

Minimization of the spherical aberration coefficients of composed of quadrupole and octupole lenses using the rectangular field distribution model

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The optimization calculations are made to find the optimum of the spherical aberration coefficients due to octupole presented. The latter can be combined with the quadrupole lens. The equations are given for composed lenses in the approximation of rectangular model of the field distribution. From the calculations it is found that the spherical aberration coefficients can be reduced to negative value. These results are similar to the results published by Fishkova, T. Ya. and Yavor, S. Ya.(1968) where the negative and positive values are at the found but the present results are the best to give rise to minimum spherical aberration coefficient. These computations have been concentrated on determined the spherical aberration coefficients in both convergence and divergence planes, and also the effects of changing the excitation of lens, the coefficient (γ) and the effective length of lens "L"(geometrical dimensions) with the changing of object and image distances studied and are taken into account.

Key word: The combined of quadrupole and octupole lenses, The spherical aberration, Rectangular model.

1.Introduction:

The invention of quadrupole - octupole and sextupole correctors in the 1990s to compensate for spherical aberration enabled atomic resolution in transmission electron microscopes and these correctors are now used in commercial high-end apparatuses[1]. There are two main classes of aberration corrector in use today. A quadrupole-octupole corrector uses quadrupoles to distort the beam and octupoles to correct the third-order aberrations [2]. The electron-optical properties of short double quadrupole, hexapole, and octupole electromagnetic lenses were analyzed. Expressions were derived for the effect of these lenses on the trajectory of an electron beam. It was shown that the use of these lenses in a system for focusing and deflecting an electron beam can correct distortion, astigmatism, and coma[3]. The chromatic aberrations are corrected by electric and magnetic quadrupole lenses, but quadrupole – octupole correctors are well suited for correcting chromatic and spherical aberrations[4]. The hexapole corrector has the simplest structure yet

eliminates only third-order spherical aberration and coma. The quadrupole - octupole corrector eliminates chromatic aberration by means of crossed electric and magnetic quadrupoles and the third-order spherical aberration by octupoles [5]. The aperture aberrations of quadrupoles, like those of round lenses, cannot change sign and complete correction of these aberrations require octupoles. These may be a separate element with eight electrodes or poles or the quadrupoles may be designed in such a way that they produce octupole fields as well as quadrupole effects. We consider these two situations separately. The octupole aberration coefficients for several situations, the thin octupole separated from and superimposed upon a rectangular model quadrupole [6]. Quadrupole lenses are commonly used for focusing electron and ion beams of high energy. They are also used in electron optical devices of moderate accelerating voltage when astigmatic focusing is required. In high-performance cathode-ray tube they are used for scan expansion to provide an increase in deflection sensitivity. In electron and mass spectrometers containing sector magnets or electrostatic cylindrical analyzers which are themselves astigmatic, quadrupole lenses enable better beam matching than conventional axially symmetrical lenses. There are also some applications of astigmatic quadrupole lenses in probe forming systems when an elliptical or linear beam spot is needed rather than around one. The spherical aberration can be overcome in quadrupole - octupole correctors but these systems are complicated both in construction [7]. The quadrupole - octupole triplets when the octupoles are excited. The chromatic aberration is not affected by exciting the octupoles and remains the same. The spherical aberration is reduced but not completely eliminated. The reason for this is, the symmetric triplet three octupoles combined with quadrupole lenses cannot provide a complete correction of the spherical aberration. The minimum spot size has been reached by gradually increasing all three octupole voltages in such a way that the beam spot in the image plane is reduced in all directions [8].

2. Theoretical part:

The spherical aberration of a quadrupole and octupole lenses in the form [9]:

$$\Delta x(z_c) = M_c x_o' (C_p x_o'^2 + C_s y_o'^2) \quad (1)$$

$$\Delta y(z_d) = M_d y_o' (D_p y_o'^2 + D_s x_o'^2) \quad (2)$$

It is assumed here that the symmetry planes coincide for the electrostatic elements and that the antisymmetry planes coincide for the magnetic elements. The quantities $\Delta x(z_c)$ and $\Delta y(z_d)$ are the aberrations of the width of line images (real and virtual) in the planes z_c and z_d respectively, M_c and M_d are the magnifications of the quadrupole lens in the convergence and divergence planes, and x_o' and y_o' are the entrance angles to the system. The coefficients C and D are the spherical aberration coefficients associated with the quadrupole and octupole elements.

If an octupole is combined with a quadrupole whose field is also approximated by a rectangle having an effective length of lens (L).

$$Cp = -\frac{\gamma L}{8\beta^4} \left[3(1 + \beta^2 a_c^2)^2 + 4\frac{a_c}{L}(1 + \beta^2 a_c^2)(1 - \cos(2\beta L)) - 4(1 - \beta^4 a_c^4) \frac{\sin(2\beta L)}{2\beta L} - \frac{a_c}{L}(1 - \beta^2 a_c^2)(1 - \cos(4\beta L)) + (1 - 6\beta^2 a_c^2 + \beta^4 a_c^4) \frac{\sin(4\beta L)}{4\beta L} \right] \quad (3)$$

$$Cs = Ds = \frac{3}{8} \frac{\gamma L}{\beta^4} \left[-2(1 + \beta^2 a_c^2)(1 - \beta^2 a_d^2) - \frac{a_c}{L}(1 - 3\beta^2 a_d^2) - \frac{a_d}{L}(1 + 3\beta^2 a_c^2) + 2(1 - \beta^2 a_d^2) \left((1 - \beta^2 a_c^2) \frac{\sin 2\beta L}{2\beta L} + \frac{a_c}{L} \cos(2\beta L) \right) + 2(1 + \beta^2 a_c^2) \left((1 + \beta^2 a_d^2) \frac{\sinh(2\beta L)}{2\beta L} + \frac{a_d}{L} \cosh(2\beta L) \right) + \left(\frac{a_c}{L}(1 + \beta^2 a_d^2) - \frac{a_d}{L}(1 - \beta^2 a_c^2) \right) \sin(2\beta L) \sinh(2\beta L) - \left(\frac{a_c}{L}(1 + \beta^2 a_d^2) + \frac{a_d}{L}(1 - \beta^2 a_c^2) \right) \cos(2\beta L) \cosh(2\beta L) - \left((1 - \beta^2 a_c^2)(1 + \beta^2 a_d^2) - 4\beta^2 a_c a_d \right) \cosh(2\beta L) \frac{\sin(2\beta L)}{2\beta L} - \left((1 - \beta^2 a_c^2)(1 + \beta^2 a_d^2) + 4\beta^2 a_c a_d \right) \cos(2\beta L) \frac{\sinh(2\beta L)}{2\beta L} \right] \quad (4)$$

Here $\beta^2 = \beta_m^2 + \beta_E^2$, β is the excitation of the quadrupole lens, β_m is the magnetic excitation of the quadrupole lens, β_E is electrostatic excitation of the quadrupole lens, the coefficient D_p is derived from equation (3) by the replacing β to $i\beta$.

Here a_c and a_d are the distances from the object to the entrance to the field in the convergence and divergence planes [8].

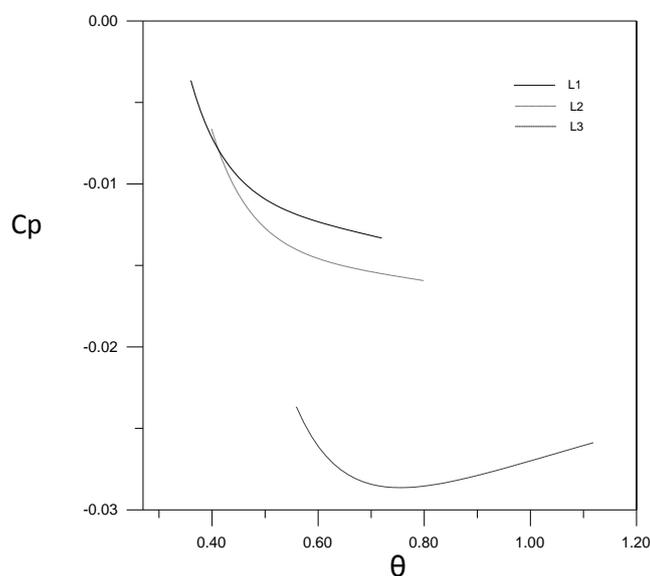
3. Results and Discussion:

The purpose of present work is to find the optimum properties of combined quadrupole and the octupole lenses to produce reduce spherical aberration coefficients in convergence and divergence planes. The model of potential rectangular model is used to find the optimum design of the lens, to find the optimum values of spherical aberrations coefficients. These effects of changing, the effective length of lens "L"(geometrical dimensions), the coefficient (γ) and excitation of lens with changing object and image distances. The present results are best to give minimum spherical aberration coefficients than that of results to give Baranova, L.A. ,Read, F.H. and Fishkova, T. Ya. ,Yavor, S. Ya.

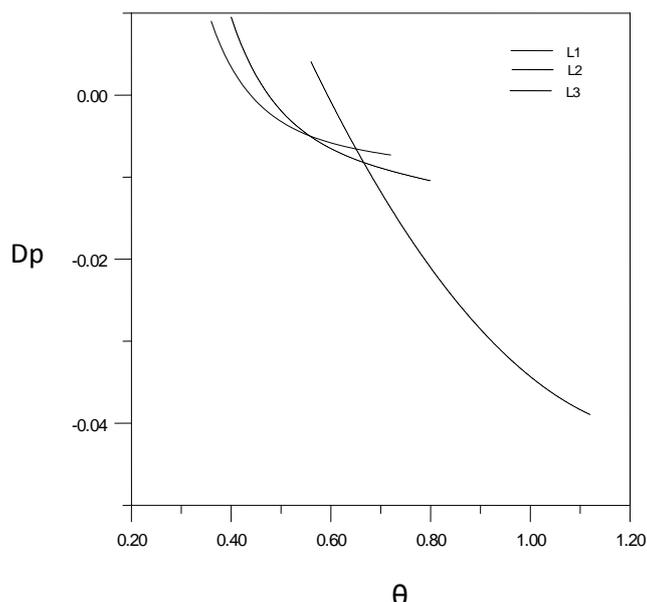
The relation between the spherical aberration coefficients and relative $\theta = \beta L$ is shown for both convergence and divergence planes in figures (1) to (4) , when (β) equal between $(0.4 \text{ mm}^{-1} \text{ to } 0.8 \text{ mm}^{-1})$ in three values of effective length of lens ($L1= 0.9 \text{ mm}$, $L2= 1\text{mm}$ and $L3=1.4\text{mm}$). In case the coefficient (γ) are related to the parameter of the octupole lens equal (0.001) .

Figure(1) the spherical aberration coefficient (C_p) is decreasing as the relative $\theta = \beta L$ increasing where the value effective length of lens ($L3$) the behavior is found for decreasing and increasing when value $(\theta = 0.8)$.

The results of spherical aberration coefficient (D_p) in case divergence plane are shown in figure (2). The calculations shows that the values of spherical aberration coefficient decrease as θ is increasing. In the region where the effective length of lens ($L3 = 1.4 \text{ mm}$) the results give the reduce spherical aberration coefficient values.

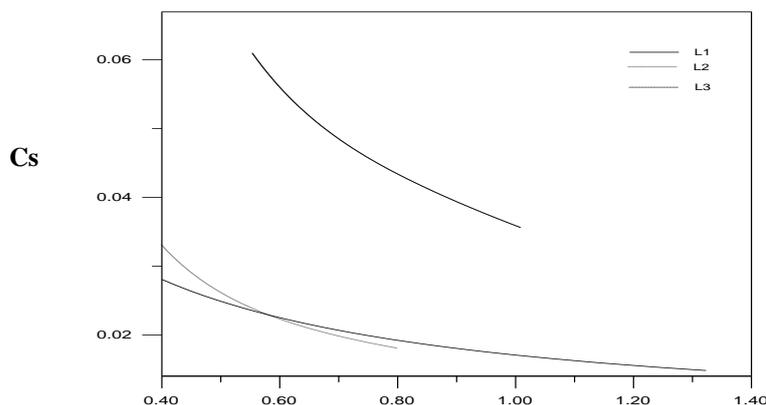


Figure(1): The relative spherical aberration coefficients C_p as a function of relative θ (βL) for the combined quadrupole and octupole lenses in convergence plane for the effective lengths of lens $L1=0.9\text{mm}$, $L2=1\text{mm}$ and $L3= 1.4\text{mm}$.

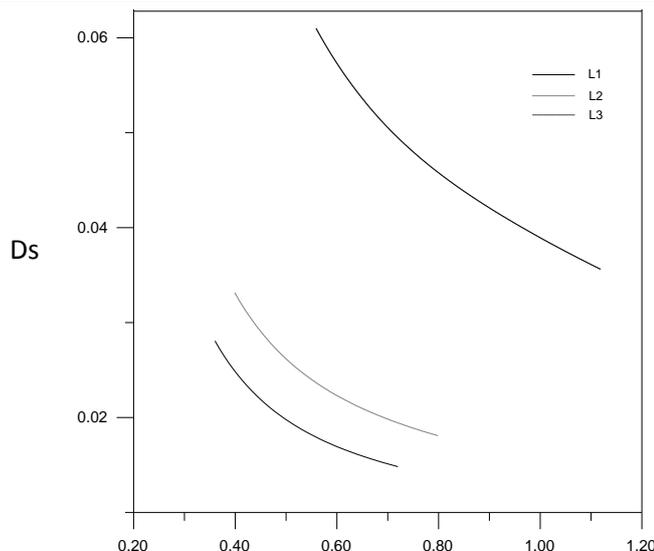


Figure(2): The relative spherical aberration coefficients D_p as a function of relative θ (βL) for the combined quadrupole and octupole lenses in divergence plane for the effective lengths of lens $L_1=0.9\text{mm}$, $L_2=1\text{mm}$ and $L_3= 1.4\text{mm}$.

In the cases of convergence and divergence planes the results are shown in figures (3) and (4) the relative coefficients (C_s) and (D_s), respectively. the relation between the spherical aberration coefficients (C_s , D_s) and (θ) is shown in figures become less as (θ) is increasing. The effective length for lens ($L_1 = 0.9 \text{ mm}$) has the values better than that which were obtained for absence spherical aberration coefficient.



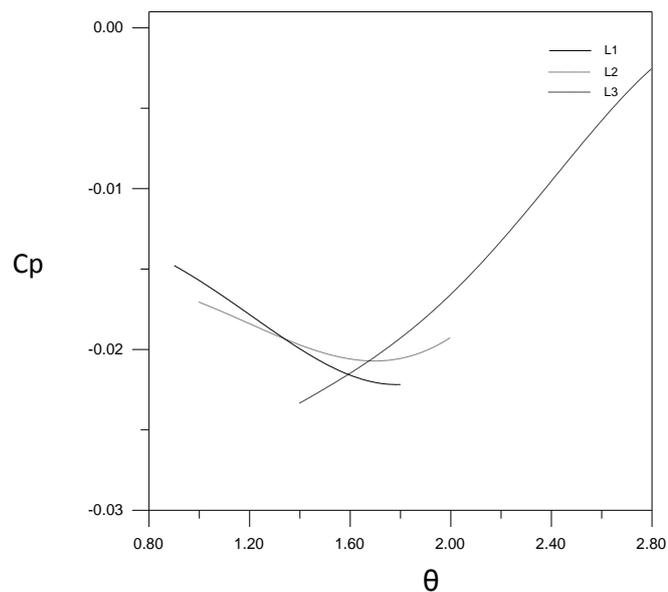
Figure(3): The relative spherical aberration coefficients C_s as a function of relative θ (βL) for the combined quadrupole and octupole lenses in convergence plane for the effective



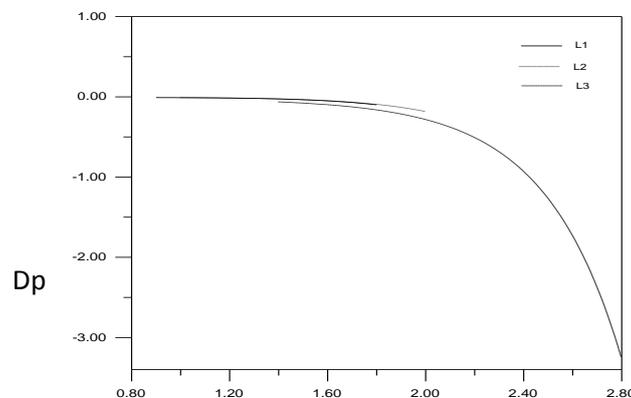
Figure(4): The relative spherical aberration coefficients D_s as a function of relative θ (βL) for the combined quadrupole and octupole lenses in divergence plane for the effective lengths of lens $L_1=0.9\text{mm}$, $L_2=1\text{mm}$ and $L_3= 1.4\text{mm}$.

The calculations for spherical aberration coefficients in both convergence and divergence planes are made for three values of effective length of lens ($L_1= 0.9\text{ mm}$, $L_2= 1\text{mm}$ and $L_3= 1.4\text{mm}$) when β equal between (1mm^{-1}) to (2mm^{-1}) to produce less spherical aberration coefficients. In case of the coefficient (γ) are related to the parameter of the octupole lens equal (0.001).

The plot between the spherical aberration coefficient and relative (θ) is shown for both convergence and divergence planes in the figures (5) and (6), respectively. The results appear in figure (5) showed the spherical aberration coefficient (C_p) decreases as (θ) is increases when the values for effective length of lens $L_1(0.9\text{mm})$ and $L_2(1\text{mm})$ and the behavior is found for the calculations of effective length of lens $L_3(1.4\text{mm})$ for spherical aberration coefficient (C_p) and the relative (θ) increases. The behavior in figure (6) is found for the calculations of spherical aberration coefficient (D_p) decrease as the relative (θ) increases. The calculation shows that the best effective length of lens is L_3 (1.4mm) can be absence spherical aberration coefficient.



Figure(5): The relative spherical aberration coefficients C_p as a function of relative θ (βL) for the combined quadrupole and octupole lenses in convergence plane for the effective lengths of lens $L_1=0.9\text{mm}$, $L_2=1\text{mm}$ and $L_3= 1.4\text{mm}$.

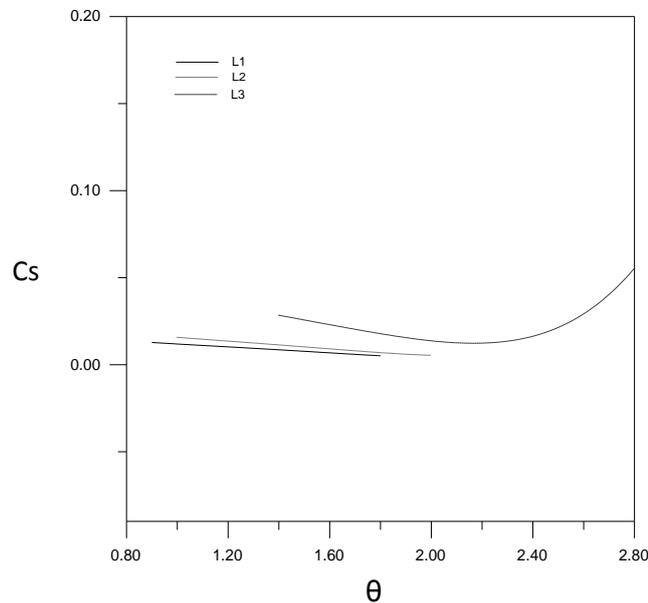


Figure(6): The relative spherical aberration coefficients D_p as a function of relative θ (βL) for the combined quadrupole and octupole lenses in divergence plane for the effective lengths of lens $L_1=0.9\text{mm}$, $L_2=1\text{mm}$ and $L_3= 1.4\text{mm}$.

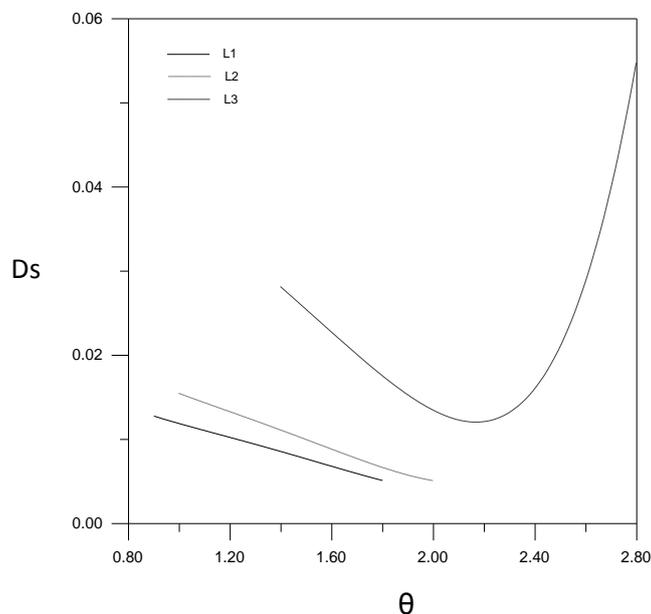
In the convergence and divergence planes, the results are explained in in figures (7) and (8). For both spherical aberration coefficients (C_s) and (D_s) inverse relation between these coefficient and relative (θ) is found as shown in figures when effective length of lens ($L_1 = 0.9\text{mm}$) and ($L_2 = 1\text{mm}$). The values of spherical aberration

coefficient (C_s) become greater with (θ) , when $(\theta = 2.4)$ is increasing as is shown in figure (7). While the behavior is found for spherical aberration coefficient (D_s)

become greater with $(\theta = 2.3)$ is increasing as is shown in figure (8).



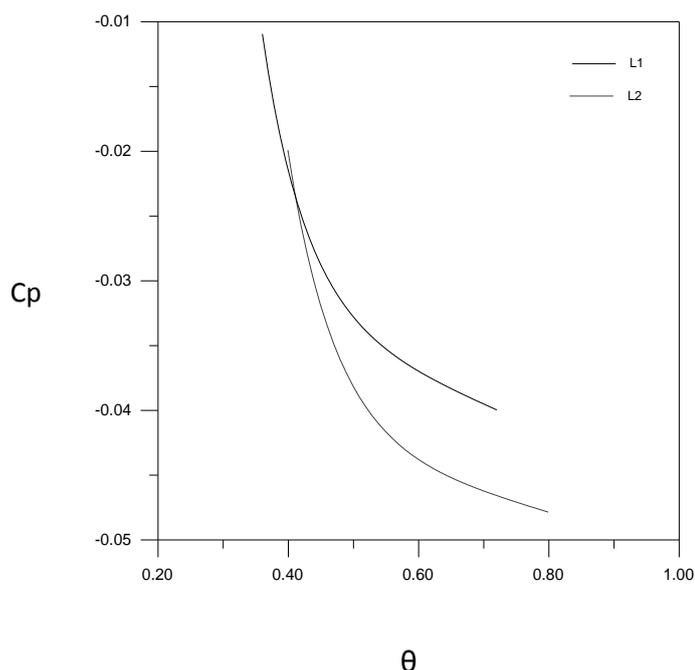
Figure(7): The relative spherical aberration coefficients C_s as a function of relative θ (βL) for the combined quadrupole and octupole lenses in convergence plane for the effective lengths of lens $L_1=0.9\text{mm}$, $L_2=1\text{mm}$ and $L_3= 1.4\text{mm}$.



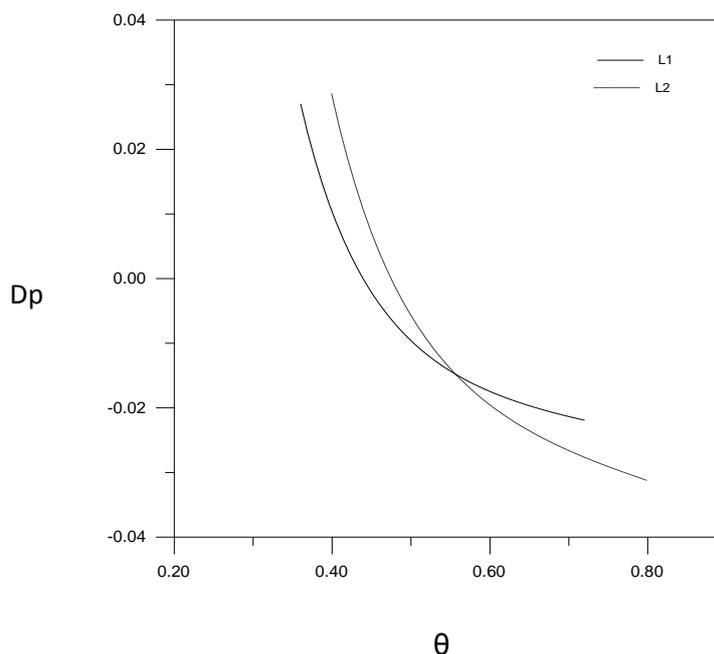
Figure(8): The relative spherical aberration coefficients D_s as a function of relative θ (βL) for the combined quadrupole and octupole lenses in divergence plane for the effective lengths of lens $L_1=0.9\text{mm}$, $L_2=1\text{mm}$ and $L_3= 1.4\text{mm}$.

shown for both convergence and divergence planes in figures (9) to (12), when β equal between (0.4mm^{-1}) to (0.8mm^{-1}) in two values of effective length of lens L1(0.9mm) and L2(1mm). In case the coefficient (γ) are related to the parameter of the octupole lens equal (0.003).

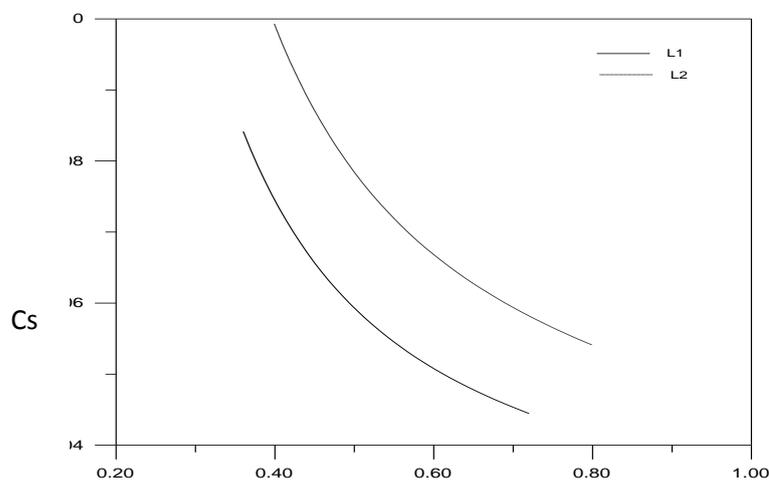
These calculations showed that the values of spherical aberrations coefficients in both figures decrease as θ is increasing. In region where the spherical aberration coefficient (C_p) the results give the best values absence spherical aberration coefficient.



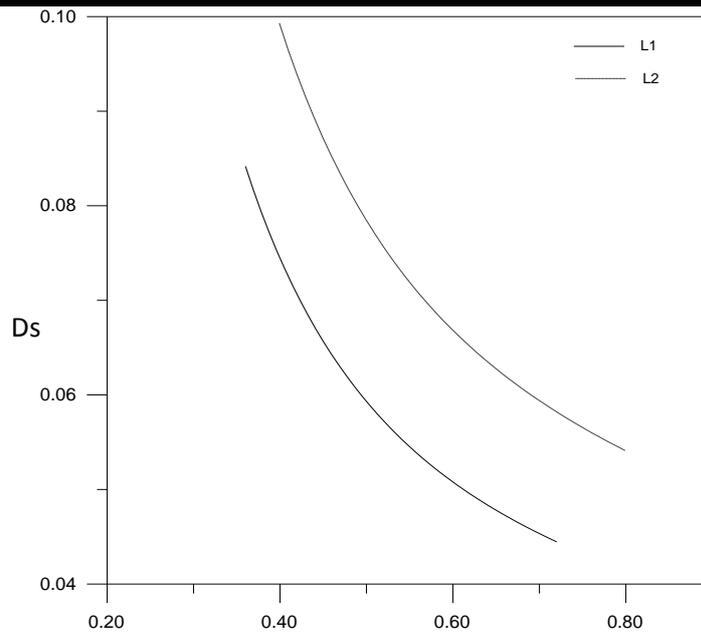
Figure(9): The relative spherical aberration coefficients C_p as a function of relative θ (βL) for the combined quadrupole and octupole lenses in convergence plane for the effective lengths of lens L1=0.9mm and L2=1mm .



Figure(10): The relative spherical aberration coefficients D_p as a function of relative θ (βL) for the combined quadrupole and octupole lenses in divergence plane for the effective lengths of lens $L_1=0.9\text{mm}$ and $L_2=1\text{mm}$.



Figure(11): The relative spherical aberration coefficients C_s as a function of relative θ (βL) for the combined quadrupole and octupole lenses in convergence plane for the effective lengths of lens $L_1=0.9\text{mm}$ and $L_2=1\text{mm}$.



Figure(12): The relative spherical aberration coefficients D_s as a function of relative θ (βL) for the combined quadrupole and octupole lenses in divergence plane for the effective lengths of lens $L_1=0.9\text{mm}$ and $L_2=1\text{mm}$.

Conculsion:

From these calculations, its appear that the rectangular model can be used to represent the axial field distribution of the combined quadrupole and octupole lenses in convergence and divergence planes to find the optimum design of the reduce spherical aberration coefficients and the parameter of the lens (β), the coefficient (γ) and the effective length of lens (L) can be used as effective parameters to reduce the values of the spherical aberration coefficients where :

1. In case the value of the effective length of lens $L_1(0.9\text{mm})$ gives the best values of the absence spherical aberration coefficients in figures (3),(4), (7) and (8).
2. In case the value of the effective length of lens $L_2(1\text{mm})$ give the lower values of the spherical aberration coefficient in figure (1).
3. The value of the effective length of lens $L_3(1.4\text{mm})$ gives the less values of the spherical aberration coefficient in figures (2),(5) and (6).
4. The calculations showed that the values of spherical aberration coefficients in figures from (9) to (12) , these results give the reduce values of spherical aberration coefficients in figure (9) when the effective length of lens $L_1(0.9\text{mm})$ and $L_2(1\text{mm})$, where the effective length of lens $L_3(1.4\text{mm})$ no give best value the absence spherical aberration coefficient.

According to the above results, the designer can choose the values of the parameter of the lens (β), the coefficient (γ) and the effective length of lens (L) to reach a favorable design.

From the calculation one can find the negative values of the aberration coefficients, therefore, the designer can use a combined quadrupole and octupole lens with these conditions, as a corrector element in the optical system to absence the total aberration of the system.

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التقليل من معاملات الزيغ الكروي تتألف من عدسات رباعية الاقطاب و octupole باستخدام نموذج التوزيع مستطيلي مجال

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الخلاصة:

حسابات الأمتلية أجريت لإيجاد معاملات الزيغ الكروي في التعامل مع العدسة octupole مع جمعها مع عدسة quadrupole. وبنظر إلى المعادلات التي أعطيت في تقريب النموذج المستطيلي لتوزيع الجهد. ومن حسابات تبين معاملات زيغ الكروي يمكنها خفضها إلى قيم سالبة. وهذه نتائج مشابهة لنتائج التي نشرت من قبل Fishkova, T. Ya. and Yavor, S. Ya.(1968) حيث تم حصول على نتائج موجبة وسالبة والنتائج التي تم الحصول عليها تعطي الحد الأدنى لمعاملات الزيغ الكروي. الحسابات ركزت على إيجاد الزيغ الكروي لكلا المستويين التقاربي والتباعدي، وكذلك تأثيرات تغير التهيج للعدسة، ومعامل (γ) وطولها الفعال مع تغير ابعاد الجسم والصورة درست وأخذت بنظر الاعتبار.

المفتاحية: العدسة رباعية الاقطاب مع عدسة octupole، الزيغ الكروي، الانموذج المستطيلي.