On Some Types Of Sc-continuous Functions

And

Sc-connected Space

By

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حول بعض انواع الدوال Sc-المستمرة و

الفضاءات Sc-المتصلة

مقدم من قبل

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Abstract :

The main aim of this work is to study some new classes of Sc- Continuous function which are(S-sc- Continuous, S^*-sc- Continuous and S^{**}-sc- Continuous) function and discussion the relation between these functions. As well as several properties of these functions are proved. Also, we introduce and study new class of Connected spaces called Sc- Connected spaces.

المستخلص:

الهدف الرئيسي من هذا العمل هو دراسة صف جديد من الدوال Sc- المستمرة تدعى الدوال (S-sc-المستمرة, S^*-sc- المستمرة و S^{**}-sc- المستمرة) وناقشنا العلاقة فيما بينها. وايضاً بعض صفات تلك الدوال برهنت .

وكذلك قدمنا ودرسنا صف جديد من الفضاءات المتصلة تدعى Sc– الفضاءات المتصلة.

***1- Introduction*:**

In 1963[5], Levin defined a set A of space X will be termed semi-open in topology and use these sets to study semi-continuity in topological space.

Joseph and Kwack[4] introduce the Concept of θ-semi open and using semi-open sets to improve the notation of S-closed space. Alias and Zanyar[1] introduce a new class of semi-open sets called sc-open sets, this class of sets lies strictly between the class of θ-semi open sets and semi-open sets also study its fundamental properties and compare it with some other types of sets.

In this paper, we introduce some types of Sc- Continuous functions namely [S-sc- Continuous functions, S^{*}-sc- Continuous functions, S^{**}-sc- Continuous functions] in topological spaces and study some of their properties. Also we study new class of connected spaces called Sc-connected spaces.

Throughout this paper (X, τ), (Y, σ) and (Z ,μ) (or simply X,Y and Z) represent non – empty topological spaces on which no separation axioms are assumed unless otherwise mentioned .For a sub sets ***A*** of a spaces *X*. ***cL(A)*** and **int *(A)*** denote the closure of *A* and the interior of *A* respectively.

*2- **Preliminaries***:

Some definition and basic concepts have been given in this section.

Definition(2-1): [5]

A subset ***A*** of a topological space(X,τ) is said to be ***semi-open*** if

$A \subseteq c\,\mathrm{L}(int(A))$ and denoted by **s-open.**

Remark (2-2): [5]

In any topological space. It is clear that every open set is s-open, but the converse is not true in general. To illustrate that consider the following example.

Example (2-1)

Consider X ={a,b,c} with the topology

$\tau = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$ and let A={a,c}, then A is s-open but not open set in a space X .

Remark (2-3): [5],[2]

In any topological space (X,τ)

1- The class of all s-open subsets of X. is denoted by ***SO(X).***

2- The union of any collection of s-open subsets of X is s-open in X

3- The intersection of two s-open sets in X is not necessary to be s-open.

Definition (2-4): [1]

A subset A of a space X is called ***sc-open*** if $A\in$ SO(X) and for each $x \in A$, there exists a closed set F such that $x \in F \subseteq A$. The class of all sc-open subsets of a topological space(X,τ) is denoted by ***SCO(X)***.

Proposition(2-5): [1]

A subset ***A*** of a space X is sc-open if and only if ***A*** is semi-open and it is the union of closed sets, i.e, $A = \cup F_\alpha$ where ***A*** is semi-open and F_α is closed for each α.

Remark (2-6): [1]

Every sc-open subsets of a space X is semi-open, but the converse is not true in general as shown in the following example.

Example (2-2)

Consider X ={a,b,c} with the topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$,

Then the family of closed sets are: $\emptyset, X, \{b,c\}, \{a,c\}, \{c\}$, we can find easily the following families:

SO(X)= $\{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}\}$,

SCO(X)= $\{X, \emptyset, \{a,c\}, \{b,c\}\}$. Then $\{a\}$ is semi-open but {a} is not sc-open set in a topology

The next example shows that an sc-open set need not be closed.

Example (2-3) :

Consider the space R with the usual topology. Now A=(0,1] such that

$A = (0,1] = \cup_{n=1}^{\infty} \left[\frac{1}{n}, 1\right]$, then A is sc- open set, but it is not closed.

Remark (2-7): [1]

1. The union of any collection of sc-open sets in a topological space (X,τ) is sc-open.
2. The intersection of two sc-open sets in X is not necessarily sc-open. The following example shows that:

Example (2-4):

Consider the space an in ***Example(2-2),*** there $\{a,b\} \in SCO(X)$ and $\{b,c\} \in SCO(X)$, but $\{a,b\} \cap \{b,c\} = \{c\} \notin SCO(X)$.

Proposition(2-8): [1]

If the family of all semi-open sets of a space X is a topology on X, then the family of sc-open is also is a topology on X.

Definition (2-9): [8]

A topological space X is said to be ***extremaly disconnected*** if

cL(G) is open for every open set G of X.

Proposition(2-10): [1]

Let *(X ,τ) be a topological space .If* X *is* extremally disconnected ,then SCO(X) forms a topology on X.

Definition (2-11): [3]

A space X is called ***locally indiscrete*** if every open subset of X is closed.

Definition (2-12): [9]

A topological space X is ***T_1-space*** iff there exist open sets G and H such that $a \in G$, $b \notin G$ *and* $b \in H$, $a \notin H$ .Then open sets G and H are not necessarily disjoint.

Proposition(2-13): [1]

If a space X is T_1-space, then $SO(X) = SCO(X)$.

Proposition(2-14): [1]

If a topological space (X,τ) is locally indiscrete, then $SO(X) = SCO(X)$.

Remark (2-15): [1] Since every open set is semi-open, it follows that if a topological space (X,τ) is T_1-space or locally indiscrete, then $\tau \subseteq SCO(X)$.

Theorem (2-16): [1],[2]

Let (X, τ) be a topological space.

1- If $A \epsilon \tau$ and $B \in SO(X)$, then $A \cap B \in SO(X)$.
2- If $A \in SCO(X)$ and B is clopen , then $A \cap B \in SCO(X)$.

Theorem (2-17): [1],[5][7]

let (Y, τ_Y) be a subspace of a space(X, τ).

1. If A is a closed subset in X and $A \subseteq Y$,then A is closed in Y.
2. If $A \in SO(X, \tau)$ and $A \subseteq Y$,then $A \in SO(Y, \tau_Y)$.
3. If $A \in SCO(X, \tau_X)$ and $A \subseteq Y$,then $A \in SCO(Y, \tau_Y)$.

Corollary(2-18): [1]

Let A and Y be any subsets of a space X If $A \in SCO(X)$ and Y is clopen subset of X, then

$A \cap Y \in SCO\ (Y)$.

Proof:

Follows from theorem(2-16)-step2- and theorem(2-17)-step3-.

Definition (2-19): [1][5]

A function $f: X \to Y$ is called

1- **S-continuous(semi- continuous**) if the inverse image of every open set in Y is a s-open in X.
2- ***Sc- continuous*** if the inverse image of every open set in Y is an sc-open in X.

Proposition(2-20): [1]

Every Sc- continuous function is S-continuous.

Definition (2-21): [6]

A space X is said to X be ***S-disconnected space*** if there exists two non-empty s-open sets A,B in X satisfy:

1- $X = A \cup B$.
2- $A \cap B = \emptyset$.

Definition (2-22):[6]

A space X is said to be ***S-connected Space*** if X is not S-disconnected.

Example (2-6):

1- Le $X = \{a, b, c\}$ with topology $\tau = \{X, \emptyset, \{a\}\}$ X is S-connected since X $\{a\}$ are the only open sets which are not disjoint.

2- Let $X = \{a, b, c\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$, X is S-disconnected since $\{a\}$ and $\{b, c\}$ are disjoint s-open sets in X such that $X = \{a\} \cup \{b, c\}$

3- Sc- continuous function types:

In this section, we introduce a new class of Sc- Continuous functions namely [S- sc-Continuous functions, S^*-sc- Continuous functions and S^{**}-sc- Continuous functions] and studying the relations between them. Also , several properties of these functions are proved.

Definition (3-1):

A function $f: X \to Y$ from a topological space X into a topological space Y is said to be ***S- sc-Continuous*** if the inverse image of every sc-open set in Y is s-open set in X .

Definition (3-2):

A function $f: X \to Y$ from a topological space X into a topological space Y is said to *be* ***S^*- sc-Continuous*** if the inverse image of every s-open set in Y is sc-open set in X.

Proposition(3-3):

Every S^*- sc- Continuous functions is S- sc- Continuous.

Proof:

Let $f: (X, \tau) \to (Y, \sigma)$ be a S^*- sc- Continuous function, and G be an sc-open set in Y, then by Remark(2-6) we get G is an s-open set in Y. Thus , $f^{-1}(G)$ is an sc-open set in X and by using Remark(2-6) we have $f^{-1}(G)$ is an s-open set in Y. Hence, $f: X \to Y$ is a S-sc- Continuous function.

But the converse is not true in general, as the following example show:

Example (3-1):

let $X = \{a, b, c\}$ with the topology $\tau = \{X, \emptyset\{a\}, \{a, b\}\}$. Then

$SO(X) = \{X, \emptyset\{a\}, \{a, b\}\{a, c\}\}$,$SCO\ (X) = \{X, \emptyset\}$.

And let $Y = \{d, e, f\}$ with the topology $\sigma = \{Y, \emptyset, \{d\}, \{e\}, \{d, e\}\}$. Then

$SO(Y) = \{Y, \emptyset\{d\}, \{e\}, \{d, e\}, \{d, f\}, \{e, f\}\}$

$SCO\ (Y) = \{Y, \emptyset, \{d, e\}, \{e, f\}\}$.

Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = f(b) = f$ and

$f(c) = d$ It is observe that f is S- sc- Continuous function but not S*- sc- Continuous, since for the s-open set $G = \{d\}$ in y but $f^{-1}(G) = f^{-1}(\{d\}) = \{c\}$ is not Sc-open set in X .

Proposition(3-4):

Every S*- sc- Continuous functions is Sc- Continuous.

Proof:

Let $f: (X, \tau) \to (Y, \sigma)$ be S*- sc- Continuous function, and let G be an open set in Y. then by using Remark(2-2) we get G is an s-open set in Y. Thus, $f^{-1}(G)$ is an sc-open set in X .Hence, a function $f: X \to Y$ isa Sc- Continuous.

The following example shows the converse is not necessarily true:

Example (3-2):

let $X = \{a, b, c\}$ with the topology $\tau = \{X, \emptyset\{a\}, \{a, b\}\}$.Then

$SO(X) = \{X, \emptyset\{a\}, \{b\}\{a, b\}\}, \{a, c\}, \{b, c\}\}$ and

$SCO\ (X) = \{X, \emptyset, \{a, c\}\{b, c\}\}$.

Define $f: (X, \tau) \to (Y, \tau)$ by $f(a) = f(c) = b$ and

$f(b) = c$ It is easily seen that f is Sc- Continuous function but not

S*- sc- Continuous, since for the s-open set $G = \{a, c\}$ in X but

$f^{-1}(G) = f^{-1}(\{a, c\}) = \{b\}$ is not sc-open set in X.

Corollary(3-5): Every S*- sc- Continuous function is S- Continuous

Proof: Follows from proposition(3-4) and proposition(2-20).

*But the converse is not necessarily true in general. It is easily seen that in **Example(3-2).** f is S-Continuous but is not S^*- sc- Continuous.*

Here in the following propositions addition the necessarily condition in order to the converse of proposition(3-3) is true:

Proposition(3-6):

If $f: X \to Y$ is a S-sc- Continuous function and X, Y are T_1-space, then f is S^*- sc- Continuous.

Proof :

Let G be an s-open set in Y and since Y is T_1-space, then by using proposition(2-13) we get G is sc-open set in Y. Thus, $f^{-1}(G)$is an s-open set in X and since X is T_1-space and by proposition(2-13) we have $f^{-1}(G)$is an sc-open set in X .Hence, A function $f: X \to Y$ is a S^*-sc- Continuous.

Remark (3-7):

If $f: X \to Y$ be a S-sc- Continuous function, then f is not necessarily Sc- Continuous. It is easily see that in ***Example (3-1)*** f is S-sc- Continuous, but not sc- Continuous, since for the open set $G = \{d\}$ in y but $f^{-1}(G) = f^{-1}(\{d\}) = \{c\}$ is not sc-open set in X.

The following proposition given the necessarily condition to make Remark(3-7) is true:

Proposition(3-8):

If $f: X \to Y$ be a S-sc- Continuous function and X , Y are T_1-space, then f *is* sc- Continuous.

Proof:

By proposition(3-6) we get $f: X \to Y$ is a S^*- sc- Continuous function, and by using proposition(3-4) we have a function $f: X \to Y$ is a Sc- Continuous.

Remark (3-9) :

If $f: X \to Y$ is a Sc- Continuous function then, f is not necessarily S-sc- Continuous.

It is easily seen that from the following example:

Example (3-3): Let $X = \{a, b\}$ with the topology $\tau = \{X, \emptyset, \{a\}, \{b\}\}$. Then

$SO(X) = SCO(X) = \{X, \emptyset, \{a\}, \{b\}\}$.Let $Y = \{a, b, c\}$ with the topology

$\sigma = \{Y, \emptyset\{a\}, \{b\}\{a, b\}\}$.$SO(Y) = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$,and

$SCO\ (Y) = \{Y, \emptyset, \{a,c\}, \{b,c\}\}$. Define $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = a, f(b) = b$ and $f(c) = c$. It is easily see that f is Sc-Continuous function. But is not S-sc- Continuous, since for the sc-open set $G = \{a,c\}$ in (Y,σ) but $f^{-1}(G) = f^{-1}(\{a,c\}) = \{a,c\}$ is not s-open set in (X,τ).

Now, we give another type of S-sc- Continuous function is called S^{**}-sc- Continuous.

Definition (3-10):

A function $f: X \to Y$ from a topological space X in to a topological space Y is said to be ***S^{**}-sc-Continuous*** if the inverse image of every sc-open set in Y is sc-open in X.

Proposition(3-11):

Every S^{**}-sc- Continuous function is S-sc- Continuous.

Proof:

Let $f: X \to Y$ be a S^{**}-sc- Continuous function and let G be an sc-open set in Y. Thus, $f^{-1}(G)$ is an sc-open set in X. And by using Remark(2-6) we get $f^{-1}(G)$ is an s-open set in X.Hence, $f: X \to Y$ is a S-sc- Continuous function.

But the converse of proposition (3-11) is not necessarily true. *It is easily see that in Example(3-1) .Since for the sc-open set $G = \{e,f\}$ in Y. But $f^{-1}(G) = f^{-1}(\{e,f\}) = \{a,b\}$* is not sc-open set in X.

Remark (3-12)

If $f: X \to Y$ be a S^{**}-sc- Continuous function, then f is not necessarily S–Continuous. It is easily seen that in ***Example(3-2)*** *, f* is S–Continuous but is not S**- sc- Continuous .

Here is the following propositions addition the necessarily condition in order to the converse of proposition(3-11) and Remark(3-12) are true.

Proposition(3-13):

If $f: X \to Y$ is a S-sc- Continuous function and X is T_1-space,then f is a S^{**}-sc- Continuous.

Proof:

Let G be an sc-open set in Y. Thus, $f^{-1}(G)$ is an s-open set in X. Since X is T_1-space, then by using proposition(2-13) we get $f^{-1}(G)$ is an sc-open set in X.Hence, a function $f: X \to Y$ is a S^{**}-sc- Continuous

Proposition(3-14):

If $f: X \to Y$ is a S^{**}-sc- Continuous function and Y is T_1-space, then f is S–Continuous.

Proof:

Let G be an open set in Y. Since Y is T_1-space and by Remark (2-15) we get G is an sc-open set in Y. Thus , $f^{-1}(G)$ is an sc-open set in X and by using Remark(2-2) we get $f^{-1}(G)$ is an s-open set in X.Hence, $f: X \to Y$ is a S- Continuous function.

Now the following proposition show the relation between S^{**}-sc- Continuous function and S^{*}-sc- Continuous function.

Proposition(3-15):

Every S^{*}-sc- Continuous function is S^{**}-sc- Continuous.

Proof:

Let $f: X \to Y$ be a S^{*}-sc- Continuous, and let G be an sc-open set in Y. then by using Remark(2-2) we get G is an s-open set in Y .Thus, $f^{-1}(G)$ is an sc-open set in X.Hence, a function $f: X \to Y$s a S^{**}-sc- Continuous.

The converse of proposition(3-15) need not be true as seen from the following example:

Example (3-4):

Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \emptyset, \{a\}, \{b\}\{a, b\}\}$. And let $Y = \{d, e, f\}$ .With the topology $\sigma = \{Y, \emptyset\{d\}, \{d, e\}\}$.Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = e,$ $f(b) = d$ and $f(c) = f$.

It is seen that f is S^{**}-sc- Continuous function, but is not S^{*}-sc- Continuous. Since for the s-open set $G = \{d\}$ in Y but $f^{-1}(G) = f^{-1}(\{d\}) = \{b\}$ is not sc-open set in X

Now, the following proposition addition the necessarily condition in order to the converse of proposition(3-15) is true.

Proposition(3-16):

If $f: X \to Y$ is a S^{**}-sc- Continuous function and Y is T_1-space, then f is S^{*}-sc- Continuous.

Proof:

Let G be an s-open set in Y. since Y is an T_1-space and by using proposition(2-13) we get G is an sc-open set in Y. Thus, $f^{-1}(G)$ is an sc-open set in$X$ .Hence , a function $f: X \to Y$ is a S^{*}-sc- Continuous.

Remark (3-17)

1- If $f: X \to Y$ is a Sc- Continuous function, then f is not necessarily S^{**}-sc- Continuous. We can see that from ***Example(3-2).***
2- If $f: X \to Y$ is a S^{**}-sc- Continuous, then f is not necessarily Sc- Continuous. It is easily see that in ***Example(3-4).***

Here, in the following proposition given the necessarily condition to make every S^{**}-sc- Continuous function is Sc- Continuous.

Proposition(3-18):

If $f: X \to Y$ is a S^{**}-sc- Continuous function and Y is T_1-space or locally indiscrete, then f is Sc- Continuous.

Proof:

Let G be an open set in Y since Y is T_1-space or locally indiscrete and using Remark(2-15) we get G is an sc-open set in Y. Thus, $f^{-1}(G)$ is an sc-open set in X.Hence, a function $f: X \to Y$ is a Sc- Continuous.

Here, in the following diagram. It shows the relation between the S-sc- Continuous function types (without using condition), where the converse is not necessarily true.

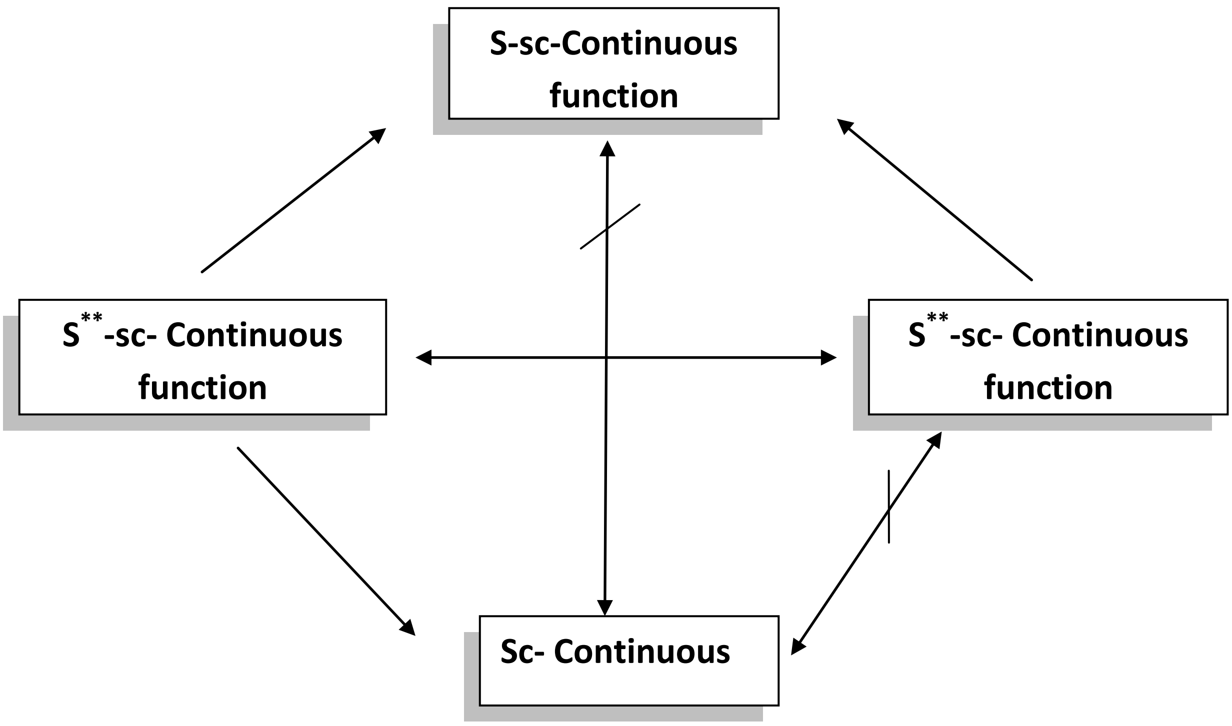

Diagram (1)

Summarized The Relation Between S-sc- Continuous Function Types

Now we give the composition of some types of S-sc- Continuous function.

Proposition(3-19):

If a function $f: X \to Y$ is S-Sc- Continuous and function.

1. $g: Y \to Z$ is S**-sc- Continuous, then the composition $gof: X \to Z$ is S-sc- Continuous.
2. $g: Y \to Z$ is S*-sc- Continuous, then the composition $gof: X \to Z$ is S-sc- Continuous.

Proof:

(1) Let G be an sc-open set in Z. Thus, $g^{-1}(G)$ is an sc-open set in Y, since f is S-sc-Continuous function. Then $f^{-1}(g^{-1}(G))$ is an s-open set in X.

But $f^{-1}(g^{-1}(G)) = (g0f)^{-1}(G)$.Then, $(g0f)^{-1}(G)$ is an s-open set in X.Hence $g0f: X \to Z$ is S-sc- Continuous function.

(2) Since $g: Y \to Z$ is a S*-sc- Continuous function . Then by proposition(3-15) we get $g: Y \to Z$ is a S**-sc- Continuous and by proposition(3-19) step-1- we obtain $g0f: X \to Z$ is S-sc- Continuous function.

Proposition(3-20):

If a function $f: X \to Y$ is S**-sc- Continuous and a function $g: Y \to Z$ is S-sc- Continuous .Then the composition $g0\,f : X \to Z \ is\ Sc -$ Continuous function.

Proof:

Let G be an open set in Z . Thus, $g^{-1}(G)$ is an sc-open set in Y. since $f: X \to Y$ is a S**-sc-Continuous function. Then we get $f^{-1}(g^{-1}(G))$ is an sc-open set in X.

But $f^{-1}(g^{-1}(G)) = (g0f)^{-1}(G)$.Then, $(g0f)^{-1}(G)$ is an sc-open set in X .

Hence, $g0f: X \to Z$ is a Sc- Continuous function.

Proposition(3-21):

If a function $f: X \to Y$ and $g: Y \to Z$ are S**-sc- Continuous, then $g0\,f : X \to Z\ is$ S**-sc-Continuous (S-sc- Continuous) respectively.

Proof:

Let G be an sc-open set in Z .Thus, $g^{-1}(G)$ is an sc-open set in Y. Then . Then we get $f^{-1}(g^{-1}(G))$ is an sc-open set in X. But $f^{-1}(g^{-1}(G)) = (g0f)^{-1}(G)$.Then, $(g0f)^{-1}(G)$ is an sc-open set in X . Hence, $g0f: X \to Z$ is S**-sc- Continuous function .by proposition(3-11) we get $g0f: X \to Z$ also S-sc- Continuous function.

Corollary(3-22):

Let $X_1, X_2, X_3, \ldots, X_n, X_{n+1}$ be a topological spaces and if $f_1: X_1 \to X_2$, $f_2: X_2 \to X_3$, ……, $f_n: X_n \to X_{n+1}$ are S** - sc –continuous function . Then $f_n 0 f_{n-1} 0 \ldots\ldots 0 f_1 : X_1 \to X_{n+1}$ is S^{**}-sc- Continuous (S-sc- Continuous) respectively.

Proof :

Let G be a sc-open set in X_{n+1} . [since f_1, f_2, …, f_n are S**-sc-continuous functions] .Then

$f_n^{-1}(G)$ is sc-open *in* X_n .Thus, $f_{n-1}^{-1}(f_n^{-1}(G))$ is sc-open in X_{n-1},

also $f_{n-2}^{-1}(f_{n-1}^{-1}(f_n^{-1}(G))$ is sc-open set in X_{n-2},…. and so on.

Then we have $f_1^{-1}(f_2^{-1}(\ldots.. f_{n-2}^{-1}(f_{n-1}^{-1}(f_n^{-1}(G))\ldots)))$ is an sc-open set in X_1.But

$f_1^{-1}(f_2^{-1}(\ldots.. f_{n-2}^{-1}(f_{n-1}^{-1}(f_n^{-1}(G))\ldots))) = (f_n o f_{n-1} o \ldots\ldots o f_1)^{-1}(G)$.Hence,

$f_n 0 f_{n-1} 0 \ldots\ldots 0 f_1 : X_1 \to X_{n+1}$ is S**-sc-continuous function,

also by proposition (3-11) we obtain $f_n 0 f_{n-1} 0 \ldots\ldots 0 f_1 : X_1 \to X_{n+1}$ is a S-sc-continuous function .

The following corollary it is easy . Thus, we omitted it

Corollary(3-23):

If a function $f: X \to Y$ is S^{**}-sc- Continuous and function $g: Y \to Z$ is a S^{**}-sc-Continuous ,then $g0f: X \to Z$ is a S^{*}-sc-Continuous[$S{**}-sc-$ Continuous and $S-sc-$ Continuous]function respectively.

Proposition(3-24):

If a function $f: X \to Y$ is S^{*}-sc- Continuous and a function.

1) $g: Y \to Z$ is $Sc-$ Continuous, then $g0f: X \to Z$ is Sc- Continuous.
2) $g: Y \to Z$ is S-sc- Continuous, then $g0f: X \to Z$ is S^{**}-sc- Continuous (S-sc- Continuous) function respectively.

Proof:

(1) Let G be an open set in Z. Thus $g^{-1}(G)$ is an sc-open set in Y and by using Remark(2-6) we get $g^{-1}(G)$ is an s-open set in Y. Since $f: X \to Z$ is S*-sc- Continuous. Then , $f^{-1}(g^{-1}(G)) is\ an\ sc-open\ set\ in\ X. But f^{-1}(g^{-1}(G)) = (g0f)^{-1}(G)$.Then, $(g0f)^{-1}(G)$ is an sc-open set in X.Hence, $g0f: X \to Z$ is a Sc- Continuous function.

(2) Let G be an sc-open set in Z. Thus $g^{-1}(G)$ is an sc-open set in Y and since $f: X \to Z$ is S*-sc- Continuous function. Then, $f^{-1}(g^{-1}(G))$ is an sc-open set in X.But $f^{-1}(g^{-1}(G)) = (g0f)^{-1}(G)$.Hence, $g0f: X \to Z$ is a S**-sc- Continuous function, and by using proposition(3-11) we get $g0f: X \to Z$ also S-sc- Continuous function.

Proposition(3-25):

If a function $f: X \to Y$ and $g: Y \to Z$ are S*-sc- Continuous, then the composition $g0f: X \to Z$ is S*-sc- Continuous.

Proof: Let G be an s-open set in X .Thus $g^{-1}(G)$ is an sc-open set in ,and by using Remark (2-6) we get $g^{-1}(G)$ is an s-open set in Y. Then, $f^{-1}(g^{-1}(G))$ is an sc-open set in X.

But $f^{-1}(g^{-1}(G)) = (g0f)^{-1}(G)$.Hence, $g0f: X \to Z$ is S*-sc- Continuous function.

Corollary(3-26):

Let $X_1, X_2, X_3,$ .... , X_n ,X_{n+1} be a topological spaces and if $f_1: X_1 \to X_2$, $f_2: X_2 \to X_3$, ..., $f_n: X_n \to X_{n+1}$ are S* - sc –continuous function . Then $f_n 0 f_{n-1} 0 \dots\dots 0 f_1 : X_1 \to X_{n+1}$ is

S*-sc- Continuous .

Proof :Let G be a s-open set in X_{n+1} . [since f_n *is* S*-sc-continuous functions] .Then

f_n^{-1} (G) is sc-open *in* X_n and by Remark(2-6) we have f_n^{-1} (G) is s-open *in* X_n .Since f_{n-1} *is* S*-sc-continuous functions we get $f_{n-1}^{-1}(f_n^{-1}(G))$ is sc-open in X_{n-1}, and by Remark(2-6) we obtain $f_{n-1}^{-1}(f_n^{-1}(G))$ is s-open in X_{n-1}. Also since f_{n-2} *is* S*-sc-continuous functions. Thus,

$f_{n-2}^{-1}(f_{n-1}^{-1}(f_n^{-1}(G))$is sc-open set in X_{n-2}, and by Remark(2-6) we have $f_{n-2}^{-1}(f_{n-1}^{-1}(f_n^{-1}(G))$ is s-open set in X_{n-2}, ….. and so on.

Then we have $f_1^{-1}(f_2^{-1}(\dots\dots f_{n-2}^{-1}(f_{n-1}^{-1}(f_n^{-1}(G))\dots)))$ is an sc-open set in X_1.But

$f_1^{-1}(f_2^{-1}(\dots\dots f_{n-2}^{-1}(f_{n-1}^{-1}(f_n^{-1}(G))\dots))) = (f_n o f_{n-1} o \dots\dots o f_1)^{-1}(G)$.Hence,

$f_n 0 f_{n-1} 0 \dots\dots 0 f_1 : X_1 \to X_{n+1}$ is S*-sc-continuous function.

Remark (3-27)

1. If $f: X \to Y$ is S-sc- Continuous function and $g: Y \to Z$ is Sc- Continuous. Then $g0f: X \to Z$ is not necessarily Sc- Continuous function.
2. If $f: X \to Y$ and $g: Y \to Z$ are S-sc- Continuous function. Then $g0f: X \to Z$ is not necessarily S- sc- Continuous function.
3. If $f: X \to Y$ is S**-sc- Continuous function and $g: Y \to Z$ is S-sc-Continuous .Then $g0f: X \to Z$ is not necessarily S^*-sc- Continuous function.

The following proposition given the necessarily condition to make Remark(3-27) is true:

Proposition(3-28):

If a function $f: X \to Y$ S-sc- Continuous , $g: Y \to Z$ is Sc- Continuous function and X is T_1-space. Then $g0f: X \to Z$ is Sc- Continuous.

***Proof*:**

Let G be an open set in Z .Thus, $g^{-1}(G)$ is an sc-open set in Y Since $f: X \to Y$ is a S-sc- Continuous function. Then we get $f^{-1}(g^{-1}(G))$ is an s-open set in X. Since X is T_1-space. By proposition(2-13) we have $f^{-1}(g^{-1}(G))$ is an sc-open set in X .

But $f^{-1}(g^{-1}(G)) = (g0f)^{-1}(G)$. Hence, a function $g0f: X \to Z$ is a Sc- Continuous.

Proposition(3-29):

If $f: X \to Y$ and $g: Y \to Z$ are S-sc- Continuous function ,and Y is T_1-space .

Then $g0f: X \to Z$ i S- sc- Continuous function

***Proof*:**

Let G be an sc- open set in Z .Thus $g^{-1}(G)$ is an s-open se in Y. Since Y is T_1-space and by proposition (2-13) we get $g^{-1}(G)\ is\ an\ sc-open\ set\ in\ Y\ .Thus\ f^{-1}(g^{-1}(G))$ is an s-open set in X . But $f^{-1}(g^{-1}(G)) = (g0f)^{-1}(G)$ Hence, a function $g0f: X \to Z$ is S-sc-Continuous

Similarly, we prove the following proposition:

Proposition(3-30):

If a function $f: X \to Y$ S^{**}-sc- Continuous,$g: Y \to Z$ is S-sc- Continuous function and Y, Z are T_1-space. Then $g0f: X \to Z$ is S^*-sc- Continuous function.

Remark (3-31):

From **[1]** if $f: X \to Y$ be S-sc- Continuous function, and let A is $clopen$ csubset of X Then the restriction $f|A: A \to Y$ is Sc- Continuous in the subspace A.

Now, we give some proposition about the restriction of some S-sc- Continuous functions types.

Proposition(3-32):

Let $f: X \to Y$ be S-sc- Continuous function. If A is an open subset of X. Then $f|A: A \to Y$ is sc- Continuous functions in the subspace A.

Proof:

Let G be an sc-open set of Y. since f is S-sc- Continuous. Then $f^{-1}(G)$ is an s-open set in X. since A is an open set of X. By theorem

(2-16) step -1- , $(f|A)^{-1}(G) = f^{-1}(G) \cap A$ is an s-open subspace of A.. This shows that $f|A: A \to Y$ is a S-sc- Continuous function.

Proposition(3-33):

Let $f: X \to Y$ be S^{**}-sc- Continuous function. If A is clopen subset of X. Then $f\ |A: A \to Y\ \ f$ is a S^{**}-sc- Continuous in the subspace A.

Proof:

Let G be an sc-open set of Y since f is S^{**}-sc- Continuous. Then $f^{-1}(G)$ is an sc-open set in X . since A is clopen subset of X . By theorem (2-16)-step-2-,$(f|A)^{-1}(G) = f^{-1}(G) \cap A$ is an sc-open subspace of A. Hence, $f\ |A: A \to Y$ is a S^{**}-sc- Continuous function.

Corollary(3-34):

Let $f: X \to Y$ be S^{**}-sc- Continuous function. If A is c$|open$ subset of X . Then f is a S-sc- Continuous function in the subspace A.

***Proof*:**

Follows directly from proposition(3-33) and proposition (3-11).

Proposition(3-35):

Let $f: X \to Y$ be a S^{*}-sc- Continuous function. If A is clopen subset of X. Then $f|A: A \to Y$ is a S^{*}-sc- Continuous function in the subspace A.

Proof:

Let G be an s-open set of Y. since f is S*-sc- Continuous. Then $f^{-1}(G)$ is an sc-open set in X. since A is c$|open$ subset of X. By theorem (2-16)-step-2- ,$(f|A)^{-1}(G) = f^{-1}(G) \cap A$ is an sc-open subspace of A. Hence$f\ |A: A \to Y$ f is a S*-sc- Continuous function.

Similarly, we prove the following corollary:

Corollary(3-36):

Let $f: X \to Y$ be S*-sc- Continuous function and A is c$|open$ subset of X. Then $f|A: A \to Y$ is a S**-sc- Continuous (S-sc- Continuous) function in the subspace A respectively.

4- Sc-Connected space:

In this section we introduce a new class of connected space called Sc-connected space and prove some of it is properties.

Definition (4-1):

A space X is ***Sc-disconnected space*** iff there exists two non empty sc-open sets G , H in X satisfy:

1- $X = G \cup H$.
2- $G \cap H = \emptyset$.

Otherwise X is called ***Sc-connected space***.

Proposition(4-2):

Every S- connected space is Sc-connected.

Proof:

Let (X, τ) be a S- connected space, suppose that X is not Sc-connected space .

Then $X = A \cup B\ where\ A$ and B are disjoint nonempty sc-open sets in (X, τ).

By Remark(2-6), A and B are s-open sets in X . Since $X = A \cup B$. Thus X is S- disconnected space [which is contradiction], since X is S- connected space .Hence X is Sc-connected.

The following example shows the converse is not necessarily true.

Example (4-3):

Let $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}\{a, b\}\}$.Then (X, τ) is a Sc- connected space but not S- connected space, because $X = \{a\} \cup \{b, c\}$ and $\{a\} \cap \{b, c\} = \emptyset$.

Remark (4-4)

The following example shows that If X a connected space. Then X is not necessarily

Sc- connected space.

Example (4-5):

Let $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}\{a, b\}\}$. Then (X, τ) is a connected space but not

a Sc- connected space.

From definition(2-21),(2-22) and above discussion , we can get the following diagram it shows the relation these types of spaces.

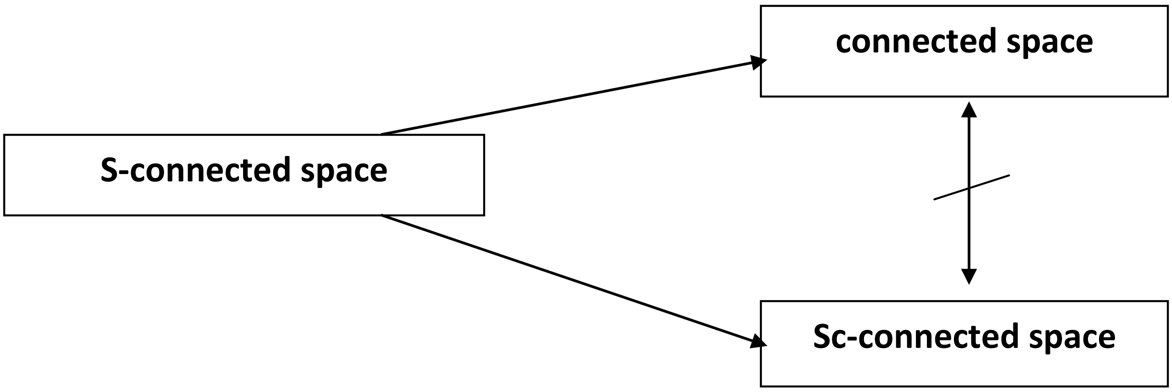

Diagram (2)

Summarized The Relation Between Some Types Of Connected Space

Proposition(4-6):

Let $f: (X, \tau) \to (Y, \sigma)$ is Sc- continuous on to function. If X is Sc- connected space. Then Y is connected.

Proof:

Suppose that Y is disconnected space. Then there exists two disjoint non-empty open sets G and H such that $Y = G \cup H$. Since f is a Sc-continuous on to function. Then, $f^{-1}(G)$, $f^{-1}(H)$ are two sc-open sets in X such that $X = f^{-1}(Y)$

$$= f^{-1}(G \cup H) = f^{-1}(G) \cup f^{-1}(H) \quad \text{and} \quad f^{-1}(G) \cap f^{-1}(H) = \emptyset$$

Thus, X is Sc- disconnected space (which are contradiction), since X is Sc- connected space.Hence, Y must be connected space.

Proposition(4-7):

Let $f\colon (X,\tau) \to (Y,\sigma)$ is a S-sc- Continuous on to function. If X is S-connected space. Then Y is sc-connected space.

Proof:

Suppose that Y is Sc-disconnected space. Then there exists two disjoint non-empty sc- open sets G and H such that $Y = G \cup H$. Since$f$ is a S-sc- continuous on to function .Then $f^{-1}(G)$,$f^{-1}(H)$ are two s-open sets in X such that: $X = f^{-1}(Y) = f^{-1}(G \cup H)$

$$=f^{-1}(G) \cup f^{-1}(H) \text{ and } f^{-1}(G) \cap f^{-1}(H) = \emptyset.$$

Thus, is S-disconnected space (which are contradiction), since X is S- connected space .Hence, Y must be Sc-connected space.

Similarly , we prove the following two proposition.

Proposition(4-8):

Let $f\colon (X,\tau) \to (Y,\sigma)$ is a S^{**}-sc- Continuous on to function. If X is S-connected space. Then Y is Sc-connected space.

Proposition(4-9):

Let $f\colon (X,\tau) \to (Y,\sigma)$ is a S^{*} -sc- Continuous on to function. If X is S-connected space. Then Y is Sc-connected space.

Proposition(4-10):

Let $f\colon (X,\tau) \to (Y,\sigma)$ is a S^{**}-sc- Continuous on to function. If X isSc-connected space. Then Y is Sc-connected space.

Proof:

Suppose that Y is Sc-disconnected space. Then there exists two disjoint non-empty sc-open sets G and H such that $Y = G \cup H$. Since f is

a S^{**}-sc- continuous on to function .Then, $f^{-1}(G)$,$f^{-1}(H)$ are two sc-open sets in X such that: $X = f^{-1}(Y) = f^{-1}(G \cup H)$

$$=f^{-1}(G) \cup f^{-1}(H) \quad \text{and } f^{-1}(G) \cap f^{-1}(H) = \emptyset$$

Thus, X is Sc-disconnected space (which are contradiction), since X is Sc- connected space. Hence, Y must be Sc-connected space.

Proposition(4-11):

Let $f: (X,\tau) \to (Y,\sigma)$ is a S^*-sc- Continuous on to function. If X is Sc-connected space. Then Y is Sc-connected space .

Proof:

Suppose that Y is Sc-disconnected space. Then there exists two disjoint non-empty sc-open sets G and H such that $Y = G \cup H$, and by Remark(2-6) we have G and H are s-open set in Y Since f is a S^{*-}sc- continuous on to function .Then, $f^{-1}(G)$,$f^{-1}(H)$ are two sc-open sets in X such that: $X = f^{-1}(Y) = f^{-1}(G \cup H)$

$$= f^{-1}(G) \cup f^{-1}(H) \quad \text{and } f^{-1}(G) \cap f^{-1}(H) = \emptyset$$

Thus, X is Sc-disconnected space (which are contradiction), since X is Sc- connected space. Hence, Y must be Sc-connected space.

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