ_i , $(2 \le i \le n)$ on X as indicated in proposotion 3.2.

2. The converse of the 1st conclusion is not true as indicated in example 3.3.

3. If A and B are separated with respect to the space (X, τ_1). Then A and B are $L_n -$separated with respect to the $L_n -$space $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ for every topologies τ_i , $(2 \le i \le n)$ on X as indicated in proposotion 3.4.

4. The converse of the 3rd conclusion is not true as indicated in example 3.5.

5. If $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ is an $L_n -$space and A$\subseteq$X such that A is $L_n -$disconnected with respect to the $(A, \tau_{1A}, \tau_{2A}, \ldots\ldots, \tau_{nA})$, then A contains a subset S which is disconnected with respect to (X, τ_1) as indicated in proposotion 3.6.

6. If $f : (X, \tau_1, \tau_2, \ldots\ldots, \tau_n) \to (Y, \tau'_1, \tau'_2, \ldots\ldots, \tau'_n)$ is an $L_n -$continuous function such that A is $L_n -$connected with respect to $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$, then f(A) is connected with respect to (Y, τ'_1) as indicated in proposotion 3.8.

7. If A and B are $L_n -$connected sets which are not $L_n -$separated with respect to the $L_n -$space $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$, then A$\bigcup$B is $L_n -$connected as indicated in proposotion 3.9.

8. If A and B are connected sets which are not separated with respect to the space (X, τ_1), then it is not necessary that A$\cup$B is L_n-connected with respect to the L_n-space $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ as indicated in example 3.10.

9. If $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ is an L_n-space such that A,B$\subseteq$X, $A \subset B \subseteq \overline{A}^{L_n}$ and A is L_n-connected, then B is L_n-connected, in particular $\overline{A}^{L_n}$ is L_n-connected as indicated in proposotion 3.11.

10. If $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ is an L_n-space such that A,B$\subseteq$X, $A \subseteq B \subseteq \overline{A}^{1}$ and A is connected with respect to the space (X, τ_1), then it is not necessary that B is L_n- connected with respect to the L_n-space $(X, \tau_1, \tau_2, \ldots\ldots, \tau_n)$ as indicated in example 3.12.

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Watermark Embedding in Color Image Using the Intermediary Frequency Coefficients

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Abstract:

The rapid development of the communication networks through the Internet and development of the electronic trade with spread of the digital media such as (images, audio, video) which can be got easily, copied, and distributed with other person names. All these led to the needs of the authentication or copyright. The most important techniques used are watermark techniques.

The watermark techniques are divided into two types, they are spatial domain and frequency domain techniques. The frequency domain techniques are more stronger than the spatial domain techniques. The proposed algorithm is used with Discrete Cosine transform (DCT). Several experiments were given to illustrates the performance of the proposed scheme. This research focus on the frequency domain and deals with the images by choosing the best locations to embed the watermark to ensure the digital watermarking requirements.

Keywords: **Image, watermark, spatial domain, frequency domain.**

1. Introduction:

The development of the World Wide Web (WWW) led to the possibility of sharing the information by many users all over the world, and the development of software and hardware led to the ability of sending and receiving many data through the networks, now it became possible to send and receive different types of animated, static images, and different types of digital media [1].

The digital media such as (images, audio, video) that can be got easily, copied, and distributed with another person names all these led to the needs of the authentication or copyright. There are many methods to protect the copyright of these media. The important one of these techniques are the watermark techniques [2].

The proposed technique embed digital watermarking in the color images, to get an image that contains a digital watermarking. This technique has the ability to safe the watermark against attacks, and in the same time to keep the good quality of the reconstructed image. So the resultant image not be recognized by the Human Visual System (HVS).

2. Digital Watermark:

Refers to embed a message or digital watermark into another digital media. The purpose is the authenticity or copyright of this digital media. The digital watermark must be very difficult to remove, or destruct from the media by the attacks [3].

The embedding techniques in the images depending on basically on the sensitivity border of human being eye or Human Visual System (HVS).

The structure of a digital watermark composes from two stages: the first stage is the watermark embedding, and the second is the watermark detection and extraction. Figure (1) shows the embedding and the extraction operations.

The digital media (original image) that contains the digital watermark is called the carrier. The watermarking (the secret message (image)) is not attach in the carrier material as a separate file or link but its an embedded information directly within the carrier material and dealing with them as one material [4].

The digital watermark should contain some features to be able to protect the digital media from the attacks, and the most important requirements are [5, 6]:

1. **Transparency.**
2. **Robustness.**
3. **Capacity.**
4. **Security.**

3 <u>Discrete Cosine Transform (DCT):</u>

The image types are binary images, grayscale images, and color images.

The proposed algorithm deals with the color images, that can be considered as a three grayscale images, that contain three basic colors red, green, and blue and each color represents (8 bits), then can be represented each pixel contains three colors by (24 bits) as shows in the Figure (2) [7].

The proposal algorithm using DCT method to the watermark embedding. In which each image is divided into blocks with n*n pixels, then transform these blocks into the transform coefficients. The transform coefficients are blocks with three levels of frequency signals. DCT are used in the image compression applications too, like Joint Photographic Experts Group JPEG [8].

Each blocks transform from spatial domain to the frequency domain using the 2D-DCT equation as shows in the equation (1) [9,10]:

$$B_{pq} = \alpha p \alpha q \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} A_{mn} \cos\frac{\pi(2m+1)p}{2M} \cos\frac{\pi(2n+1)q}{2N} \quad \text{.....(1)}$$

$$0 <= p <= M-1$$

$$0 <= q <= N-1$$

$$\alpha_p = \begin{cases} 1/\sqrt{M}, & p = 0 \\ \sqrt{2/M}, & 1 \le p \le M-1 \end{cases} \qquad \alpha_q = \begin{cases} 1/\sqrt{N}, & q = 0 \\ \sqrt{2/N}, & 1 \le q \le N-1 \end{cases}$$

where: M and N are the row and column size of A

***p, q* =0, 1, ……., n-1**

This method produces block matrix that contains the transform coefficients to each pixel, the image signal changes slowly from point to point. Therefore, the

block energy is concentrated in number of low frequency coefficients that exists in the upper left corner from the matrix. While the high frequency coefficients exists in the lower right corner. There are intermediary frequency coefficients that exist in the middle of block as shows in Figure (3) , and these coefficients have a real values.

On the other hand, the Inverse DCT (IDCT) retrieves the original information from frequency domain to the spatial domain as shows in the equation (2) [9,10].

$$Amn = \sum_{p=0}^{M-1}\sum_{q=0}^{N-1} \alpha p \alpha q Bpq \cos\frac{\pi(2m+1)p}{2M}\cos\frac{\pi(2n+1)q}{2N} \quad \text{...(2)}$$

$$\alpha_p = \begin{cases} 1/\sqrt{M}, & p = 0 \\ \sqrt{2/M}, & 1 \le p \le M-1 \end{cases} \qquad \alpha_q = \begin{cases} 1/\sqrt{N}, & q = 0 \\ \sqrt{2/N}, & 1 \le q \le N-1 \end{cases}$$

beds the watermark image into the original image. And the second one to extracts the watermark image from the original image and these two algorithms described as follows:

The Embedding Algorithm:

1. Read the Original Image (Carrier). And Read the watermark image.
2. Divide the Carrier into blocks of n*n pixels.
3. Divide the watermark into blocks of n*n pixels.
4. Transform the carrier blocks into frequency domain using DCT.
5. Choose the intermediary frequency coefficients and then replace it with the pixel values of the watermark blocks.
6. Transform the carrier blocks from frequency domain to the spatial domain using IDCT.
7. Assemble the carrier blocks to produce an image with watermark (Water-Image). As shows in Figure (4).

The Extraction Algorithm:

1. Read the Water-Image produced from the embedding algorithm. And Read the original image (carrier).
2. Divide the Water-Image into blocks of n*n pixels.
3. Divide the Carrier into blocks of n*n pixels.
4. Transform the Water-Image blocks into frequency domain using DCT.

5. **Transform the carrier blocks into frequency domain using DCT.**
6. **Take the chosen intermediary frequency coefficients and then subtract the carrier intermediary coefficients from the Water-Image frequency coefficients to produce the original watermark blocks.**
7. **Assemble the watermark blocks to produce the watermark image. As shows in Figure (5).**

5. The Experimental Work and Analyses:

Peak signal-to-noise ratio (PSNR) is the standard method for quantitatively comparing a reconstructed image with the original image. For an 8-bit grayscale image, the peak signal value is 255. Hence the PSNR of an M×N 8-bit grayscale image x and its reconstruction $\hat{x}$ is calculated as [11]:

$$PSNR = 10\log_{10}\frac{255^2}{MSE} \qquad \text{......(3)}$$

where the Mean Square Error (MSE) is defined as:

$$MSE = \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left[x(m,n) - \hat{x}(m,n)\right]^2 \qquad \text{......(4)}$$

Figure (6) shows the original image and the watermarking image using the proposed algorithm. Figure(7) shows the first & second experiment using the carrier images with two watermarks and the PSNR that results from this experiment, while Figure (8) shows the third & forth experiment using the carrier images with two watermarks and the PSNR that results from this experiment .

The Experiments:

There are four experiments each one have original image with one watermark images where the first watermark image is embedded in the first original image and calculates the result of PSNR to show the effect of the embedding operation on the original image.

Then perform the second experiment on the first original image with the second original watermark and calculates the PSNR, …etc.

The result of the experiments are showed in the Table (1):

Experiment No	Original Image No.	Watermark No.	PSNR (dB)
1	1	1	30.0576
2	1	2	35.7086
3	2	1	19.858
4	2	2	25.1018

Table (1) The Results of experiments

6. Conclusions:

The experiments show the ability of the proposed algorithm to embed the watermark image in an efficient manner by achieving the watermarking requirements that showed in the following:

1. Transparency: The proposed algorithm provide a high degree of transparency that the HVS does not recognize the watermark that embedded in the original image. And this can be achieved by choosing the embedded locations which are far from the low frequency locations (blocks energy).
2. Capacity: The proposed algorithm has a high capacity that can reach the half of the original image size. The watermark can be embed in the (4, 8, 16) locations at least in each block (8*8=64 locations).
3. Security: The important requirement that must be provided in the system by embed the watermark in the appropriate frequency domain coefficients and produce an embed image with good PSNR.

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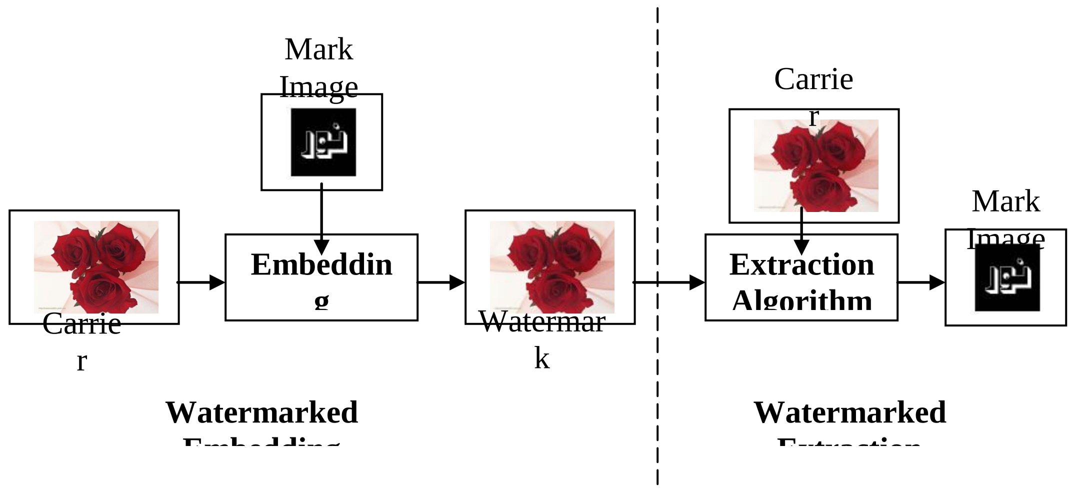

Figure (1) The embedding and the extraction

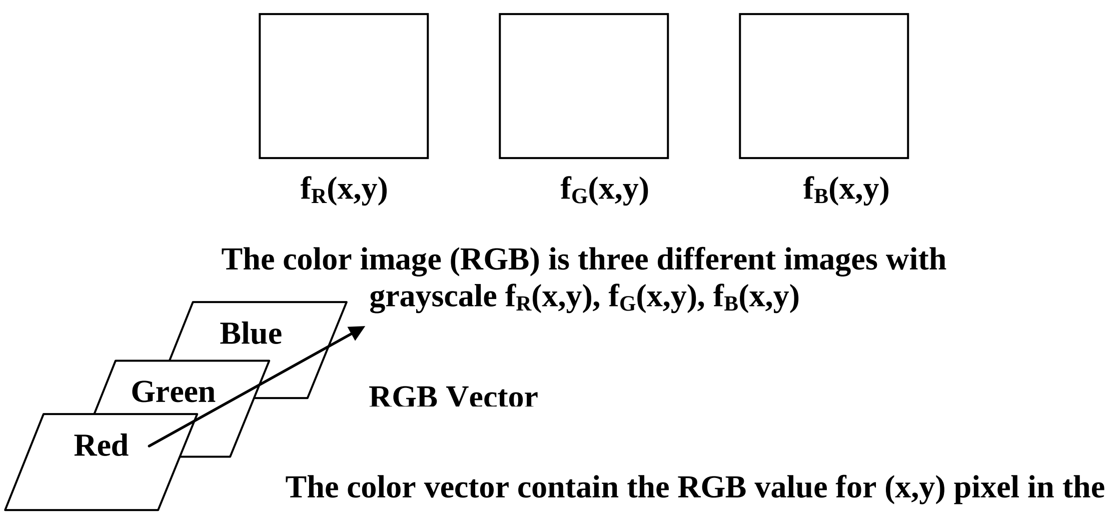

Figure (2) Representation of color image.

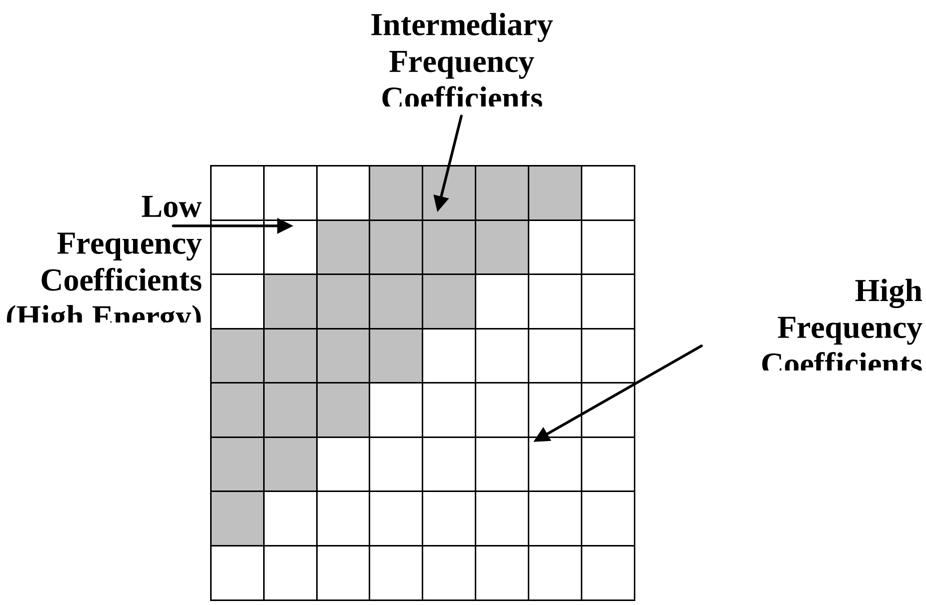

Figure (3). The different frequency regions

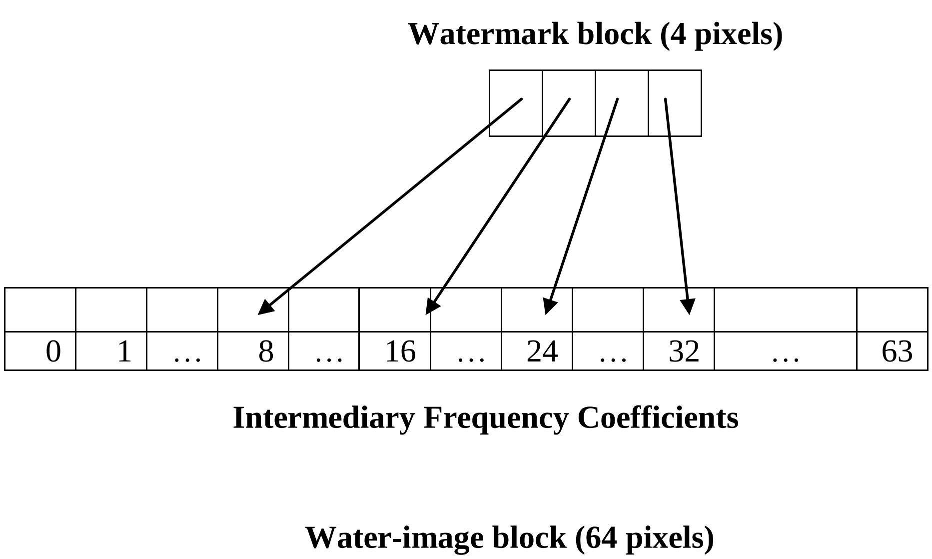

Figure (4). The embedding stage.

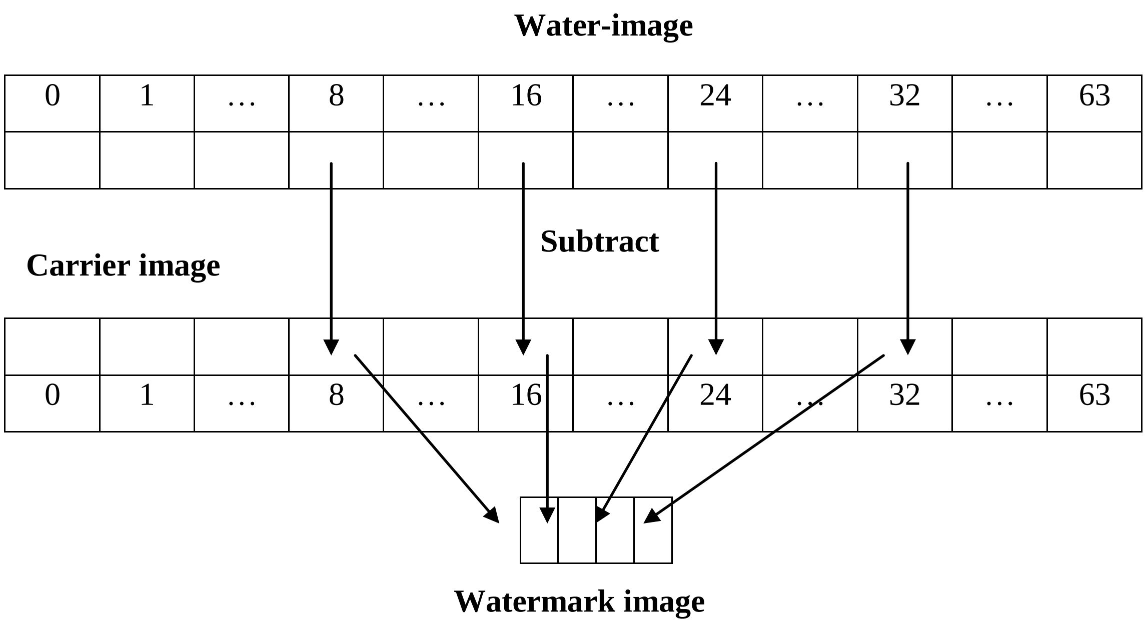

Figure (5). The extraction stage.

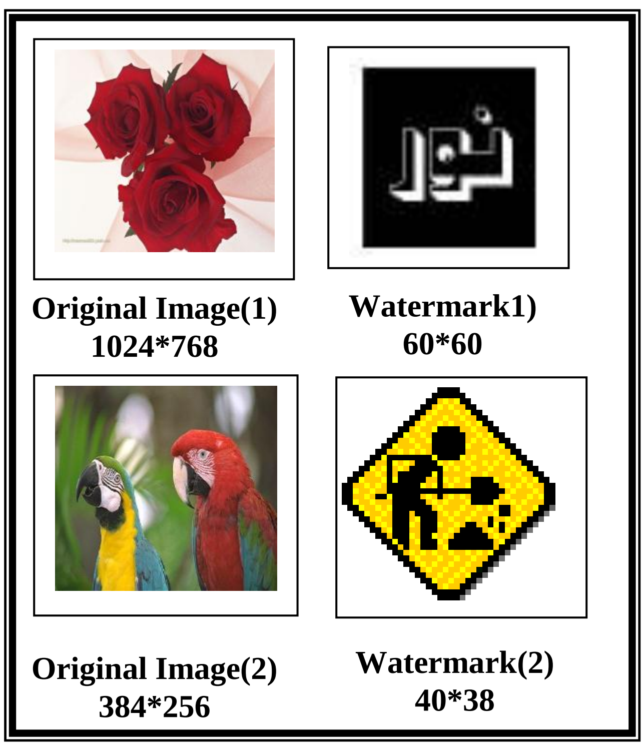

Figure (6) The original and watermark images.

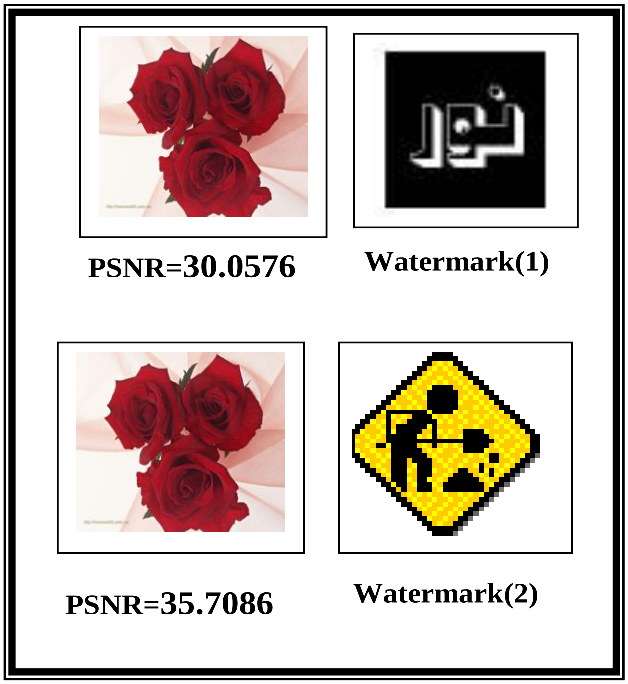

Figure (7) The result of experiment (1,2).

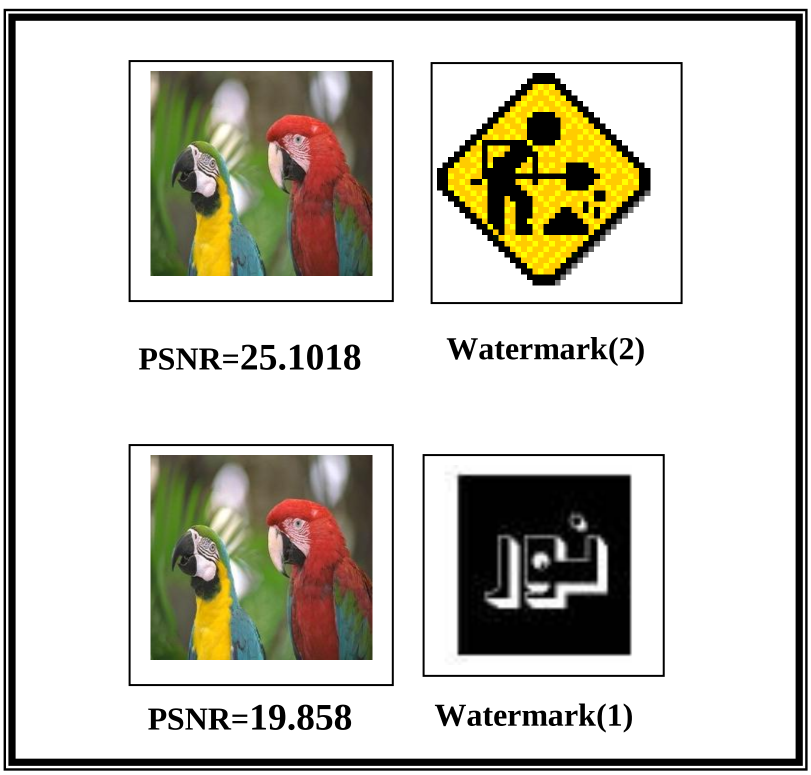

Figure (8) The result of experiment (3,4).

تضمين العلامة المائية
في الصورة الملونة باستعمال معاملات الترددات الوسطية

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المستخلص:

النمو السريع لشبكات الاتصال عبر الإنترنت وتطور التجارة الالكترونية وانتشار الأوساط الرقمية المختلفة مثل (الصور، الصوت، الفيديو) والتي أصبح من السهل الحصول عليها ونسخها وتوزيعها بأسماء أشخاص آخرين كل هذا أدى إلى خلق حاجة ملحة لحماية حقوق النشر واثبات الملكية وغيرها. وأحدى أهم التقنيات المستعملة في هذا المجال هي تقنيات العلامة المائية الرقمية.

تنقسم تقنيات التضمين إلى قسمين رئيسيين هما تقنيات التضمين في المجال المكاني وتقنيات التضمين في المجال الترددي وعادة ما تكون تقنيات التضمين في المجال الترددي أكثر قوة من تقنيات التضمين في المجال المكاني وهذا ما تم التركيز عليه في هذا البحث وذلك باختيار أفضل مواقع للتضمين في الصور الملونة لتحقق متطلبات تضمين العلامة المائية الرقمية.

الكلمات المفاتيح: Keywords

Image, watermark, spatial domain, frequency domain