Recognizing Human Face Images Using their

Energy levels and B(E2) values for Mg-26 isotope of O(6) dynamical symmetry using IBM-1

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Abstract:

In this work interacting boson model (IBM-1) has been used to study the nuclear structure of Mg(A=26) isotope where it belong to dynamical symmetries (6). The energy levels, the electric quadrupole probabilities B(E2), reduced matrix elements <I_f||$T^{(E2)}$||I_i> and intrinsic moments Q_o has been predicted.

Two main programs were used in the present calculation, (IBSS1.for) to calculate the energy levels and (IBMT.for) to calculate the B(E2) , <I_f||$T^{(E2)}$||I_i> and Q_o. Our results are compared with available experiment data where they show good agreement.

مستويات الطاقة والانتقالات الكهربائية رباعية القطب لنظير Mg-26 ذات التناظر الديناميكي O(6) باستخدام نموذج IBM-1

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الخلاصة:

استخدم في هذا البحث نموذج البوزونات المتفاعل الأول (IBM-1) لدراسة التركيب النووي لنظير المغنيسيوم Mg(A=26) والذي ينتمي الى التناظر الديناميكي O(6)، إذ تم حساب مستويات الطاقة واحتمالية الانتقالات الكهربائية رباعية القطب B(E2) وعناصر المصفوفة المختزلة <I_f|| $T^{(E2)}$ ||I_i> وتم كذلك حساب العزم الذاتي الكهربائي Q_o.

وقد استخدم برنامجين جزئيين من البرنامج الرئيسي لهذا النموذج في إيجاد النتائج النظرية وهما كل من (IBSS1.For) الخاص بحساب مستويات الطاقة وبرنامج (IBMT.For) الخاص بحساب كل من B(E2)

وتم مقارنة النتائج التي حصلنا عليها مع القيم العملية المتوفرة فكانت متوافقة بشكل جيد. . Q_o ،$||I_i>$ $T^{(E2)}$ $<I_f||$،

Introduction:

Arima and Iachello (1979) [1] have developed the interacting boson model (IBM), which is based on the well-known shell model and on geometrical collective model of the atomic nucleus.

This model is to describe nuclear properties such as spins, energies of the levels, decay probabilities for the emission of gamma quanta, probabilities of electromagnetic transition and their reduced matrix elements for different transitions multipole moment and mixing ratios [1, 2].

The (IBM-1) is used in the present work, this model represents very important step formed in the description of collective nuclear excitations. The underlying U(6) group structure of model basis leads to a simple Hamiltonian which is capable of describing the three specific limits of collective structure vibrational U(5), rotational SU(3) and gamma unstable O(6) [1,3].

In the simplest version of the interacting boson model (IBM-1), its assumed that low-lying collective states in even-even nuclei away from closed shells are dominated by excitation of the valence protons and the valence neutrons (particles outside the major closed shell) while the closed shell core is inert. Furthermore, its assumed that the particle configurations which are most important in shaping the properties of the low-lying states are these in which identical particles are coupled together forming pairs of angular momentum 0 and 2 [1,2].

Brown B. and Wildenthal B. (1983) [3], they used IBM-1 to calculate the energy levels, μ_I and B(M1) for Mg-24 of O(6) dynamical symmetry which they show a good agreement of Hamiltonian equation for internal levels in (A=17-39) isotopes.

Zhang J. et al. (1997) [4], study of ^{24}Mg and ^{28}Si angular momentums, branching ratios and B(E2) comparing with experimental data.

Theoretical part:

1- The Hamiltonian operator of the (IBM-1)

The Hamiltonian operator according IBM-1can be written as follows [1, 2]:

$$\hat{H} = \sum_{i=1}^{N} \varepsilon_i + \sum_{i<j}^{N} V_{ij} \quad \text{............(1)}$$

Where ε_i: is the boson energy.

V_{ij}: is the boson-boson interacting energy.

N: is the total number of bosons.

The most commonly used from of the (IBM-1) Hamiltonian [5]:

$$\hat{H} = \in \hat{n}d + a_o(\hat{P}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4 + \hat{T}_4) \quad \text{.............(2)}$$

Where $\in = \in s + \in d$ **is the boson energy.**

The operators:

$$\left.\begin{array}{ll} \hat{n}_d = (\hat{d}^\dagger.\hat{\tilde{d}}) & \text{The boson number operator} \\ \hat{P} = 1/2(\hat{\tilde{d}}.\hat{\tilde{d}}) - 1/2(\hat{\tilde{S}}.\hat{\tilde{S}}) & \text{The pairing bosons operator} \\ \hat{L} = \sqrt{10}[\hat{d}^\dagger x\hat{\tilde{d}}]^{(1)} & \text{The angular momentum operator} \\ \hat{Q} = [(\hat{d}^\dagger x\hat{\tilde{S}}) + (\hat{S}^\dagger xd)]^{(2)} - CHI[\hat{d}^\dagger x\hat{\tilde{d}}]^{(2)} & \text{The quadrapole operator} \\ \hat{T}_3 = [\hat{d}^\dagger x\hat{\tilde{d}}]^{(3)} & \text{The octupole operator} \\ \hat{T}_4 = [\hat{d}^\dagger x\hat{\tilde{d}}]^{(4)} & \text{The hexadecapole operator} \end{array}\right\} \quad \text{................(3)}$$

$CHI = -1/2\sqrt{7}$ **The rotational dynamical symmetry and equal zero for vibration and γ-soft dynamical symmetry. And a_o,a_1,a_2,a_3,a_4 are the phenomenological parameters.**

2- Gamma unstable Dynamical symmetry O(6)
For O(6) dynamical symmetry, the Hamiltonian operator becomes[6]:

$$\hat{H} = a_o(\hat{P}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_3(\hat{T}_3.\hat{T}_3) \quad \text{...........(2)}$$

The electric quadrupole transition operator in the IBM-1 can be written as[2]:

$$\hat{T}^{(E2)} = \alpha_2\left[\hat{d}^+ \times \hat{\tilde{s}} + \hat{s}^+ \times \hat{\tilde{d}}\right]_\mu^{(2)} + \beta_2\left[\hat{d}^+ \times \hat{\tilde{d}}\right]_\mu^{(2)} \quad \text{........(3)}$$

Where α_2 is the effective charge of boson.
β_2 is the effective of one particle in d-boson.

While the electric quadrupole transition probability can calculated by[7]:

$$B(E2; I_i \rightarrow I_f) = \frac{1}{2I+1} \left| \left\langle I_f \left\| \hat{T}^{(E2)} \right\| I_i \right\rangle \right|^2 \ldots\ldots\ldots\ldots(4)$$

And the electric quadrupole moment (Q_I) is[1,2]:

$$Q_I = \sqrt{\frac{16\pi}{5}} \begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix} \left\langle I_f \left\| \hat{T}^{(E2)} \right\| I_i \right\rangle \ldots\ldots\ldots\ldots(5)$$

<u>Results & Discussion:</u>

In this work we have studied the nuclear structure of even-even Mg (A=26) isotope which is classified to gamma unstable dynamical symmetry O(6) by comparing the energy ratios E_4/E_2=2.39 , E_6/E_2=4.53 , E_8/E_2=7.29 with ideal values for three dynamical symmetries U(5), O(6) and SU(3) of IBM-1. This study past over energy levels, γ-transitions, electric quadrupole transition probabilities, reduced matrix elements and intrinsic quadrupole moments. Table (1) presents the isotope used in the present work according to its atomic mass number, total number of boson and the corresponding Hamiltonian parameters according to gamma unstable dynamical symmetries O(6).

Table (1): The parameters of Hamiltonian operator and electric transitions for Mg (A=26) isotope used in present work of IBM-1.

Isotopes	N_π	N_υ	N	$\hat{P}.\hat{P}$ (MeV)	$\hat{I}.\hat{I}$ (MeV)	$\hat{T}_3.\hat{T}_3$ (MeV)	α2 (eb)	β2 (eb)
^{26}Mg	2	3	5	0.5980	0.1325	0.5520	0.0600	0.1910

Tables (2) presents values of the energy levels (present work), according to energy bands (g, β, and γ bands) in comparison with available experimental data. This table list the new energy levels belong to, β_1, β_2, β_3 and β_4 bands with their spins and parties. The results show that, the β-band is a large extent emerge than the γ-band for the dynamical symmetry U(5) and SU(3), while the emergence of γ-band is increasing for the isotopes having the transitional dynamical symmetry U(5)-O(6). The β-band is not difficult to see it in the dynamical symmetry O(6), in the low spin states, but it is difficult to find it due to the high spin state.

Figure (1) shows the gamma energy transitions comparison with experimental data as a function of spin sequences, it is noted that the sequences (0_4---2_1 and 8_1---6_1) have a high probability of gamma transitions.

Figure (2) shows energy levels for ^{26}Mg isotope present work corresponding to the typical energy band spectrum, in comparisons with available experimental data. The energy levels in β_2, β_3, β_4 and γ_1, γ_2 are shown in this figure. Table (3) list values of intrinsic quadrupole moments, gamma energy transitions, electric quadrupole transitions probability and reduce matrix elements which show a good agreement comparison with a available experimental data for Mg-26 isotope.

Figure (3) explains the relationship between electric quadrupole transitions probability and it's reduce matrix elements as a function of spin sequences, the transitions (0_2---2_2, 0_3---2_1) show an inverse relation between them by using IBM-1 and the transitions (0_2---2_2) has a very high probability of electric quadrupole transitions. Figure (4) shows the electric quadrupole moments as a function of angular momentum by using "IBMT.for" program of IBM-1 for Mg-26 isotope.

Table (2): The energy levels of the ^{26}Mg isotope according to energy bands (g,β,γ-band) in comparison with available experimental data[8,9].

Level / B.E	0^+	2^+	4^+	6^+	8^+	10^+
	2^+	3^+	4^+	5^+	6^+	7^+
g. exp.	0.0000	1.8087	4.3184	8.2010	13.1900	-
g. IBM-1	0.0000	1.5678	4.3060	8.2146	13.2936	19.5430
β_1. exp.	3.5888	4.8343	5.4737	-	-	-
β_1. IBM-1	3.5880	5.1558	5.5140	10.9746	16.6056	-
β_2. exp.	4.9003	-	-	-	-	-
β_2. IBM-1	4.9680	6.8118	7.8940	11.8026	-	-
β_3 exp	6.2560	-	-	-	-	-
β_3 IBM-1	5.9800	7.5478	9.2740	-	-	-
β_4 exp	-	-	-	-	-	-
β_4 IBM-1	8.5560	11.5038	12.5860	-	-	-

γ_1. exp.	2.9384	-	-	-	-	-
γ_1. IBM-1	3.2238	5.8956	10.1020	13.3590	14.2866	15.3688
γ_2. exp.	-	-	-	-	-	-
γ_2. IBM-1	8.1918	9.4836	-	10.0470	-	-

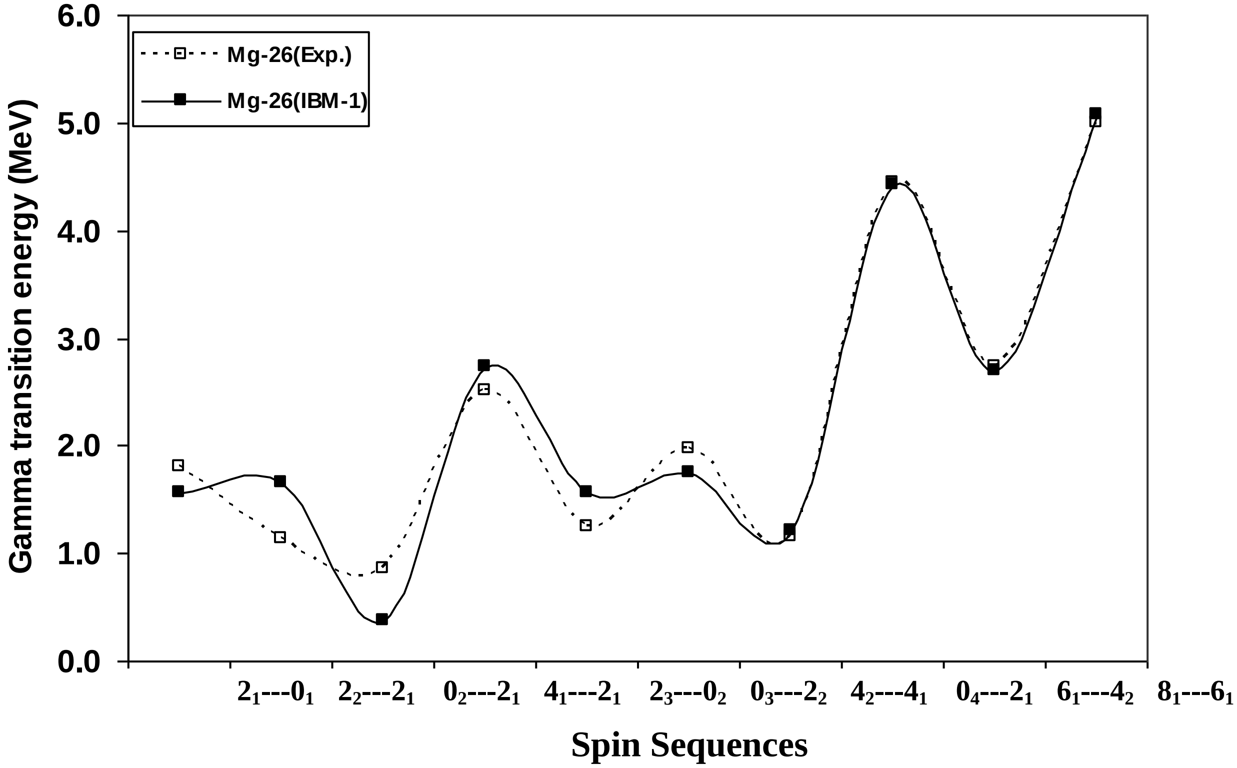

Figure (1): gamma energy transitions comparison with experimental data as a function of spin sequences in (IBM-1) for Mg-26.

Table (3): electric moments, gamma transitions, electric quadrupole transitions probability and reduce matrix elements comparison with experimental data(8,9,10,11) for Mg-26 isotope.

I^{π}	Q_I (b)	$I_i^+ — I_f^+$	E_γ (MeV)		B(E2) $(eb)^2$	$<I_f\|\|T^{(E2)}\|\|I_i>$
			EXP(8,9).	IBM-1		
0_1^+	——	——	——	——	——	——
2_1^+	0.5491 Exp.(10)0.5540	$2_1^+ — 0_1^+$	1.8087	1.5678	0.3240×10^{-1} Exp.(11) 0.0305	-0.4025
2_2^+	-0.2034	$2_2^+ — 2_1^+$ $2_2^+ — 0_1^+$	1.1297 2.9384	1.6560 3.2238	0.4114×10^{-1} 0.2606×10^{-1}	-0.4536 0.3610
0_2^+	——	$0_2^+ — 2_2^+$	0.8504	0.3642	0.7296×10^{-1}	0.2701
4_1^+	0.9539	$4_1^+ — 2_1^+$ $4_1^+ — 2_2^+$	2.5097 1.3800	2.7382 1.0822	0.4114×10^{-1} 0.1471×10^{-1}	0.6085 -0.3638

2_3^+	0.6101	$2_3^+ - 2_1^+$ $2_3^+ - 0_2^+$	3.0256 1.2455	3.5880 1.5678	0.3574×10^{-2} 0.1512×10^{-1}	0.1337 -0.2750
0_3^+	——	$0_3^+ - 2_1^+$ $0_3^+ - 2_2^+$	3.0916 1.9619	3.4002 1.7442	0.3185×10^{-1} 0.3960×10^{-1}	0.1785 -0.1990
4_2^+	0.3217	$4_2^+ - 2_1^+$ $4_2^+ - 2_2^+$ $4_2^+ - 4_1^+$	3.6650 2.5353 1.1553	3.9462 2.2902 1.2080	0.1668×10^{-1} 0.2074×10^{-1} 0.1886×10^{-1}	-0.3875 0.4321 -0.4120
3_1^+	——	$3_1^+ - 2_2^+$ $3_1^+ - 4_1^+$	—— ——	2.6718 1.5896	0.2829×10^{-1} 0.1131×10^{-1}	-0.4450 -0.2814
0_4^+	——	$0_4^+ - 2_1^+$	4.4473	4.4122	0.1561×10^{-15}	-0.00001
2_4^+	-0.2441	$2_4^+ - 0_2^+$ $2_4^+ - 2_3^+$	—— ——	3.2238 1.6560	0.1946×10^{-1} 0.1646×10^{-1}	0.3119 -0.2869
2_5^+	0.8542	$2_5^+ - 0_4^+$	——	1.5678	0.3600×10^{-2}	0.1342
4_3^+	1.11450	$4_3^+ - 4_1^+$ $4_3^+ - 2_3^+$	—— ——	3.5880 2.7382	0.3033×10^{-2} 0.1646×10^{-1}	0.1652 0.3849
2_6^+	0.2644	——	——	——	——	——
6_1^+	1.4351	$6_1^+ - 4_1^+$ $6_1^+ - 4_2^+$	3.8826 2.7273	3.9086 2.7006	0.3960×10^{-1} 0.2414×10^{-1}	0.7175 -0.5602
4_4^+	-0.3660	——	——	——	——	——
5_1^+	0.6687	$5_1^+ - 4_1^+$ $5_1^+ - 6_1^+$	—— ——	5.7410 1.8324	0.1216×10^{-1} 0.7426×10^{-2}	-0.3657 -0.2858
4_5^+	0.4163	$4_5^+ - 2_1^+$	——	8.5342	0.6066×10^{-4}	-0.0234
6_2^+	0.8357	$6_2^+ - 4_2^+$ $6_2^+ - 6_1^+$	—— ——	5.4606 2.7600	0.2142×10^{-1} 0.9997×10^{-2}	0.5277 -0.3605
2_7^+	-0.1830	$2_7^+ - 2_2^+$	——	8.2800	0.5097×10^{-15}	0.00001
6_3^+	1.8572	——	——	——	——	——
4_6^+	0.6201	——	——	——	——	——
8_1^+	——	$8_1^+ - 6_1^+$	4.9890	5.0790	0.3142×10^{-1}	0.7308
5_2^+	-0.4629	——	——	——	——	——
6_4^+	0.2496	——	——	——	——	——

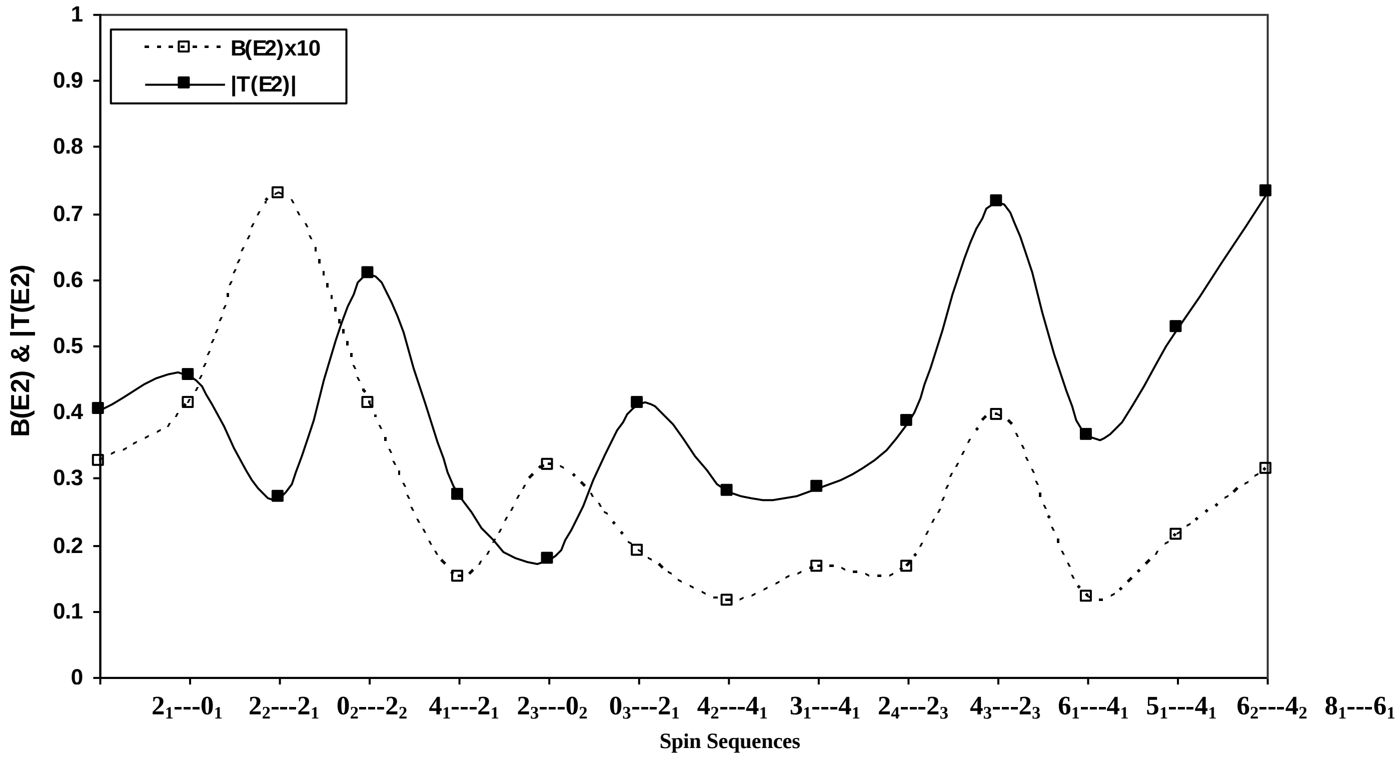

Figure (3): electric quadrupole probabilities and its reduce matrix elements as a function of spin sequences in (IBM-1) for Mg-26.

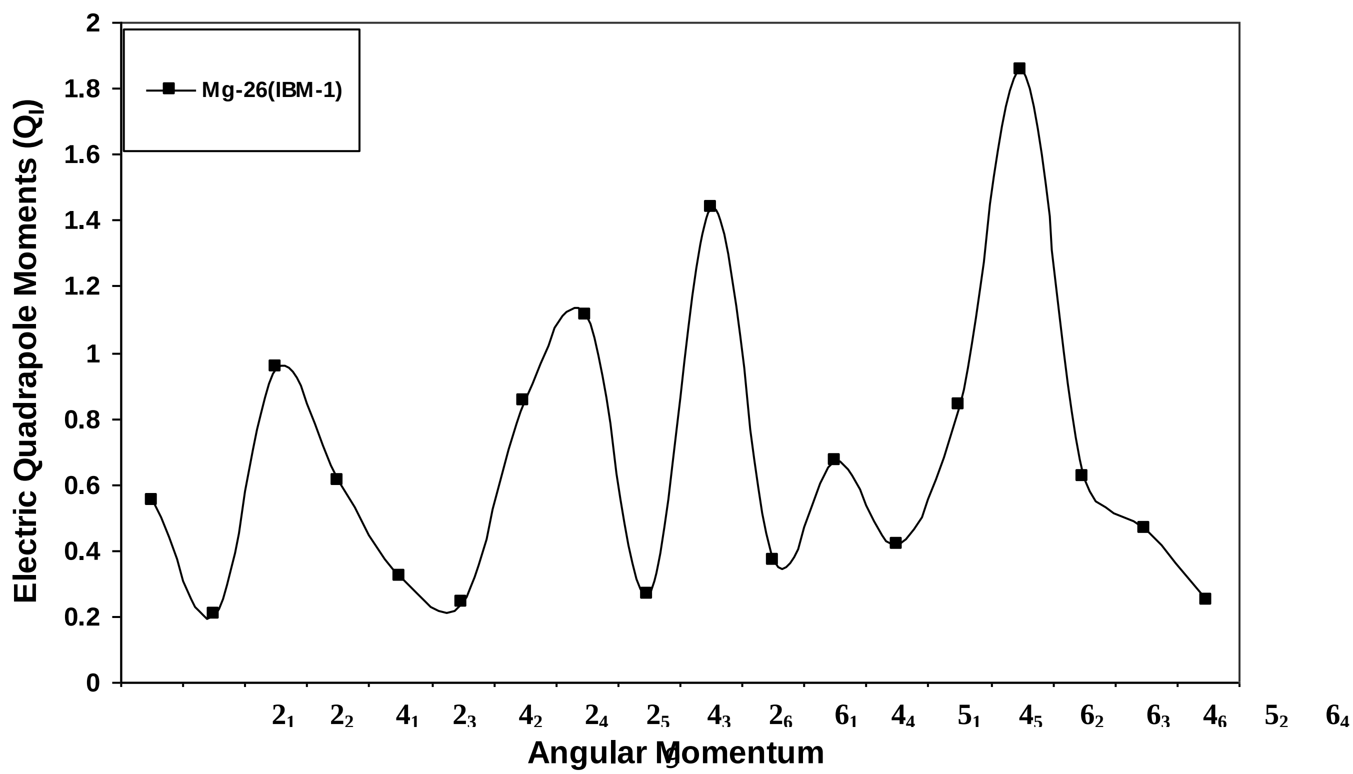

Figure (4): electric quadrupole moments as a function of angular momentum in IBM-1 for Mg-26.

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