Stopping Power for Proton Interacting with Aluminum, Beryllium and Carbon Using Different Formulas

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Abstract

Stopping power for proton interaction with Aluminum, Beryllium and Carbon for energies range from 1 to 12 MeV has been studied. The numerical calculations and analysis of Bethe, Ziegler equations, SRIM and PSTAR software was done using Matlab program. Results of the four methods are presented as tables and figures. We formulate three different power equations expressing the stopping power values in terms of the studied energy range.

Keywords: stopping power, SRIM, PSTAR, Bethe equation, Ziegler equation energy loss

1-Introduction

Fast charged particles passing through matter, ionize the atoms or molecules which are encounter, they lose their energy gradually in many small steps. Stopping power is defined as, the average energy loss of the particle per unit path length (S(E)= - dE/dx). Very often they use mass stopping power which are represent the linear stopping power S(E) per the density of the material in order to expressed its units like MeV/(mg/cm^2), then expressed as the energy loss per unit thickness of the attenuator measured in g/cm^2[1] . Thus the mass stopping power is nearly independent of the density of the material. Stopping power is very important for many parts of basic science, especially for all technological applications and medical treatments [2] .

2-Proton interaction with matter

Protons traversing matter lose energy through successive collisions with the atoms and molecules of the material. With respect to energy loss, the most important interaction is between the proton and the atomic or molecular electrons. The interaction between a proton and an atomic nucleus is called a nuclear reaction it affects the proton flux and eventually the proton trajectory (elastic and inelastic scattering), but the most important parameter characterizing the energy loss of an incident proton is the stopping power [1]. A full description of the proton energy loss process, however, requires more detailed information than is provided by the stopping power alone. The amount of energy transferred from a proton to an atomic electron, as well as the number of interactions that occur per unit path length has a probability distribution. This causes statistical fluctuations in the energy deposition. The most important contribution to proton scattering comes from the electromagnetic interaction with the nucleus. This gives rise to small scattering angles, but since there are a large number of collisions their effect are considerable. If the impact parameter

is small the hadronic interaction contributes to elastic scattering, and in addition inelastic interactions can occur, these can either be an inelastic scattering process during which the incident proton transfers energy to the nucleus (which will then be in an excited state and decay by γ emission) or a nuclear reaction process such as (p,n), (p,d), (p,2p) or (p,3p) where the incident proton will disappear. In case of scattering of protons by very light nuclei, such as protons in hydrogen, also the recoil nucleus can travel a considerable length before its energy is fully deposited [3]. For protons of energy T_p , the stopping power is the same as that of electrons of energy $T_e = T_p /1830$ and of deuterons of energy $T_d = 2T_p$; for an α-particle (z=2) of the same velocity and of energy $T_\alpha = 4T_p$, it is four times greater [4] .As the radiation dose deposited by photons drops off exponentially with penetration depth, the dose deposited by protons increases very slowly for about three-quarters of its range of travel in the medium before increasing sharply, reaching a maximum value before rapidly dropping off to zero. The depth at which the maximum energy is deposited by protons is called the Bragg peak, the peak's position is proportional to the energy of the proton beam [5].

3-Bethe formula

An accelerated beam traversing a target will lose energy. This energy loss is due to scattering at electrons and nuclei, with the electronic stopping most prominent in nearly all energy regions. The energy loss itself depends on the energy or more precisely the velocity of the beam. The energy loss at a given energy is usually expressed in terms of the stopping power, which can be defined as the energy loss per given amount of material. The rate at which a particle loses energy per unit path length is known as the stopping power of the medium. A quantum-mechanical derivation including relativistic effects is known as the Bethe - Bloch formula :[6]

$$-\frac{dE}{dx} = \left(\frac{z\,e^2}{4\pi\varepsilon_0}\right)^2 \frac{4\pi Z\rho N_A}{Amv^2}\left[\ln\left(\frac{2mv^2}{I}\right) - \ln(1-\beta^2) - \beta^2\right] \qquad (1)$$

Where

$v = \beta c$ **is the ion velocity,**

ze = ion charge,

m = electron mass,

I= mean ionization energy,

N_A **=Avogadro's number,**

A, Z, ρ **are the mass number, atomic number, and density of the stopping material, respectively [6]**

4-Ziegler formula

To bridge the gap between the high- and low-energy theories, interpolation formulas of different levels of complexity were proposed by Varelas and Biersack[7] :

With main expressions as:

$$(\boldsymbol{s})^{-1} = (\boldsymbol{S_{LOW}})^{-1} + (\boldsymbol{S_{HIGH}})^{-1} \quad \textbf{(2)}$$

Or

$$\mathbf{S} = \boldsymbol{S_{LOW}S_{HIGH}}/(\boldsymbol{S_{LOW}} + \boldsymbol{S_{HIGH}}) \quad \textbf{(3)}$$

Where S_{LOW}(low Energy stopping), with

$$\boldsymbol{S_{LOW}} = \boldsymbol{A_1 E^{1/2}} \quad \textbf{(4)}$$

And S_{HIGH}(High Energy Stopping), with

$$\boldsymbol{S_{HIGH}} = \frac{\boldsymbol{A_2}}{\boldsymbol{E}} \ln\left(\mathbf{1} + \frac{\boldsymbol{A_3}}{\boldsymbol{E}} + \boldsymbol{EA_4}\right) \quad \textbf{(5)}$$

Here A_1, A_2,$A_3$and A_4 are fitting constants.

The fitting formula eq.(3) asymptotically agrees with eq(1) at high energy. As the Varelas-Biersack formula furthermore is much simpler and has the right asymptotic behavior both at high and low energies, Eq. (3) is used usually for the fitting approximation curves [7].

5-SRIM (Stopping and Range of Ions in Matter)

SRIM is a software package concerning the Stopping and Range of Ions in Matter. Since its introduction in 1985, major upgrades are made about every six years. Currently, more than 700 scientific citations are made to SRIM every year. A recent textbook ‘‘SRIM – The Stopping and Range of Ions in Matter” describes in detail the fundamental physics of the software. Since this time, corrections have been made based on new experimental data. Major changes occur in SRIM about every six years. The last major changes were in 1995 and 1998 and 2003. In 1995 a complete overhaul was made of the stopping of relativistic light ions with energies above 1 MeV/u. In 1998, special attention was made to the Barkas Effect and the theoretical stopping of Li ions. For SRIM-2010, the following major improvements have been made:

(1) About 2800 new experimental stopping powers were added to the database, increasing it to over 28,000 stopping values.

(2) Improved corrections were made for the stopping of ions in compounds.

(3) New heavy ion stopping calculations have led to significant improvements on SRIM stopping accuracy.

(4) A self-contained SRIM module has been included to allow SRIM stopping and range values to be controlled and read by other software applications.

(5) Individual interatomic potentials have been included for all ion/atom

collisions, and these potentials are now included in the SRIM package. The SRIM code simulates the transport of heavy ions of than 2 GeV/u in matter. The models are based on quantum mechanical treatments and, thus statistical, in the sense that the ions makes macroscopic movements during their collision thus the collision results are averaged. This procedure, common to most charged-particle transport algorithms [8]. A full catalog of stopping power plots can be downloaded at www.SRIM.org. Over 500 plots presented the accuracy of the stopping and ranges produced by SRIM along with 27,000 experimental data points are indicated. References to the citations which reported the experimental data are included. The principle authors of the SRIM series are James F. Ziegler and Jochen P.Biersack, although others have contributed. [9]

6-PSTAR

A PC package is documented for calculating stopping powers and ranges of electrons, protons and helium ions in matter for energies from 1 keV up to 10 GeV, These databases can also calculate similar results at any other energy grid between these limits, Energies are specified in MeV. Stopping powers and ranges for electrons can be calculated for any element, compound or mixture. Stopping powers and ranges of protons and helium ions can be calculated for 74 materials (26 elements and 48 compounds and mixtures).

The ESTAR, PSTAR and ASTAR calculate stopping powers and ranges for electrons, protons and alpha particles (helium ions). A detailed description of the methods used in these programs can be found in ICRU Reports 37 and 49 .[10]

7-Data Reduction and Analysis

The total stopping power elements for protons interact with element were calculated using the following four methods. The Bethe formula, Ziegler formula with their corresponding coefficients presented in reference and the two computer program PSTAR and SRIM software.

8- Stopping power for proton interaction with Al, C and Be

8.1- Bethe formula

The total stopping power for proton interact with Be, C and Al for energy range from 1 to 12 MeV were calculated using the reduced form of Bethe formula [7]

$$-\frac{dE}{dx}\left(\frac{MeV}{10^{21}\frac{atom}{cm^2}}\right) = 0.239114\,\frac{Z_p}{E_p}\,ln\left[2.178\times 10^{-3}\,\frac{E_p}{I_{ave}}\right] \quad (6)$$

The calculation was done using the Matlab computer program. The calculated ionization potential of the three considered elements are I_{AL}=162.9984 eV, I_{Be}=84.2241 eV, I_C=100.3939 eV.

8.2- Ziegler formula

The total stopping power for proton interact with Be, C, Al for energy range from 1 to 12 MeV were calculated according to eqs. (2-6) using Matlab computer program.

8.3- PSTAR program

The total stopping power for proton in Be, C and Al for energy range from 1 to 12 MeV were calculated using PSTAR program.

8.4- SRIM program

The total stopping power for proton in Be, C and Al for energy range from 1 to 12 MeV were calculated using SRIM program .

All data are presented in table (1 and 2) and figures(1-3).

9-Conclusion

The Ziegler and the computer programs PSTAR, SRIM are very well representing the Beth equation. Results for Carbon and Beryllium is a little higher than the Beth equation (2% and 1.5% respectively)while for Aluminum it's a little lower (1.2%). The stopping power values in this range are remarkably close which indicate the validity of the two software.

We can represent the calculated stopping power (the curves 1 - 3) in the considered energy interval by a power equation of the type ; $y=a\,x^b+c$,

For Beryllium we obtain as equation:

$$-\frac{dE}{dx} = 3.384\,x^{-0.6986} - 0.1231 \qquad (7)$$

For Carbon we obtain as equation:

$$-\frac{dE}{dx} = 4.783\,x^{-0.6787} - 0.197 \qquad (8)$$

For Aluminum we obtain as equation:

$$-\frac{dE}{dx} = 8.415\,x^{-0.63} - 0.4732 \qquad (9)$$

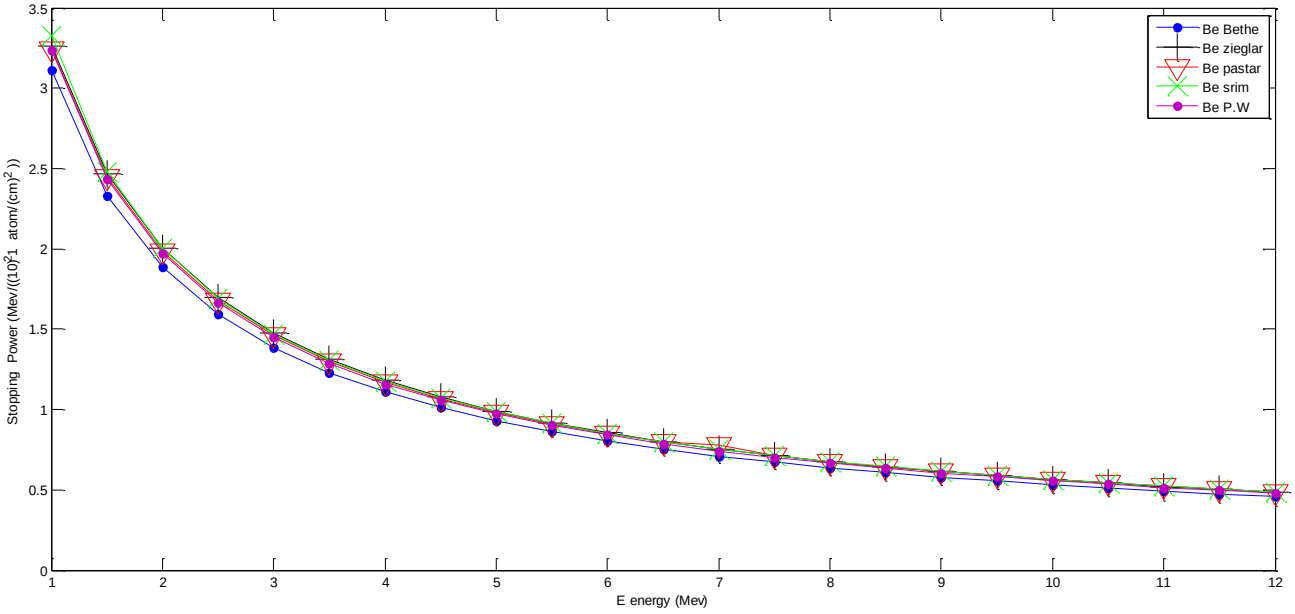

Fig (1) Stopping power Graph for Beryllium

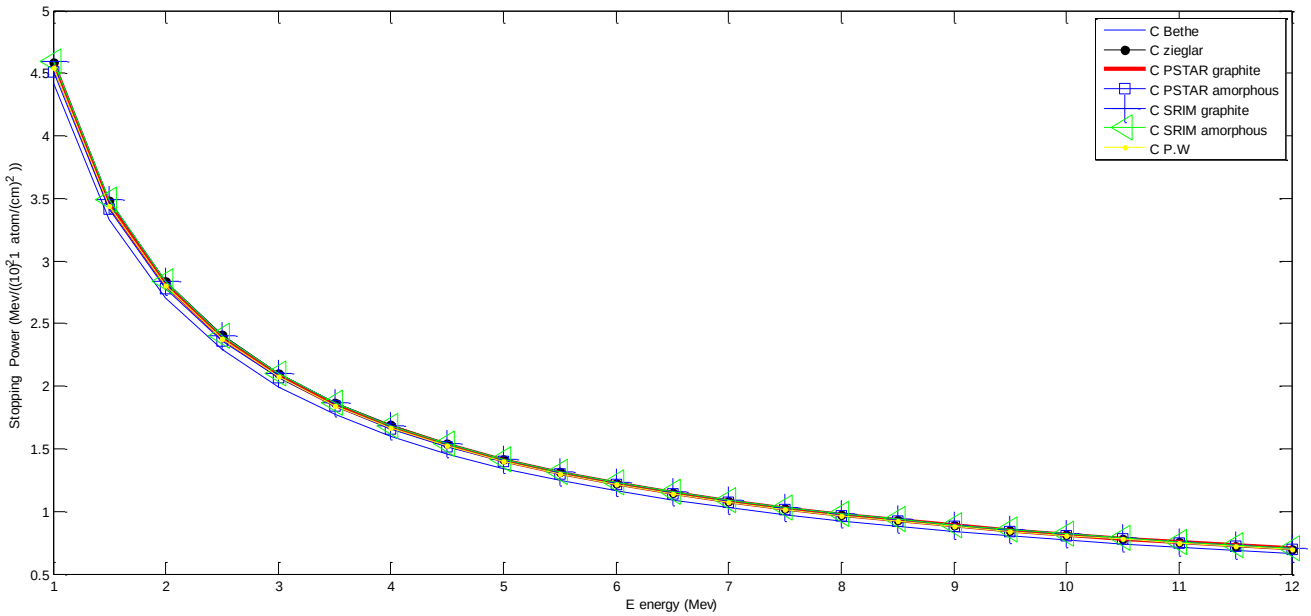

Fig (2) Stopping power Graph for Carbon

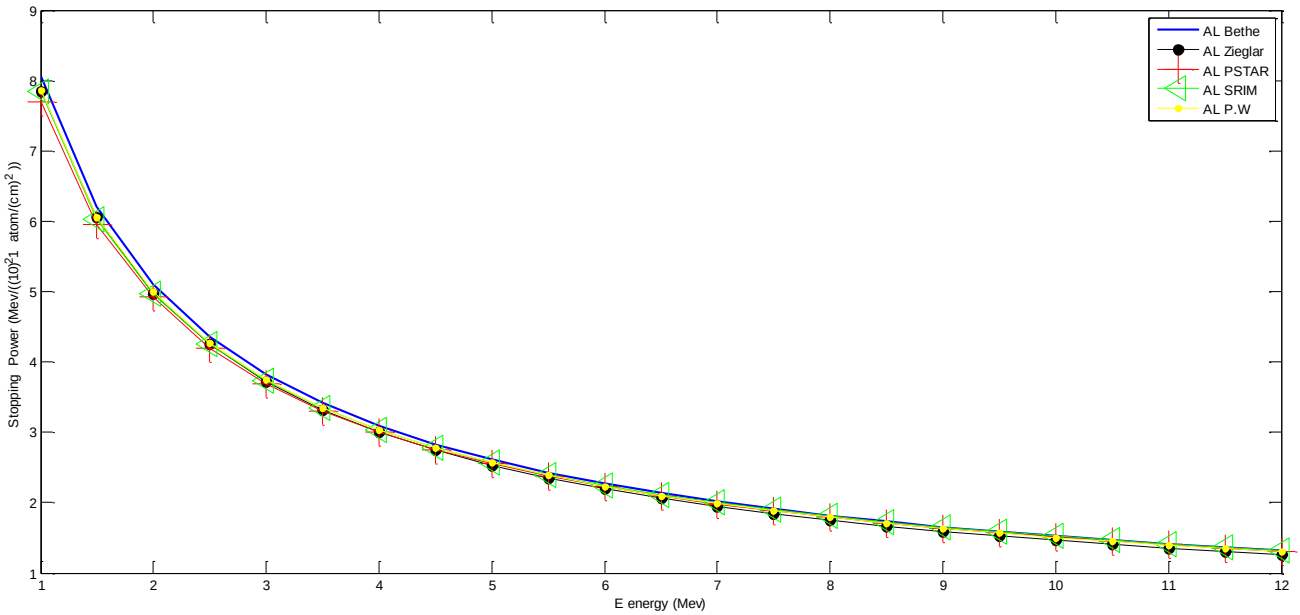

Fig (3) Stopping power Graph for Aluminum

Table (1): stopping power of Be, C for protons in unit of (MeV cm^2/ 10^21 atoms)

Ep MeV	$^{9}_{4}Be$					$^{12}_{6}c$						
	Bethe	zieglar	pastar	Srim	P.W	Bethe	zieglar	pastar graphite	pastar amorphous	srim graphite	srim amorphous	P.W
1.0000	3.1110	3.2644	3.2449	3.3279	3.2370	4.4146	4.5846	4.5791	4.5113	4.5960	4.5960	4.5469
1.5000	2.3326	2.4688	2.4475	2.4783	2.4318	3.3309	3.4856	3.4508	3.4129	3.4871	3.4871	3.4424
2.0000	1.8870	2.0040	1.9852	2.0020	1.9695	2.7045	2.8408	2.8088	2.7809	2.8337	2.8337	2.8004
2.5000	1.5950	1.6963	1.6800	1.6908	1.6655	2.2917	2.4118	2.3842	2.3643	2.4044	2.4044	2.3768
3.0000	1.3873	1.4762	1.4633	1.4707	1.4494	1.9969	2.1038	2.0832	2.0653	2.0982	2.0982	2.0743
3.5000	1.2312	1.3103	1.2999	1.3056	1.2868	1.7748	1.8708	1.8542	1.8386	1.8670	1.8670	1.8454
4.0000	1.1092	1.1805	1.1717	1.1756	1.1592	1.6009	1.6878	1.6743	1.6606	1.6859	1.6859	1.6659
4.5000	1.0110	1.0757	1.0683	1.0725	1.0569	1.4606	1.5400	1.5292	1.5171	1.5398	1.5398	1.5211
5.0000	0.9301	0.9893	0.9832	0.9865	0.9723	1.3447	1.4178	1.4092	1.3982	1.4187	1.4187	1.4012
5.5000	0.8621	0.9168	0.9117	0.9146	0.9013	1.2473	1.3150	1.3081	1.2982	1.3167	1.3167	1.3003
6.0000	0.8041	0.8549	0.8508	0.8534	0.8408	1.1642	1.2272	1.2220	1.2129	1.2296	1.2296	1.2142
6.5000	0.7541	0.8014	0.7980	0.8005	0.7885	1.0923	1.1512	1.1473	1.1389	1.1546	1.1546	1.1398
7.0000	0.7103	0.7546	0.7790	0.7543	0.7428	1.0295	1.0848	1.0821	1.0741	1.0885	1.0885	1.0746
7.5000	0.6718	0.7135	0.7115	0.7159	0.7032	0.9740	1.0261	1.0245	1.0171	1.0338	1.0338	1.0182
8.0000	0.6375	0.6769	0.6755	0.6775	0.6668	0.9247	0.9740	0.9730	0.9663	0.9791	0.9791	0.9660
8.5000	0.6068	0.6441	0.6431	0.6468	0.6352	0.8806	0.9273	0.9274	0.9210	0.9351	0.9351	0.9211
9.0000	0.5792	0.6146	0.6141	0.6161	0.6060	0.8408	0.8852	0.8859	0.8799	0.8911	0.8911	0.8790
9.5000	0.5541	0.5879	0.5878	0.5909	0.5802	0.8047	0.8470	0.8484	0.8427	0.8550	0.8550	0.8421
10.0000	0.5313	0.5635	0.5639	0.5656	0.5561	0.7718	0.8123	0.8142	0.8088	0.8189	0.8189	0.8075
10.5000	0.5105	0.5413	0.5420	0.5446	0.5346	0.7417	0.7804	0.7829	0.7777	0.7887	0.7887	0.7767
11.0000	0.4913	0.5208	0.5220	0.5235	0.5144	0.7141	0.7512	0.7541	0.7492	0.7585	0.7585	0.7476
11.5000	0.4737	0.5020	0.5034	0.5056	0.4962	0.6886	0.7242	0.7276	0.7228	0.7327	0.7327	0.7214
12.0000	0.4573	0.4845	0.4862	0.4877	0.4789	0.6650	0.6993	0.7031	0.6985	0.7070	0.7070	0.6966

Table (2): stopping power of AL for protons in unit of (MeV cm^2/ 10^21 atoms)

Ep MeV	$^{27}_{13}Al$				
	Bethe	zieglar	pastar	Srim	P.W
1.0000	8.0585	7.8460	7.6985	7.8436	7.8617
1.5000	6.2126	6.0566	5.9529	6.0340	6.0640
2.0000	5.1066	4.9770	4.9235	4.9671	4.9936
2.5000	4.3627	4.2483	4.1984	4.2526	4.2655
3.0000	3.8245	3.7201	3.6926	3.7372	3.7436
3.5000	3.4151	3.3180	3.3077	3.3449	3.3464
4.0000	3.0919	3.0007	3.0033	3.0347	3.0327
4.5000	2.8298	2.7433	2.7527	2.7835	2.7773
5.0000	2.6123	2.5299	2.5513	2.5754	2.5672
5.5000	2.4287	2.3499	2.3767	2.3983	2.3884
6.0000	2.2714	2.1957	2.2245	2.2472	2.2347
6.5000	2.1349	2.0621	2.0947	2.1161	2.1020
7.0000	2.0153	1.9450	1.9828	2.0000	1.9858
7.5000	1.9096	1.8416	1.8843	1.9035	1.8847
8.0000	1.8153	1.7495	1.7904	1.8069	1.7905
8.5000	1.7307	1.6668	1.7098	1.7289	1.7091
9.0000	1.6543	1.5923	1.6382	1.6508	1.6339
9.5000	1.5849	1.5246	1.5710	1.5868	1.5668
10.0000	1.5216	1.4629	1.5129	1.5228	1.5050
10.5000	1.4636	1.4063	1.4547	1.4687	1.4483
11.0000	1.4102	1.3544	1.4054	1.4147	1.3962
11.5000	1.3609	1.3064	1.3562	1.3682	1.3479
12.0000	1.3152	1.2620	1.3114	1.3216	1.3025

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قدرة الايقاف لتفاعل البروتونات مع الالمنيوم والبريليوم والكاربون باستخدام مختلف المعادلات

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الملخص

تم دراسة قدرة الايقاف لتفاعل البروتونات مع عناصر البيريليوم والكاربون والالومنيوم لمدى طاقي من 1 الى 12 ميكا الكترون فولط . اجريت الحسابات العددية وتحليل النتائج باستخدام برنامج الماتلاب لمعادلتي بيث وزيكلر (Bethe and Ziegler equations) وبرنامجي بيستار وسريم (PSTAR and SRIM). وقد تم عرض كافة القيم والنتائج وكما هو موضح في الجداول والاشكال البيانية, تم استنباط ثلاثة معادلات جبرية (نوع معادلات قوى) للتعبير عن قدرة الايقاف للبروتونات للعناصر الثلاثة.