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On (i,j) -prw Closed Set and (i,j)-prwContinuousfunction in BitopologicalSpaces

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Abstract:

In this paper we introduce and study the a new class of closed set called (i,j)-preregular weakly closed set (briefly (i,j)-prw closed).and (i,j)- pre weakly closed set (briefly (i,j)-pw closed).Moreover the notions of (i,j)-prw- Continuous function and (i,j)- pw- Continuous function are introduced ,and study the relationships among them, are studied.

***Keywords**: (i,j) -prw closed , (i,j)-pw closed, (i,j)-prw-Continuous function, (i,j)- pw- Continuous function.*

1.Introduction and Preliminaries:

A triple (X, T_i, T_j) *where X is non-empty set and* T_i *and* T_j *are topologies on X is called a bitopological space . Kelly [9] initiated the study of such spaces . In 1985,Fukutake [5] introduced the concepts of g-closed set in bitopological spaces then, several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. S.S.Benchalli and RS Wali [2]introduced new class of sets called regular weakly-closed (briefly rw-closed set) in topological spaces which lies between the notions of w-closed and regular g-closed sets. M.karpagadovi and A.pushralath [10] introduced a new class of sets called (i,j)-rw-closed) in bitopological spaces which lies between (i,j)-w-closed and (i,j)- regular g-closed. If A is a subset of X with topology T, then the closure of A is denoted by T-cl(A) orcl(A), the interior of A is denoted by T-int(A) or int(A) and the* *complement of A in X is denoted by* A^c *. The intersection of all pre closed subset of X containing A is called pre-closure of A and denoted by* $pcl(A)$*.Observe that* $pcl(A) = A \cup cl(int(A))$*[3].Now, we need to recall the following definitions.*

Definition 1.1:*A subset A of a topological space* (X, T) *is called:*

1- *A generalized closed (briefly, g-closed) [11] if* $cl(A) \subseteq U$ *wherever* $A \subseteq U$ *,U is open in X.*
2- *A generalized open (briefly, g-open) [11] if* A^c *is g-closed in X.*
3- *A regular closed [15] if* $cl\,\mathrm{int}(A) = A$*.*
4- *A pre closed [13] if* $cl\,\mathrm{int}(A) \subseteq A$*.*
5- *A semi open [12] if* $A \subseteq cl\,\mathrm{int}(A)$*.*
6- *A strongly generalized closed (briefly,* g^**-closed) [16] if* $cl(A) \subseteq U$ *wherever* $A \subseteq U$ *is g- open in X.*

Notation : We use the relation (i,j)that denotes to the pair of bitopologies (T_i, T_j) *.*

Definition 1.2: *A sub set A of a bitopological space* (X, T_i, T_j) *is called:*

1- *(i,j)-regular open, [4] if* $i\,\mathrm{int}[jcl(A)] = A$*.*
2- *(i,j)-pre-open, [8] if* $A \subseteq j\ int[i\ cl(A)]$ *.*
3- *(i,j)-semi-open, [12] if* $A \subseteq jcl[i\ int(A)]$ *.*
4- *(i,j) –g-closed, [5] if* $T_j - cl(A) \subseteq U$ *wherever* $A \subseteq U$*and U is open in*T_i*.*
5- *(i,j) weakly closed,(brieflyw-closed) [6] if* $T_j - cl(A) \subseteq U$ *wherever* $A \subseteq U$ *and U is semi open in*T_i*.*
6- *(i,j) weakly generalized closed, (briefly wg-closed) [7] if* $T_j - cl[T_i\,\mathrm{int}(A)] \subseteq U$ *wherever* $A \subseteq U$ *and U is open in*T_i*.*
7- *(i,j) regular generalized closed, (briefly rg-closed) [1] if* $T_j - cl(A) \subseteq U$ *wherever* $A \subseteq U$ *and U is regular open in*T_i*.*
8- *(i,j) generalized pre regular closed, (briefly gpr-closed) [6] if* $T_j - pcl(A) \subseteq U$ *wherever* $A \subseteq U$ *and U is regular open in*T_i*.*

9- *(i,j) –g^*-closed [14] if* $T_j - cl(A) \subseteq U$ *wherever* $A \subseteq U$ *and* U *is g-open in*T_i.

2. (i,j) –prw-Closed Set and (i,j)-pw Closed Set in BitopologicalSpaces.

In this section, we introduce a new class of sets in bitopological spaces.

Definition 2.1:*A subset A of a bitopological space* (X, T_i, T_j)*is called (i,j) –prw-closed if* $T_j - pcl(A) \subseteq U$ *wherever* $A \subseteq U$ *and* U *is regular preopen in*T_i.

Definition 2.2: *A subset A of a bitopological space* (X, T_i, T_j)*is called (i,j) –pw-closed if* $T_j - cl(A) \subseteq U$ *wherever* $A \subseteq U$ *and* U *is preopen in*T_i.

Theorem 2.3:*Every* $T_j - closed$ *set in* (X, T_i, T_j) *is (i,j) –prw-closed .*

Proof : Assume that A is $T_j - closed$, $A \subseteq U$ *and U is regular preopen in* T_i*;Since A is* $T_j - closed$*,then* $cl(A) = A$*; Thus, we have* $T_j - pcl(A) \subseteq U$. *Hence A is (i,j) –prw-closed.*

Remark 2.4:*Observe that, the converse of the theorem 2.3 need not be true in general as show by the following example.*

Example 2.5:*Let* $X = \{1,2,3\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{1\}\}$, $T_i = \{\emptyset, X, \{2,3\}\}$. *Then the set* $\{2,3\}$ *is* $(i,j) - prw\ closed\ set$ *but not* $T_j - closed$.

Theorem 2.6:*Every* $T_j - closed$ *set in* (X, T_i, T_j) *is (i,j) –pw-closed set.*

Proof : Assume that A is $T_j - closed$, $A \subseteq U$ *and U is pre-open in* T_i *, since A is* $T_j - closed$*, then* $cl(A) = A$*;Thus, we have* $T_j - cl(A) \subseteq U$. *Hence A is (i,j) –pw-closed.*

Remark 2.7:*Observe that, the converse of the theorem 2.6 need not be true in general as show by the following example.*

Example 2.8: *Let* $X = \{a, b, c, d\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{c, b, d\}, \{a, b, d\}\}$, $T_j =$

$\{\emptyset, X, \{c\}, \{b\}, \{b, c\}\}$. *Then the set* $\{a\}$ *is* $(i, j) - pw\ closed\ set$ *but not* $T_j - closed$.

Theorem 2.9:*Every* $(i, j) - regular\ closed$ *set in* (X, T_i, T_j) *is (i,j) – prw-closed set.*

Proof : Assume that A is $(i, j) regular - closed$, $A \subseteq U$ *and U is regular pre-open in* T_i , *since A is* $T_j - regular\ closed$, $cl(A) = A$ *;Thus, we have* $T_j - pcl(A) \subseteq U$. *Hence A is (i,j) –prw-closed.*

Remark 2.10:*Observe that, the converse of the theorem 2.9 need not be true in general as show by the following example.*

Example 2.11: *Let* $X = \{a, b, c, d\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{c, b, d\}, \{a, b, d\}\}$, $T_j = \{\emptyset, X, \{c\}, \{b\}, \{b, c\}\}$. *Then the set* $\{c, d\}$ *is* $(i, j) - prw\ closed\ set$ *but not (i,j) regular closed .*

Theorem 2.12:*Every* $(i, j) - regular\ closed$ *set in* (X, T_i, T_j) *is (i,j) – pw-closed set.*

Proof : Assume that A is $(i, j) regular - closed$, $A \subseteq U$ *and U is pre-open in* T_i , *since A is* $T_j - regular\ closed$, $cl(A) = A$*;Thus, we have* $T_j - cl(A) \subseteq U$. *Hence A is (i,j) –pw-closed.*

Remark 2.10: *Observe that, the converse of the theorem 2.12 need not be true in general as show by the following example.*

Example 2.11:*Let* $X = \{a, b, c, d\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{c, b, d\}, \{a, b, d\}\}$, $T_j = \{\emptyset, X, \{c\}, \{b\}, \{b, c\}\}$. *Then the set* $\{c, d\}$ *is* $(i, j) - pw\ closed\ set$ *but not (i,j) regular closed.*

Remark 2.12:*The following example shows that (i,j) –prw-closed set is independent of (i,j)-rg-closed set .*

Example 2.13: *Let* $X = \{a, b, c, d\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{d\}, \{a, c\}, \{a, d\}, \{a, c, d\}\}$, $T_j = \{\emptyset, X, \{c, d\}, \{a, c, d\}\}$.

Then the set $\{a, b\}$ *is* $(i, j) - rg$ *closed set but not (i,j)-prw closed set .*

Example 2.14: *Let* $X = \{a, b, c, d\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{c, b, d\}, \{a, b, d\}\}$ *,* $T_j = \{\emptyset, X, \{a, b, d\}, \{a, c, d\}, \{a, d\}\}$ *. Then the set* $\{a\}$ *is not* $(i, j) - rg$ *closed set but is (i,j)-prw closed set.*

Remark 2.15:*The following examples show that (i,j) –pw-closed set is independent of (i,j)-rg-closed set .*

Example 2.16: *Let* $X = \{a, b, c, d\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{c, b, d\}, \{a, b, d\}\}$ *,* $T_j = \{\emptyset, X, \{c\}, \{b\}, \{b, c\}\}$ *.*

Then the set $\{a\}$ *is* $(i, j) - pw$ *closed set but not (i,j)-rg closed set .*

Example 2.17: *Let* $X = \{a, b, c, d\}$ *, be a bitopological space with* $T_i = \{\emptyset, X, \{a\}, \{d\}, \{a, c\}, \{a, d\}, \{a, c, d\}\}$ *,* $T_j = \{\emptyset, X, \{c, d\}, \{a, c, d\}\}$ *.*

Then the set $\{a, b\}$ *is* $(i, j) - rg$ *closed set but not (i,j)-pw closed set .*

Theorem 2.18: *Every* $(i, j) - prw$ *closed set in* (X, T_i, T_j) *if and only if is (i,j) –gpr-closed set*

Proof : Assume that A is $(i, j)prw - closed$ *,* $A \subseteq U$ *and U is regular pre-open in* T_i*,sinc every* $T_i - regular\ open$ *is* $T_i - regular\ pre\ open$ *, then* $pcl(A) \subseteq T_j - pcl(A) \subseteq U$ *;Thus, we have* $(i.j) - pcl(A) \subseteq U$ *. Hence A is (i,j) –gpr closed. And the other proof is in the same way.*

Theorem 2.19: *Every* $(i, j) - gpr$ *closed set in* (X, T_i, T_j) *is (i,j) –pw-closed set.*

Proof: : Assume that A is $(i, j)gpr - closed$ *,* $A \subseteq U$ *and U is reguler open in*T_i *,sinc every* $T_i - regular\ open$ *is* $T_i - pre\ open$ *, then* $pcl(A) \subseteq T_j - cl(A) \subseteq U$ *, there for* $(i.j) - pcl(A) \subseteq U$ *. Hence A is (i,j) –pw closed set.*

Remark 2.20:*Observe that, the converse of the theorem 2.19 need not be true in general as show by the following example.*

Example 2.21: *Let* $X = \{a, b, c, d\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{c, b, d\}, \{a, b, d\}\}$ *,* $T_j = \{\emptyset, X, \{c\}, \{b\}, \{b, c\}\}$ *.Then the set* $\{a, c, d\}$ *is* $(i, j) - pw\ closed\ set$ *but not (i,j)-gpr closed set .*

Theorem 2.22: *Every(* $i, j) - prw\ closed\ set$ *in* (X, T_i, T_j) *is (i,j) –pw-closed set.*

Proof : Assume that A is $(i, j) - prw\ closed$ *,* $A \subseteq U$ *and U is regular pre-open in* T_i *,sinc every* $T_i - regular\ pre\ open$ *is* $T_i - pre\ open$ *, then there for . Hence A is (i,j) – pw closed set.*

Remark 2.23:*Observe that, the converse of the theorem 2.22 need not be true in general as show by the following example.*

Example 2.24: *Let* $X = \{a, b, c, d\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{c, b, d\}, \{a, b, d\}\}$ *,* $T_j = \{\emptyset, X, \{c\}, \{b\}, \{b, c\}\}$ *. Then the set* $\{c, d\}$ *is* $(i, j) - pw\ closed\ set$ *but not (i,j) -prw closed.*

Remark 2.25: *Every* $(i, j) - prw\ closed\ set$ *and every* $(i, j) - pw\ closed\ set$ *in* (X, T_i, T_j) *is not (i,j) w-closed set .*

Example 2.24: *Let* $X = \{a, b, c, d\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{c, b, d\}, \{a, b, d\}\}$ *,* $T_j = \{\emptyset, X, \{c\}, \{b\}, \{b, c\}\}$ *. Then the set* $\{a\}$ *is* $(i, j) - prw\ closed\ set$ *and* $(i, j) - pw\ closed\ set$ *but not (i,j) -w closed set.*

Theorem 2.25:*Every* $(i, j) - wg\ closed\ set$ *in* (X, T_i, T_j) *is (i,j) –prw-closed set.*

Proof : Assume that A is $(i, j) - wg\ closed$ *,* $A \subseteq U$ *and U is regular pre-open in*T_i *, since* $cl(A) = A$*;Thus, we have* $T_j - pcl(A) \subseteq U$ *. Hence A is (i,j) –prw-closed.*

Remark 2.26: *Observe that, the converse of the theorem 2.25 need not be true in general as show by the following example.*

Example 2.27: *Let* $X = \{a, b, c, d\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{c, b, d\}, \{a, b, d\}\}$ *,* $T_j =$

$\{\emptyset, X, \{c\}, \{b\}, \{b, c\}\}$. *Then the set* $\{a\}$ *is* $(i,j) - prw$ *closed set a but not (i,j) -wg closed set.*

Theorem 2.28:*Every* $(i,j) - wg$ *closed set in* (X, T_i, T_j) *is (i,j) –pw-closed set .*

Proof : Assume that A is $(i,j) - wg$ *closed* , $A \subseteq U$ *and U is regular pre-open in*T_i *, since* $cl(A) = A$*;Thus, we have* $T_j - cl(A) \subseteq U$*. Hence A is (i,j) –pw-closed.*

Remark 2.29: *Observe that, the converse of the theorem 2.28 need not be true in general as show by the following example.*

Example 2.30: *Let* $X = \{a, b, c, d\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{c, b, d\}, \{a, b, d\}\}$, $T_j = \{\emptyset, X, \{c\}, \{b\}, \{b, c\}\}$. *Then the set* $\{c\}$ *is* $(i,j) - pw$ *closed set a but not (i,j) -wg closed set.*

Remark 2.31:*The following example shows that (i,j) –pw-closed set is independent of (i,j)-semi closed set .*

Example 2.32: *Let* $X = \{a, b, c, d\}$*, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b\}, \{a, b, c\}\}, T_j = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$.

Then the set $\{a\}$ *is* $(i,j) - semi\ closed\ set$ *a but not (i,j) -pw closed set.*

The set $\{a, b\}$ *is* $(i,j) - pw$ *closed set a but not (i,j) -* $semi\ closed\ set$.

Remark 2.33:*The following example shows that (i,j) –prw-closed set is independent of (i,j)-semi closed set .*

Example 2.34: *Let* $X = \{a, b, c, d\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b\}, \{a, b, c\}\}$, $T_j = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$.

Then the set $\{a, c, d\}$ *is* $(i,j) - semi\ closed\ set$ *a but not (i,j) -prw closed set.The set* $\{a, b\}$ *is* $(i,j) - prw$ *closed set a but not (i,j) -* $semi\ closed\ set$.

Remark 2.35:*The following example shows that (i,j) –prw-closed set is independent of (i,j)-*g^**closed set .*

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Example 2.36: *Let* $X = \{a, b, c, d\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b\}, \{a, b, c\}\}$, $T_j = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$.

Then the set $\{d\}$ *is* $(i, j) - g^*$ *closed set a but not (i,j) -prw closed set,*

The set $\{a, b\}$ *is* $(i, j) - prw$ *closed set a but not (i,j) -*g^* *closed set.*

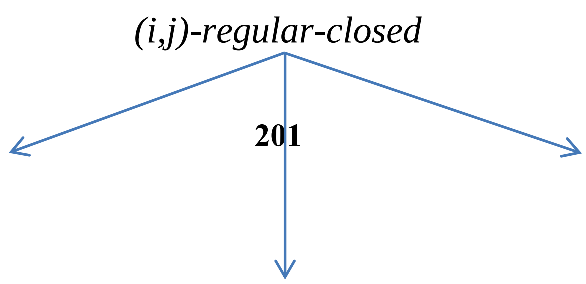
Remark 2.37:*The following example shows that (i,j) –pw-closed set is independent of (i,j)-*g^**closed set .*

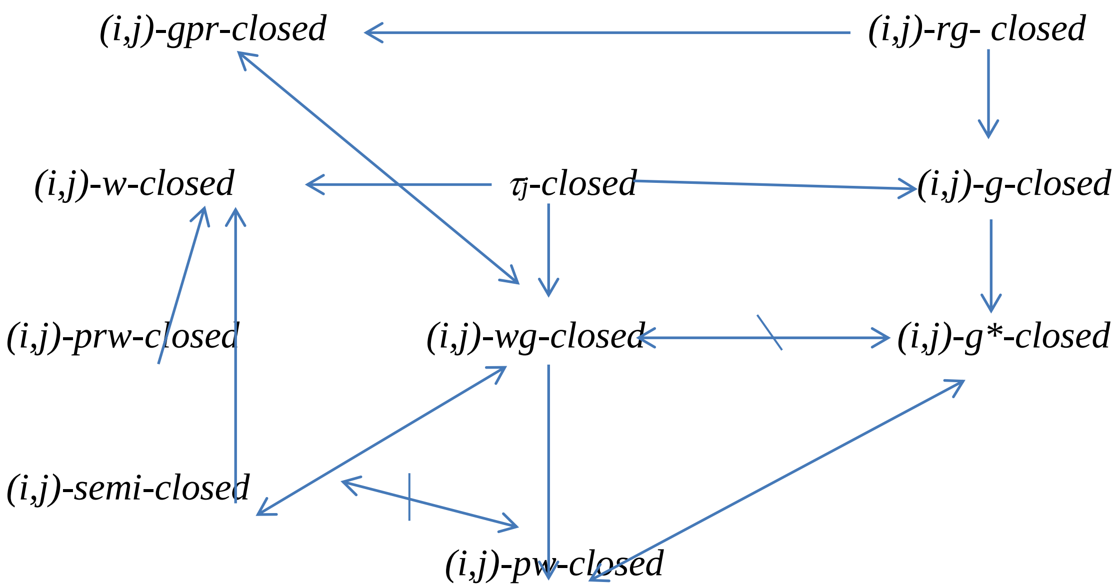
Example 2.38: *Let* $X = \{a, b, c, d\}$ *, be a bitopological space with topologies* $T_i = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b\}, \{a, b, c\}\}$, $T_j = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$.

Then the set $\{d\}$ *is* $(i, j) - g^*$ *closed set a but not (i,j) -pw closed set,*

The set $\{a, b\}$ *is* $(i, j) - pw$ *closed set a but not (i,j) -*g^* *closed set.*

The following diagram shows explains the relations among all the concepts introduced in the above section :





3.(i,j) –prw- Continuousand (i,j)-pw- Continuous in BitopologicalSpaces.

In this section we introduce (i,j) prw- continuity and (i,j) pw-continuity in bitopological spaces .

Definition 3.1: *A function* $\boldsymbol{f}:\left(\boldsymbol{X},\boldsymbol{T}_{\boldsymbol{i}},\boldsymbol{T}_{\boldsymbol{j}}\right) \longrightarrow (\boldsymbol{Y},\boldsymbol{\sigma}_{\boldsymbol{i}},\boldsymbol{\sigma}_{\boldsymbol{j}})$ *is*

1-(i,j) w- Continuous if inverse image of any (i,j) closed set in Y is (i,j) w-closed set in X.

2-(i,j) wg- Continuous if inverse image of any (i,j) closed set in Y is (i,j) wg closed set in X.

3-(i,j) rg- Continuous if inverse image of any (i,j) closed set in Y is (i,j) rg closed set in X.

4-(i,j) gpr- Continuous if inverse image of any (i,j) closed set in Y is (i,j) gpr closed set in X.

5-(i,j) semi- Continuous if inverse image of any (i,j) closed set in Y is (i,j) semi closed set in X.

6-(i,j) g- Continuous if inverse image of any (i,j) closed set in Y is (i,j) g- closed set in X.

7-(i,j) g- Continuous if inverse image of any (i,j) closed set in Y is (i,j) g*- closed set in X.*

8 -(i,j) regular - Continuous if inverse image of any (i,j) closed set in Y is (i,j) regular- closed set in X.

Definition 3.2:*A function* $\boldsymbol{f}:\left(\boldsymbol{X},\boldsymbol{T_i},\boldsymbol{T_j}\right) \longrightarrow (\boldsymbol{Y},\boldsymbol{\sigma_i},\boldsymbol{\sigma_j})$ *is called (i,j) prw- Continuous if inverse image of any (i,j) closed set in Y is (i,j) prw closed set in X.*

Definition 3.3:*A function* $\boldsymbol{f}:\left(\boldsymbol{X},\boldsymbol{T_i},\boldsymbol{T_j}\right) \longrightarrow (\boldsymbol{Y},\boldsymbol{\sigma_i},\boldsymbol{\sigma_j})$ *is called (i,j) pw- Continuous if inverse image of any (i,j) closed set in Y is (i,j) pw closed set in X.*

Theorem 3.4*:Every* $(\boldsymbol{i},\boldsymbol{j})-\boldsymbol{prw}$ *Continuous function is* $(\boldsymbol{i},\boldsymbol{j})-\boldsymbol{pw}$*Continuous function.*

Proof : Let $\boldsymbol{f}:\boldsymbol{X} \longrightarrow \boldsymbol{Y}$ *be a* $(\boldsymbol{i},\boldsymbol{j})-\boldsymbol{prw}$ *Continuous function from the spaces X to a spaces Y, and U be an closed set in Y. By* $(\boldsymbol{i},\boldsymbol{j})-\boldsymbol{prw}$ *Continuity of f , we have* $\boldsymbol{f^{-1}}(\boldsymbol{U})$ *is (i,j) prw- closed set in set in X. By (theorem 2.22), we obtain that* $\boldsymbol{f^{-1}}(\boldsymbol{U})$ *is (i,j) pw-closed set in X. There for,* $\boldsymbol{f}$ ***is*** $(\boldsymbol{i},\boldsymbol{j})-\boldsymbol{pw}$*–Continuous function .*

Remark 3.5:*Observe that, the converse of the theorem 3.4 need not be true in general as show by the following example.*

Example 3.6: *Let* $X=\{a,b,c,d\}$, $Y=\{1,2,3,4\}$*be a bitopological spaces with topologies* $T_i=\{\emptyset,X,\{a\},\{a,b\},\{a,c,d\}\}$, $T_j=\{\emptyset,X,\{a\},\{d\},\{a,d\}\}$.$\sigma_1=\sigma_2=\{\emptyset,Y,\{2\},\{1,3\},\{1,2,3\}\}$, *and let* $f:X \longrightarrow Y$ *be function defined by* $f(a)=f(b)=\{4\}$, $f(c)=\{1\}$, $f(d)=\{2\}$ *, then* f *is* $(i,j)-pw$*- Continuous but is not is* $(i,j)-prw$*Continuous. Because the* $cl\ int\,\{a,b,c\}\cup\{a,b,c\}=\{a,b,c\}\notin$ *regular preopen set in* T_1.

Remark 3.7:*The following example shows that* $(i,j)-pw$ *continuity is not* $(i,j)-gpr$ *continuity.*

Example 3.8*: Let* $X=\{a,b,c,d\}$, $Y=\{1,2,3,4\}$*be a bitopological spaces with topologies* $T_i=\{\emptyset,X,\{a\},\{a,b\},\{a,c,d\}\}$, $T_j=\{\emptyset,X,\{a\},\{d\},\{a,d\}\}$.$\sigma_1=\sigma_2=\{\emptyset,Y,\{2\},\{1,3\},\{1,2,3\}\}$, *and let* $f:X\longrightarrow Y$ *be function defined by* $f(a)=f(b)=\{4\}$, $f(c)=\{1\}$, $f(d)=\{2\}$ *, then* f *is* $(i,j)-pw$*- Continuous but is not is* $(i,j)-gpr$*- Continuous. Because the* $cl\ int\ \{a,b,c\}\cup\{a,b,c\}=\{a,b,c\}\notin$ *regular open set in* T_1.

Theorem 3.9:*Every* $(i,j)-wg$ *-Continuous is* $(i,j)-pw$*-Continuous*

Proof : Let U be closed set in Y then $\boldsymbol{f^{-1}(U)}$ *is (i,j) wg- closed set because* $\boldsymbol{f}$ *is* $\boldsymbol{(i,j)-wg-}$ *Continuous .Then* $\boldsymbol{f^{-1}(U)}$ *is (i,j) pw-closed set by (theorem 2.28) ,then* $\boldsymbol{f}$ $\boldsymbol{is\ (i,j)-pw}$*- continuity.*

Theorem 3.10:*Every* $(i,j)-wg$ *- Continuous is* $(i,j)-prw$*-Continuous .*

Proof : Let U be closed set in Y then $f^{-1}(U)$ *is (i,j) wg- closed set because* f *is* $(i,j)-wg-$ *Continuous.Then* $f^{-1}(U)$ *is (i,j) prw-closed set by (theorem 2.28) ,then* f *is* $(i,j)-prw$*- continuity.*

Remark 3.11*: Observe that, the converse of the theorem 3.10 need not be true in general as show by the following example*

Example 3.12*:Let* $X=\{a,b,c\}$,$Y=\{a,b,c\}$*be a bitopological spaces with topologies*$T_i=\{\emptyset,X,\{a\},\{b,c\}\}$,$T_j=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$.$\sigma_i=\left\{\emptyset,Y,\{a\}\right\}$, $\sigma_j=\{\emptyset,Y,\{a\},\ \{a,b\}\}$, *and let* $f:X\longrightarrow Y$ *be function defined by* $f(a)=\{a\}$, $f(b)=\{b\}$, $f(c)=\{a\}$ *, then* f *is* $(i,j)-prw$*- Continuous but is not is* $(i,j)-wg$*- Continuous. Because the cl* $\{,c\}=\{,c\}\notin T_1$.

Theorem 3.13*:Every* $(i,j)-regular-Continuous$ *is* $(i,j)-pw$ *continuity*

Proof : Let U be closed set in Y then $f^{-1}(U)$ *is (i,j) regular - closed set because* f *is* $(i,j)-regular-$ *Continuous .Then* $f^{-1}(U)$ *is (i,j) pw- closed set by (theorem 2.12) ,then* f *is* $(i,j)-$ *pw- continuity.*

Theorem 3.14*:Every* $(i,j)-regular$ *Continuous is* $(i,j)-prw$*- Continuous*

Proof : Let U be closed set in Y then $f^{-1}(U)$ *is (i,j) regular - closed set because* f *is* $(i,j)-regular-$ *Continuous .Then* $f^{-1}(U)$ *is (i,j) prw- closed set by (theorem 2.9) ,then* f *is* $(i,j)-$ *pw-continuity .*

Theorem 3.15*:Every* T_j- *Continuous is* $(i,j)-pw$ *Continuous.*

Proof : Let U be closed set in Y then $f^{-1}(U)$ *is* T_j*- closed set because* f *is* T_j- *Continuous .Then* $f^{-1}(U)$ *is (i,j) pw- closed set by (theorem 2.6) ,then* f *is* $(i,j)-pw$*- continuity .*

Remark 3.16:*Observe that, the converse of the theorem 3.15 need not be true in general as show by the following example*

Example 3.17*:Let* $X=\{a,b,c\}$, $Y=\{a,b,c\}$*be a bitopological spaces with topologies* $T_i=\{\emptyset,X,\{a\},\{b,c\}\}$, $T_j=\{\emptyset,X,\{a,c\},\{c\}\}$.$\sigma_i=\{\emptyset,Y,\{a\}\}$, $\sigma_j=\{\emptyset,Y,\{a\},\{a,b\}\}$, *and let* $f\colon X\longrightarrow Y$ *be function defined by* $f(a)=\{a\}$, $f(b)=\{b\}$, $f(c)=\{c\}$ *, then* f *is* $(i,j)-pw$*Continuous but is not is* T_{j-} *Continuous. Because the* $\{b,c\}\notin T_j$*.*

Theorem 3.18:*Every* T_j- *Continuous is* $(i,j)-prw$*- Continuous.*

Proof : Let U be closed set in Y then $f^{-1}(U)$ *is* T_j*- closed set because* f *is* T_j- *Continuous .Then* $f^{-1}(U)$ *is (i,j) prw- closed set by (theorem 2.3) ,then* f *is* $(i,j)-prw$*- Continuous.*

Remark 3.19:*Observe that, the converse of the theorem 3.18 need not be true in general as show by the following example.*

***Example 3.20**: Let* $X=\{a,b,c\}$, $Y=\{a,b,c\}$*be a bitopological spaces with topologies* $T_i=\{\emptyset,X,\{a\},\{b,c\}\}$, $T_j=\{\emptyset,X,\{a,c\},\{c\}\}$.$\sigma_i=\{\emptyset,Y,\{a\}\}$, $\sigma_j=\{\emptyset,Y,\{a\},\ \{a,b\}\}$, *and let* $f:X\longrightarrow Y$ *be function defined by* $f(a)=\{a\}$, $f(b)=\{b\}$, $f(c)=\{c\}$, *then* f *is* $(i,j)-prw$*Continuous but is not is* T_{j-} *Continuous. Because the* $\{b,c\}\notin T_j$.

***Theorem 3.21:**Every* $(1,2)-prw-Continuous$ *in* (X,T_1,T_2) *is (i,j) –gpr- continuity , Every* $(1,2)-gpr-Continuous$ *is (i,j) –prw - Continuous.*

Proof : Let U be closed set in Y then $\boldsymbol{f^{-1}(U)}$ *is (i,j) prw- closed set because* $\boldsymbol{f}$ *is* $(\boldsymbol{i},\boldsymbol{j})-\boldsymbol{prw}-$ *Continuous .Then* $\boldsymbol{f^{-1}(U)}$ *is (i,j) gpr-closed set by (theorem 2.18) ,then* $\boldsymbol{f}$ $\boldsymbol{is}\ (\boldsymbol{i},\boldsymbol{j})-\boldsymbol{gpr}$*-Continuous.now let U be closed set in Y then* $\boldsymbol{f^{-1}(U)}$ *is (i,j) gpr-closed set because* $\boldsymbol{f}$ *is* $(\boldsymbol{i},\boldsymbol{j})-\boldsymbol{gpr}-$ *Continuous .Then* $\boldsymbol{f^{-1}(U)}$ *is (i,j) prw- closed set by (theorem 2.18) ,then* $\boldsymbol{f}$ $\boldsymbol{is}\ (\boldsymbol{i},\boldsymbol{j})-\boldsymbol{prw}$*-Continuous.*

***Remark 3.22:**The following example shows that* $(i,j)-pw$ *continuity and* $(i,j)-prw$ *continuity is not* $(i,j)-rg-$ *continuity.*

***Example 3.23**: Let* $X=\{a,b,c\}$, $Y=\{a,b,c\}$*be bitopological spaces with topologies* $T_i=\{\emptyset,X,\{a\},\{b,c\}\}$, $T_j=\{\emptyset,X,\{a,b\},\{b\}\}$.$\sigma_i=\{\emptyset,Y,\{a\}\}$, $\sigma_j=\{\emptyset,Y,\{a\},\ \{a,b\}\}$, *and let* $f:X\longrightarrow Y$ *be function defined by* $f(a)=\{a\}$, $f(b)=\{b\}$, $f(c)=\{c\}$, *then* f *is* $(1,2)-prw$ *continuity and* $(1,2)-pw$ *continuity but is not is* $(1,2)-rg-$ *continuity. Because the* $cl\{c\}=\{c\}\notin regular\ open\ set\ in\ T_i$.

***Remark 3.24:**The following example shows that* $(i,j)-pw$ *continuity and* $(i,j)-prw$ *continuity is not* $(i,j)-w-$ *continuity.*

***Example 3.25:** Let* $X=\{a,b,c\}$, $Y=\{a,b,c\}$*be bitopological spaces with topologies* $T_i=\{\emptyset,X,\{a\},\{b,c\}\}$, $T_j=\{\emptyset,X,\{a,b\},\{b\}\}$.$\sigma_i=\{\emptyset,Y,\{a\}\}$, $\sigma_j=\{\emptyset,Y,\{a\},\ \{a,b\}\}$, *and let*

$f:X \longrightarrow Y$ *be function defined by* $f(a)=\{a\}$, $f(b)=\{b\}$, $f(c)=\{c\}$, *then* f *is* $(i,j)-prw$- *Continuous and* $(i,j)-pw$-*Continuous but is not* $is(i,j)-rg-$ *Continuous. Because the* $\{c\} \notin$ $semi\ open\ set\ in\ T_i$.

Remark 3.26:*The following examples shows that* $(i,j)-pw$ *continuity independent with* $(i,j)-semi-$ *continuity.*

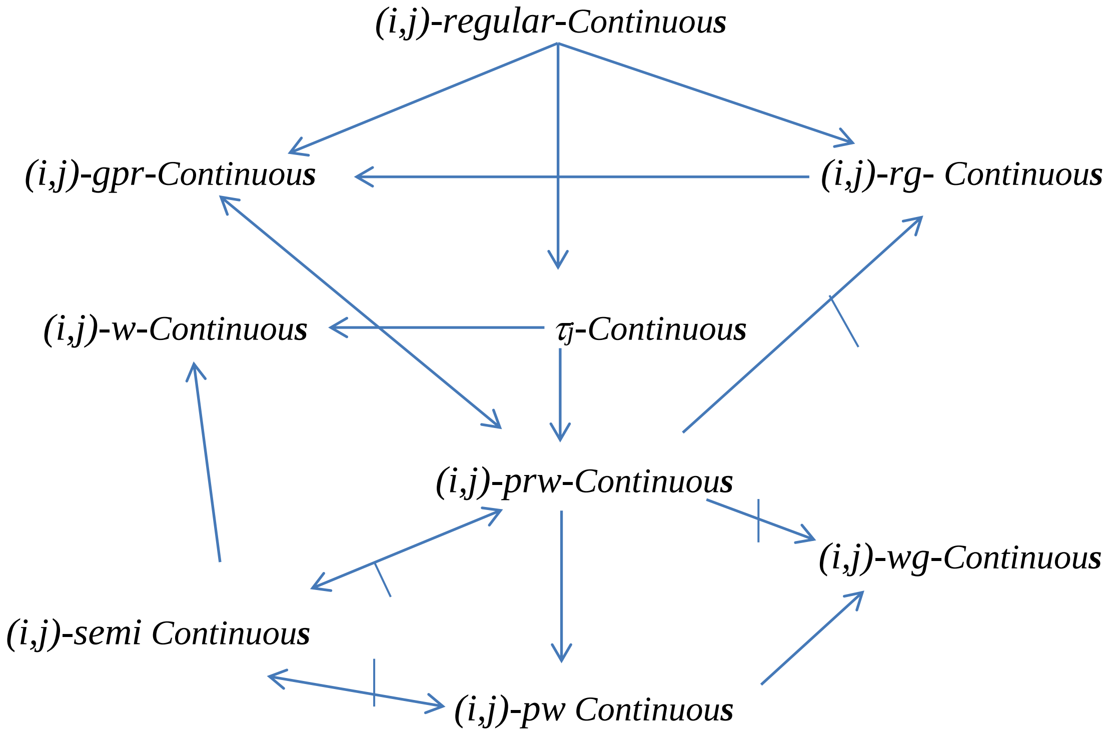
Example 3.27:*Let* $X=\{a,b,c,d\}$, $Y=\{1,2,3,4\}$*be bitopological spaces with topologies*$T_i=\{\emptyset,X,\{a\},\{a,b\},\{b\},\{a,b,c\}\}$, $T_j=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$.$\sigma_i=\sigma_j=\{\emptyset,Y,\{2\},\{1,3\},\{1,2,3\}\}$, *and let* $f:X \longrightarrow Y$ *be function defined by* $f(a)=\{1\}$, $f(b)=f(d)=\{4\}$, $f(c)=\{2\}$, *then* f *is* $(i,j)-semi$*Continuous but is not is* $(i,j)-pw$*Continuous. Because the* $cl\,\{d\}=\{c,d\} \notin$ *regular open set in* T_i.

Example 3.28: *Let* $X=\{a,b,c,d\}$, $Y=\{1,2,3,4\}$*be bitopological spaces with topologies* $T_i=\{\emptyset,X,\{a\},\{a,b\},\{b\},\{a,b,c\}\}$, $T_j=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$.$\sigma_i=\sigma_j=\{\emptyset,Y,\{2\},\{1,3\},\{1,2,3\}\}$, *and let* $f:X \longrightarrow Y$ *be function defined by* $f(a)=f(b)=\{4\}, f(c)=\{1\}$, $f(d)=\{2\}$, *then*f*is* $(i,j)-pw$- *Continuous but is not*$is(i,j)-$ $semi$ *Continuous. Because the* $\{a,b,c\} \notin$*semiclosed set in* T_i.

Remark 3.29:*The following example shows that* $(i,j)-semi$ *continuity is not* $(i,j)-prw$ *continuity.*

Example 3.30*:Let* $X=\{a,b,c,d\}$, $Y=\{1,2,3,4\}$*be a bitopological spaces with topologies* $T_i=\{\emptyset,X,\{a\},\{a,b\},\{b\},\{a,b,c\}\}$, $T_j=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$.$\sigma_i=\sigma_j=\{\emptyset,Y,\{2\},\{1,3\},\{1,2,3\}\}$, *and let* $f:X \longrightarrow Y$ *be function defined by* $f(a)=\{1\}$ $f(b)=\{4\}$, $f(c)=f(d)=\{2\}$, *then* f *is* $(i,j)-semi$ *Continuous but is not is* $(i,j)-prw$ *Continuous. Because the* $\{d\}is\ not(i,j)-$ prw*closed set in* T_i.

The following diagram shows explains the relations among all the concepts introduced in the above section :



Reference

[1] Arockiarani, I., Studies on generalizations of generalized closed sets and maps in topological spaces, Ph.D., Thesis, Bharathiar Univ., Coimbatore,1997.

[2] Benchalli,S.S. and Wali.R.S., "On RW-closed sets in topological spaces", Bull.Malays,.Math.Sci.Soc(2) 30(2) (2007), 99-110.

[3] Binoy, B. K.and Janaki , C.," On pgrα- closed sets in topological spaces", International Journal of Math. Archive-3(5), 2012, 2114-2121.

[4] Bose.S., Sinha.D.," Almost open, almost closed, Θ-continuous and almostcompact mappings in bitopological spaces ",.Bull.Calcutta Math.Soc.73(1981),345-354.

[5] Fukutake, T.," On generalized closed sets in bitopological spaces", Bull.Fukuoka Univ. Ed. Part III, 35 (1986), 19–28.

[6] Fukutake, T., Sundaram, P. and M. Sheik John, "w-closed sets, w-open sets and w-continuity in bitopological spaces", Bull. Fukuoka Univ. Ed.Part III, 51 (2002),1–9.

[7] Futake, T., Sundaram, P., and Nagaveni, N., "On weakly generalized closed sets weakly generalized continuous maps and Twg spaces in bitopological spaces", Bull.Fukuoka Univ.Ed.Part III, 48(1999), 33-40.

[8] Jelic,M., "A decomposition of pairwise continuity",J. Inst.Math.Comp.Sci.,3,25-29(1990).

[9] Kelly, J. C.,"*Bitopological spaces*", Proc. London Math. Society, 13(1963), 71–89

[10] Karpagadevi, M. and Pushpalatha,A.,"(i,j) –RW-closed set in Bitopological spaces", International Journal of Computer Applications, vol.47,No.6,2012.

[11] Levine, N., " Generalized closed sets in topology" Rend, Gen. Math. Palermo (2) 19 89- 96. (1970)

[12] Maheswari, S.N., and Prasad, R., "Semi open sets and semi continuous functions in bitopological spaces", Math Notae, 26, (1977/78),29-37.

[13] Mashhour.A.S., Abd El-Monsef .M.E. and El-Deeb.S.N."On pre continuous mappings and week precontinuous mappings", Proc Math, Phys.Soc.Egypt 3(1982),47-53.

[14] Sheik John, M. and Sundaram, P.," g∗-closed sets in Bitopologicalspaces",Indian J. Pure Appl. Math., 35(1) (2004), 71–80.

[15] Stone.M., "Application of the Theory of Boolean rings to general Topology", Trans.Amer.Math.Soc.41(1937),374- 481.

[16] Veera Kumar, M.K.R.S. , "Between closed sets and g-closed sets", Mem. Fac. Sci. Kochi Univ.(Math.) 21(2000), 1–19.

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الخلاصة :-

في هذا البحث تم تقديم ودراسة انواع جديدة من المجموعات المغلقة، تسمى المجموعات المغلقة القبلية الضعيفة والمجموعات القبلية المنتظمة الضعيفة وكذلك تم تقديم الدوال المستمرة القبلية الضعيفة والدوال المستمرة القبلية المنتظمة الضعيفة، ودراسة العلاقات بينهم .

مفتاح الكلمات:

(i,j)- المجموعة المغلقة القبلية المنتظمة الضعيفة ، (i,j)- المجموعة المغلقة القبلية الضعيفة ،(i,j)- الداله المستمرة المغلقة القبلية المنتظمة الضعيفة، (i,j)- الداله المستمرة المغلقة القبلية الضعيفة.