Sufficient Optimality Criteria for nonlinear Programming Problems Involving Semi-Differentiable Functions

Muhenned A. Abdul-Sahib
Computer Department
College of Science
Thi-Qar University
Thi-Qar /Iraq

Abstract

In this paper, a nonlinear programming problem has been considered, where the function involved are $\eta-$ semidifferentiable. Kaul and Layall were defined $\eta-$convexity, $\eta-$quasiconvexity and $\eta-$ pseudoconvexity for semidifferentiable functions.New classes of functions $\eta_1\eta_2-$convexity, $\eta_1\eta_2-$quasiconvexity and $\eta_1\eta_2-$ pseudoconvexity are defined on η_1- semidifferentiable.Some properties involving these function are discussed.Sufficient optimality criteria for nonlinear programming problems involving these functions are given.

الخلاصة

العالمان كول و لال عرفا اصناف الدوال
$\eta-$ convexity, $\eta-$quasiconvexity and $\eta-$ pseudoconvexity
للدوال التي هي semidifferentiable .

درست البرمجه غير الخطيه في حالة دالة الهدف والقيود
$\eta-$ Semidifferentiable (=η_1- Semidifferentiable) اصناف الدوال $\eta_1\eta_2-$ convexity, $\eta_1\eta_2-$quasiconvexity and $\eta_1\eta_2-$ pseudoconvexity عرفت
للدوال التي هي η_1- semidifferentiable
درست صفاتها وكذلك الشرط الكافي للامثليه اعطي للبرمجه الغير خطيه المتضمنه لهذه الدوال .

1. Introduction

The importance of convex functions is every local minima of convex function defined on convex set are also global minima, some generalization of convex functions and their properties were investigated by Hanson [1].He defined invexity for differentiable function as a very broad generalization of convexity.
A mathematical program of the form

Min $f(x)$ **subject to** $g(x) \le 0 \ , x \in C \subseteq R^n$ **is invex if there exists a function** $\eta : CXC \to R^n$ **such that for all** $x, u \in C$**,**

$$f(x) - f(u) \ge \eta(x,u)\nabla f(u)$$

and

$$g(x) - g(u) \ge \eta(x,u)\nabla g(u)$$

It may be noted here that the convex case corresponding to $\eta(x,u) = x - u$**.**

Kaul and Lyall[2]defined $\eta -$**convexity,** $\eta -$**quasiconvexity and pseudoconvexity for functions which the right differential of the function in the direction** $(x - \bar{x})$ **exist. Preda and Minasian[3] defined functions which are** $\eta -$**semidifferentiable numerical function defined on** $\eta -$**locally starshaped set i.e. those functions for which the right differential of the functions in the direction** $\eta(x, \bar{x})$ **exist at each point of the** $\eta -$**locally starshaped set on which the functions are defined.**

In this paper, a nonlinear programming problem is considered; where the functions involved are $\eta -$**semidifferentiable .sufficient optimality conditions are obtained. The class of semilocally preinvex is special case of** $\eta_1\eta_2 -$**convex in case** $\eta_1 -$**semidifferentiable function.**

Section 2 introduce the terms $\eta_1\eta_2 -$**convexity,** $\eta_1\eta_2 -$**quasiconvexity,** $\eta_1\eta_2 -$ **pseudoconvexity. Sections 3 contains some of the properties possessed by these functions. Section 4 shows sufficient optimality criteria for nonlinear programming problem involve** $\eta_1\eta_2 -$**convexity.**

2. DEFINITIONS AND PRELIMINARIES

Let R^n **be the n-dimensional Euclidian Space and** f **be a numerical function defined on a set** $C \subseteq R^n$ **.**

Definitintion2.1 [1]: The set C is invex at $\bar{x} \in C$ **if** $\bar{x} + \lambda\eta(x, \bar{x}) \in C$ **for any** $x \in C$ **and any** $\lambda \in [0,1]$**.**

The set C is invex if C is invex at any $x \in C$**.**

If $\eta(x, \bar{x}) = x - \bar{x} \quad \forall x \in C$ **then C is a convex set.**

The following two definitions from Kaul and Kaur.

Definition2.2: The right differential of f **at** $\bar{x} \in C$ **in the direction of** $x - \bar{x}$ **denoted by** $df^+(\bar{x}, x - \bar{x})$ **is defined as**

$$df^{+}(\bar{x}, x-\bar{x}) = \lim_{\lambda \to 0^{+}} \frac{f((1-\lambda)\bar{x}+\lambda x) - f(\bar{x})}{\lambda}$$

provided the limit exists.
If the right differential exists at each $\bar{x} \in c$, then f is said to be semidifferentiable on C

Definition2.3: A subset $C \subseteq R^{n}$ is said to be locally starshaped at $\bar{x} \in$ C if corresponding to $\bar{x}$ and each $x \in C$, there exists a maximum positive number $a(x, \bar{x}) \leq 1$ such that

$$(1-\lambda)\bar{x} + \lambda x \in C \qquad 0 < \lambda < a(x, \bar{x})$$

If C is locally starshaped fore each $\bar{x} \in C$, then C is a locally starshaped set at each of it's points

Definition2.4: The set $C \subseteq R^{n}$ is an η-locally starshaped set at $\bar{x}(\bar{x} \in C)$ if for any $x \in C$ there exists a vector function η and a positive number $a_{\eta}(x, \bar{x})$ with $0 < a_{\eta}(x, \bar{x}) \leq 1$ such that $\bar{x} + \lambda \eta(x, \bar{x}) \in C$ for any $\lambda \in [0, a_{\eta}(x, \bar{x})]$.
We say that the set C is $\eta-$locally starshaped if C is $\eta-$locally starshaped at any $x \in C$
Definition2.5 [3]: Let $f: C \to R$ be a function, where $C \subseteq R^{n}$ is an $\eta-$ locally starshaped set at $\bar{x} \in C$, we say that f is

i- Semilocally preinvex(slpi) at $\bar{x}$ if corresponding to $\bar{x}$ and each $x \in C$ there exists a positive number $d_{\eta}(x, \bar{x}) \leq a_{\eta}(x, \bar{x})$ such that

$$f(\bar{x} + \lambda \eta(x, \bar{x})) \leq \lambda f(x) + (1-\lambda) f(\bar{x}) \qquad 0 < \lambda < d_{\eta}(x, \bar{x})$$

ii- The function f is strictly semilocally preinvex (sslpi) at $\bar{x} \in C$ if for each $x \in C$, $x \neq \bar{x}$ the inequality is strict

Definition2.6 [2]: A semidifferentiable numerical function f define on a set $C \subseteq R^{n}$ is said to be $\eta-$convex at $\bar{x}$ if there exists a numerical function $\eta(x, \bar{x})$ define on CXC such that

$$f(x) - f(\bar{x}) \geq \eta(x, \bar{x}) df^{+}(\bar{x}, x-\bar{x}) \qquad \forall x \in C$$

f is said to be $\eta-$convex on C if there exists a numerical function $\eta(x_1, x_2)$ defined on CXC such that

$$f(x_1) - f(x_2) \geq \eta(x_1, x_2) df^{+}(x_2, x_1 - x_2) \qquad \forall x_1, x_2 \in C$$

When the relation is satisfied as a strict inequality f is said to be a strictly $\eta-$ convex function.

Definition2.7 [3]: Let $f: C \to R$ be a function, where $C \subseteq R^{n}$ is an $\eta-$locally starshaped set at $\bar{x} \in C$.We say that f is $\eta-$semidifferentiable at $\bar{x}$ if $df^{+}(\bar{x}, \eta(x, \bar{x}))$ exists for each $x \in C$, where

$$df^{+}(\bar{x}, \eta(x, \bar{x})) = \lim_{\lambda \to 0^{+}} \frac{[f(\bar{x} + \lambda \eta(x, \bar{x})) - f(\bar{x})]}{\lambda}$$

If f **is** $\eta -$ **semidifferentiable at any** $\bar{x} \in C$ **, then** f **is said to be** $\eta -$ **semidifferentiable on** C

Definition2.8 : $\eta_1 -$ **semidifferentiable numerical function** f **defined on an** $\eta_1 -$ **locally starshaped set C is said to be** $\eta_1\eta_2 -$ **convex at** $\bar{x} \in C$ **if there exist a numerical function** $\eta_2(x,\bar{x})$ **defined on** CXC **such that**

$$f(x) - f(\bar{x}) \geq \eta_2(x,\bar{x}) df^{+}(\bar{x}, \eta_1(x,\bar{x})) \qquad \forall x \in C$$

f **is said to be** $\eta_1\eta_2 -$ **convex on C if there exists a numerical function** $\eta_2(x_1,x_2)$ **define on** CXC **such that**

$$f(x_1) - f(x_2) \geq \eta_2(x_1,x_2) df^{+}(x_2, \eta_1(x_1,x_2)) \qquad \forall x_1, x_2 \in C$$

when the inequality satisfied strictly then f **is said to be a strictly** $\eta_1\eta_2 -$ **convex functions .**

Theorem2.1 [4]:

Let $f : C \to R$ **be an** $\eta_1 -$ **semidifferentiable on** $\eta_1 -$ **locally starshaped set C**

i- The function f **is slpi at** $\bar{x} \in C$ **iff** $df^{+}(\bar{x}, \eta(x,\bar{x}))$ **exists and**

$$f(x) - f(\bar{x}) \geq df^{+}(\bar{x}, \eta_1(x,\bar{x})) \qquad \forall x \in C$$

ii-The function f **is strictly semi locally preinvex at** $\bar{x} \in C$ **iff** $df^{+}(\bar{x}, \eta(x,\bar{x}))$ **exists and** $f(x) - f(\bar{x}) \geq df^{+}(\bar{x}, \eta_1(x,\bar{x})) \qquad \forall x \neq \bar{x}$

This theorem shows that if $f : C \to R$ **is** $\eta_1 -$ **semidifferentiable function, semi locally preinvex is special case of** $\eta_1\eta_2 -$ **convex.**

The converse is not true

Example 2.1: Consider a function

$f : [0, \frac{\pi}{2}[\to R$ **defined by**

$$f(x) = \begin{cases} \sin x & 0 \leq x < \frac{\pi}{6} \\ 2\sin x - \frac{1}{2} & \frac{\pi}{6} \leq x < \frac{\pi}{2} \end{cases}$$

It's clear that the function is not differentiable at $x = \frac{\pi}{6}$ **, take** $\eta_1(x_1,x_2) = x_1 - x_2$

$$\eta_2(x_1.x_2) = \begin{cases} \dfrac{\sin x_1 - \sin x_2}{(x_1 - x_2)\cos x_2} & x_1 \neq x_2 \\ 1 & x_1 = x_2 \end{cases}$$

$$df^+(x_2,x_1-x_2)=\begin{cases}(x_1-x_2)\cos x_2 & 0\le x_1<\frac{\pi}{6},0\le x_2<\frac{\pi}{6}\\ 2(x_1-x_2)\cos x_2 & \frac{\pi}{6}\le x_1<\frac{\pi}{2},\frac{\pi}{6}<x_2<\frac{\pi}{2}\\ (x_1-\frac{\pi}{6})\cos\frac{\pi}{6} & 0\le x_1<\frac{\pi}{6},x_2=\frac{\pi}{6}\\ 2(x_1-\frac{\pi}{6})\cos\frac{\pi}{6} & \frac{\pi}{6}\le x_1<\frac{\pi}{2},x_2=\frac{\pi}{6}\end{cases}$$

It can be easily verified that the inequality

$f(x_1)-f(x_2)\ge\eta_2(x_1,x_2)df^+(x_2,\eta_1(x_1,x_2))$ **holds in** $[0,\frac{\pi}{2}[$ **hence the function is** $\eta_1\eta_2-$**convex but if we take** $x_1=\frac{\pi}{12},$ **and** $x_2=\frac{\pi}{18}$ **we observe that**

$f(x_1)-f(x_2)<df^+(x_2,\eta_1(x_1,x_2))$

Hence the function f **is not semi locally preinvex (slpi)**

Definition2.9: η_1**-semidifferentiable function** f **defined on an** η_1**-locally starshaped set** $C\subseteq R^n$ **is said to be** $\eta_1\eta_2-$**qusiconvex at** $\bar{x}\in C$ **, if there exists a numerical function** $\eta_2(x,\bar{x})$ **defined on** $C\mathrm{X}C$ **such that**

$$f(x)\le f(\bar{x})\Rightarrow\eta_2(x,\bar{x})df^+(\bar{x},\eta_1(x,\bar{x}))\le 0\qquad\forall x\in C$$

f **is said to be** $\eta_1\eta_2-$**qusiconvex on C if there exists a numerical function** $\eta_2(x,\bar{x})$ **defined on** $C\mathrm{X}C$ **such that**

$$f(x_1)\le f(x_2)\Rightarrow\eta_2(x_1,x_2)df^+(x_2,\eta_1(x_1,x_2))\le 0\qquad\forall x_1,x_2\in C$$

The function f **is called strictly** $\eta_1\eta_2-$**qusiconvex when**

$$f(x_1)\underset{x_1\ne x_2}{<}f(x_2)\Rightarrow\eta_2(x_1,x_2)df^+(x_2,\eta_1(x_1,x_2))<0\qquad\forall x_1,x_2\in C$$

Definition2.10: An η_1-**semidifferentiable numerical function** f **defined on an** η_1-**locally starshaped set** $C\subset R^n$ **is said to be** $\eta_1\eta_2-$**pesudoconvex at** $\bar{x}\in C$ **if there exists a numerical function** $\eta_2(x,\bar{x})$ **defined on** $C\mathrm{X}C$ **such that**

$$\eta_2(x,\bar{x})df^+(\bar{x},\eta_1(x,\bar{x}))\ge 0\Rightarrow f(x)\ge f(\bar{x}),\qquad\forall x\in C.$$

also f **is** $\eta_1\eta_2-$**pseudo convex on C if there exists a numerical function** $\eta_2(x_1,x_2)$ **defined on** $C\mathrm{X}C$ **such that**

$$\eta_2(x_1,x_2)df^+(x_2,\eta_1(x_1,x_2))\ge 0\Rightarrow f(x_1)\ge f(x_2),\qquad\forall x_1,x_2\in C.$$

Remark: Every $\eta_1\eta_2-$**convex function is** $\eta_1\eta_2-$**qusiconvex for the same function** $\eta_1\eta_2-$**respectively but the converse is not true**

from the definition of $\eta_1\eta_2-$**convexity we find that**

$$f(x_1)-f(x_2)\ge\eta_2(x_1,x_2)df^+(x_2,\eta_1(x_1,x_2))\qquad\forall x_1,x_2\in C$$

Therefore

$$f(x_1)-f(x_2)\le 0\Rightarrow\eta_2(x_1,x_2)df^+(x_2,\eta_1(x_1,x_2))\le 0\qquad\forall x_1,x_2\in C$$

This shows that $\eta_1\eta_2-$**convex is** $\eta_1\eta_2-$**qusiconvex.**

The following example shows that , the converse is not always true
Example 2.2: Consider the function $f:[1,4[\to R$ **defined as follows**

$$f(x)=\begin{cases} x^3 & 1\le x<2 \\ 2x^2 & 2\le x<4 \end{cases}$$

Clearly this function is not differentiable at x=2.the computation of the η_1- **semiddefential of the function take** $\eta_1(x_1,x_2)=x_1-x_2$ **yields**

$$df^+(x_2,x_1-x_2)=\begin{cases} 3(x_1-x_2)x_2^2 & 1\le x_1<2, 1\le x_2<2 \\ 4(x_1-x_2)x_2 & 2<x_1<4, 2<x_2<4 \\ 12(x_1-2) & 1\le x_1<2, x_2=2 \\ 8(x_1-2) & 2\le x_1<4, x_2=2 \end{cases}$$

Let us choose $\eta_2(x_1,x_2)=x_1/2$ **, then it can be easily seen that in all the above ranges of** x_1,x_2 **we have**

$$f(x_1)-f(x_2)\le 0 \Rightarrow \eta_2(x_1,x_2)df^+(x_2,\eta_1(x_1,x_2))\le 0 \qquad \forall x_1,x_2\in C$$

Hence the function is $\eta_1\eta_2-$**quasiconvex .but this function is not** $\eta_1\eta_2-$**convex because at** $x_1=\frac{5}{4}, x_2=\frac{3}{2}$ **the inequality**

$$f(x_1)-f(x_2)\ge \eta_2(x_1,x_2)df^+(x_2,\eta_1(x_1,x_2))$$

doesn't hold

3. Theorem 3.1:Every η_1-**semidifferentiable numerical function** f **defined on an** η_1-**locally starshaped set** $C\subseteq R^n$ **is** $\eta_1\eta_2-$**convex iff**

$$x_1,x_2\in C \text{ and } df^+(x_2,\eta_1(x_1,x_2))=0 \Rightarrow f(x_1)\ge f(x_2)$$

Proof: let f **is** $\eta_1\eta_2-$**convex.**

Therefore there exists a numerical function $\eta_2(x_1,x_2)$ **such that**

$$f(x_1)-f(x_2)\ge \eta_2(x_1,x_2)df^+(x_2,\eta_1(x_1,x_2)) \qquad \forall x_1,x_2\in C$$

It follows from this inequality that

$$f(x_1)-f(x_2)\ge 0 \Rightarrow f(x_1)\ge f(x_2) \qquad \forall x_1,x_2\in C$$

Conversely: we have to show that

$$f(x_1)-f(x_2)\ge \eta_2(x_1,x_2)df^+(x_2,\eta_1(x_1,x_2)) \qquad \forall x_1,x_2\in C$$

If $df^+(x_2,\eta_1(x_1,x_2))=0$ **then the inequality is satisfied, we take** $\eta_2(x_1,x_2)$ **any function**

If $df^+(x_2,\eta_1(x_1,x_2))\ne 0$ **then choose** $\eta_2(x_1,x_2)=\dfrac{f(x_1)-f(x_2)}{df^+(x_2,\eta_1(x_1,x_2))}$ **and inequality is again verified**

Hence f **is** $\eta_1\eta_2-$**convex**

Theorem3.2: Let f **be a numerical function defined on an** η_1-**locally starshaped set** $C\subseteq R^n$ **and** η_1- **semidifferentiable at** $\bar{x}\in C$ **.suppose there exists**

a positive numerical function $\eta_2(x,\bar{x})$ **defined on** CXC **and maximum positive number** $a_{\eta_1}(x,\bar{x})$ **and** $d_{\eta_1}(x,\bar{x})$ **such that**

$$\bar{x}+\lambda\eta_2(x,\bar{x})\eta_1(x,\bar{x})\in C \qquad 0<\lambda<a_{\eta_1}(x,\bar{x})$$

and

$$f(\bar{x}+\lambda\eta_2(x,\bar{x})\eta_1(x,\bar{x}))\le(1-\lambda)f(\bar{x})+\lambda f(x) \qquad \forall x\in C, 0<\lambda<d_{\eta_1}(x,\bar{x})$$

where

$$a_{\eta_1}(x,\bar{x})\le 1 \text{ and } d_{\eta_1}(x,\bar{x})\le a_{\eta_1}(x,\bar{x})$$

Then f **is** $\eta_1\eta_2-$**convex at** $\bar{x}$

Proof: We have

$$f(\bar{x}+\lambda\eta_2(x,\bar{x})\eta_1(x,\bar{x}))\le(1-\lambda)f(\bar{x})+\lambda f(x) \qquad \forall x\in C, 0<\lambda<d_{\eta_1}(x,\bar{x})$$

Therefore

$$\frac{[f(\bar{x}+\lambda\eta_2(x,\bar{x})\eta_1(x,\bar{x}))-f(\bar{x})]}{\lambda\eta_2(x,\bar{x})}\mathrm{X}\eta_2(x,\bar{x})\le f(\bar{x})-f(x) \qquad \forall x\in C, 0<\lambda<d_{\eta_1}(x,\bar{x})$$

Taking the limit as $\lambda\to 0^+$ **, immediately yields the inequality**

$$\eta_2(x,\bar{x})df^+(\bar{x},\eta_1(x,\bar{x}))\le f(x)-f(\bar{x}) \qquad \forall x\in C$$

Hence f **is** $\eta_1\eta_2-$**convex**

Theorem3.3: Let f **be a numerical function defined on an** η_1-**locally starshaped set** $C\subseteq R^n$ **and** η_1- **semidifferentiable at** $\bar{x}\in C$ **.suppose there exists a positive numerical function** $\eta_2(x,\bar{x})$ **defined on** CXC **, maximum positive number** $a_{\eta_1}(x,\bar{x})$ **and** $d_{\eta_1}(x,\bar{x})$ **such that**

$$\bar{x}+\lambda\eta_2(x,\bar{x})\eta_1(x,\bar{x})\in C \qquad 0<\lambda<a_{\eta_1}(x,\bar{x})$$

and

$$f(x)\le f(\bar{x})\Rightarrow f(\bar{x}+\lambda\eta_2(x,\bar{x})\eta_1(x,\bar{x}))\le f(\bar{x}) \qquad \forall x\in C, 0<\lambda<d_{\eta_1}(x,\bar{x})$$

where

$$a_{\eta_1}(x,\bar{x})\le 1 \text{ and } d_{\eta_1}(x,\bar{x})\le a_{\eta_1}(x,\bar{x})$$

Then f **is** $\eta_1\eta_2-$**qusiconvex at** $\bar{x}$

Proof: From the assumption

$$f(x)\le f(\bar{x})\Rightarrow f(\bar{x}+\lambda\eta_2(x,\bar{x})\eta_1(x,\bar{x}))\le f(\bar{x}) \qquad \forall x\in C, 0<\lambda<d_{\eta_1}(x,\bar{x})$$

we get

$$f(\bar{x}+\lambda\eta_2(x,\bar{x})\eta_1(x,\bar{x}))-f(\bar{x})\le 0 \qquad \forall x\in C, 0<\lambda<d_{\eta_1}(x,\bar{x})$$

That is

$$\frac{[f(\bar{x}+\lambda\eta_2(x,\bar{x})\eta_1(x,\bar{x}))-f(\bar{x})]}{\lambda\eta_2(x,\bar{x})}\mathrm{X}\eta_2(x,\bar{x})\le 0 \qquad \forall x\in C, 0<\lambda<d_{\eta_1}(x,\bar{x})$$

Taking the limit as $\lambda\to 0^+$ **, we get**

$$\eta_2(x,\bar{x})df^+(\bar{x},\eta_1(x,\bar{x})) \le 0 \quad , \quad \forall x \in C$$

Hence the function f **is** $\eta_1\eta_2-$**qusiconvex at** $\bar{x}$ **.**

4. Sufficient Optimality Criterion

Consider the nonlinear programming problem

(P) **Min** $f(x)$

Subject to $g(x) \le 0$

$x \in C$

where f **and** g **are** η_1-**semidifferentiable numerical and m-dimensional vector function respectively defined on** η_1-**locally starshaped set** $C \subseteq R^n$

Let $X = \{x \in C, g(x) \le 0\}$ **be the set of all feasible solution of (P).**

Theorem4.1: Let $\bar{x} \in C$ **and let** f **and** g **be** $\eta_1\eta_2-$**convex at** $\bar{x}$ **with respect to the same functions** $\eta_1\eta_2-$**respectively .We assume that at** $\bar{x}, f, g$ **are** η_1- **semidifferentiable .If there exist** $u_0^* \in R$ **and** $u^* \in R^m$ **such that** $(\bar{x}, u_0^*, u^*)$ **satisfy the following conditions**

$$u_0^*\eta_2(x,\bar{x})df^+(\bar{x},\eta_1(x,\bar{x})) + \eta_2(x,\bar{x})u^* dg^+(\bar{x},\eta_1(x,\bar{x})) \ge 0 \qquad \forall x \in X \qquad \textbf{(4.1)}$$

$$g(\bar{x}) \le 0 \qquad \textbf{(4.2)}$$

$$u^{*'}g(\bar{x}) = 0 \qquad \textbf{(4.3)}$$

$$(u_0^*, u^*) \ge 0 \qquad \textbf{(4.4)}$$

$$u_0^* > 0 \qquad \textbf{(4.5)}$$

Then $\bar{x}$ **is an optimal solution of (P).**

Proof: Since f **is** $\eta_1\eta_2-$**convex at** $\bar{x}$ **, therefore for any** $x \in X$

$$f(x) - f(\bar{x}) \ge \eta_2(x,\bar{x})df^+(\bar{x},\eta_1(x,\bar{x}))$$

$$\ge \frac{-\eta_2(x,\bar{x})u^{*'}dg^+(\bar{x},\eta_1(x,\bar{x}))}{u_0^*} \qquad \textbf{by(4.1)\&(4.5)}$$

$$\ge \frac{u^{*'}}{u_0^*}[g(\bar{x}) - g(x)]$$

$$= \frac{-u^{*'}}{u_0^*}g(x) \qquad \textbf{(4.6)}$$

Making use of (4.4) and the fact that $x \in X$ **in (4.6) yields the inequality**

$$f(x) \ge f(\bar{x}) \qquad \textbf{(4.7)}$$

(4.2) shows that $\bar{x}$ **is feasible for the problem (P), therefore it follows from (4.7) that** $\bar{x}$ **is indeed optimal**

5.Conclusion

Sufficient Optimality Criteria for Nonlinear programming problems in which the object function and constraint are not differentiable but η-semidifferentiable are given .some properties of this class are studied

References

[1]-**M.A.Hanson "*On Sufficiency of The Kuhn-Tucker Conditions*"** Journal of Mathematical Analysis and Applications**, vol.236 (1999), pp.594-640**.

[2]-**R.N.Kaul and Vinod Lyall," *Nonconvex and Semidifferentiable Functions*",** Indian Journal of Pure and Applied Mathematics**, Vol. 19(1988), pp.1167-1179**

[3]-**V.Preda, I.M.Stancu Minasain andA.Batatorescu," *Optimality and Duality in Nonlinear Programming Involving Semilocally Preinvex and Related Functions*"** Journal Information & Optimization Sciences**, Vol. 17(1996), pp.585-596.**

[4]-**I.M.Stancu Minasain," *Optimality Conditions and Duality in Fractional Programming Involving Semi locally Preinvex and Related Functions*",** Journal of Information & Optimization Sciences**, Vol.23 (2002), pp.185-201.**