On Types of S_β- Functions

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Abstract

In this paper we introduce and study another types of functions in a topological spaces namely , S_β-compact and S_β-coercive by using the concept of S_β -open sets . Also we investigate some properties of these concepts and the relation between them.

Keywords: S_β-open, S_β-closed, S_β-compact space , S_β-continuous , S_β-compact, S_β-coercive.

الخلاصة

في هذا البحث قدمنا انواع من الدوال في الفضاءات التبولوجية أسميناها الدوال (المرصوصة -S_β) و(الاضطرارية S_β)- بأستخدام مفهوم المجموعة ($S_\beta$open-) وقدمنا بعض الخواص المبرهنات حول هذه الدوال والعلاقة بينهما.

الكلمات المفتاحية: المفتوحة- $S\beta$ ، مغلقة - $S\beta$ ،الفضاء المضغوط- $S\beta$ ، المستمر- $S\beta$ ، المدمجة- $S\beta$ ، القسرية- $S\beta$.

Introduction

In 1963, Levine initiated the notation of semi-open (briefly S-open) sets and study their properties of this concept. Throughout the properties in topological spaces. In 1982 , AbdEl-monsef defined the class of β-open set. Also in 2012, Kalaf B. and others were introduced a new concept denoted by S_β-open set. Finally, in 2013 , Kalaf B. and Ahmed K. wre introduced a new type of compact spaces namely S_β-compact. Present paper, (X,T) and (Y,T') (or simply X and Y) denote topological spaces . The closure (resp. interior) of a subset A of a space X which denoted by $cl(A)$ (resp. $int(A)$) . A subset A of X is called S-open (resp. β-open) , if $A \subseteq cl(int(A))$ (resp. $A \subseteq cl(int(cl(A)))$) . The complement of S-open (resp. β-open) set is called S-closed (resp. β-closed) set.

Finally in section two , we give some basic properties of new types of functions and relation between them .

1. Basic definitions and notations:

We introduce some elementary concept which we need in our work.

1.1. Definition: [Al-Sheikhly, 2003]

A topological space X is called:

i. Locally indiscrete if every open subset of X is closed.

ii. Hyper-connected if every non-empty open subset of is dense.

1.2. Definition: [Kalaf, 2012]

i. A S-open subset A of a topological space X is called S_β-open if for all $x \in A$, there is a β-closed set F such that $x \in F \subseteq A$. A complement of S_β-open is called $S\beta$-closed.

ii. A subset A of a topological space X is called S_β-open if and only if A is S-open and it is a union of β-closed sets.
iii. A subset N of a topological space X is called S_β-neighborhood of a subset A of X, if there is a S_β-open set U such that $A \in U \subseteq N$. When $A = \{x\}$ we say that N is S_β- neighborhood of x .
iv. A topological space X is called S_β-compact , if for every S_β-open cover of X has finit subcover. Clearly, every S-compact space is S_β-compact.

<u>1.3. Remark:</u> [Kalaf, 2013]
i. If X is a T_1-space, then every S_β-open subset in X is S-open.
ii. A space X is hyper-connected if and only if a S_β-open subsets of X are $\emptyset$ and X .
iii. If a space X is locally indiscrete, then every S-open subset of X is S_β-open.

<u>1.4. Theorem:</u> [Kalaf , 2012]
i. If B is clopen subset of a topological space X and A is open , then $A \cap B$ is S_β-open .
ii. Let $A \subseteq Y \subseteq X$, if A is S_β-open subset in X and Y is open subset in X , then A is S_β-open subset in Y .
iii. Let $A \subseteq Y \subseteq X$, if A is S_β-open subset in Y and Y is clopen subset in X , then A is S_β -open subset in X .

<u>1.5.Definition:</u> [Al-Sheikhly, 2003;Kalaf, 2013]
Let $f: X \to Y$ be a function , then f is called:
i. S-compact if the inverse image for every compact set in Y is S-compact set in X .
ii. S_β-closed if the image for every closed set in Y is S_β-closed set in X .
ii. S_β-irresolute if the inverse image for every $S\beta$-open set in Y is S_β-open set in X .

2. Type of $S\beta$-Functions:

In this section , we introduce a new S_β-functions called S_β-compact and S_β-coercive functions.

<u>2.1. Definition:</u>

A function $f: X \to Y$ is said to be S_β-compact if the inverse image for every compact set in Y is S_β-compact set in X .

<u>2.2. Example:</u>
i. An identity function $f: (X,T) \to (X,T')$ with $X = \mathcal{R}$, $T = \{\emptyset, X, \{0\}\}$ and $T' = T_{ind}$ is S_β- compact.
ii. Let $X = \{1, 2, 3\}$ and $Y = \{2, 4, 6\}$ with topologies $T = T_{ind}$, $T' = \{\emptyset, Y, \{4\}\}$ resp. A function $f: X \to Y$ defined by $f(x) = 2x, \forall x \in X$ is S_β-compact, since X is locally indiscrete space [by using remark (1.3.iii)] .

The following example shows that not every function is S_β-compact.

2.3. Example:

Consider a countable set X with co-countable topology, then the constant function from X into any space Y is not S_β-compact.

2.4. Theorem:

Every S-compact function is S_β-compact .

Proof: By using definition (1.2.iv) , this is just the condition of our theorem.

The converse of the Theorem above is not true in general as the example shows:

2.5. Example:

Let $X = \mathcal{R}$ with topology $T = \{\emptyset, X, \{0\}\}$, then a function $f: (X,T) \to (X,T_U)$ which defined by $f(x) = 0, \forall x \in X$ is S_β-compact, but not S-compact.

In general a S_β-compact and compact functions are independent as the following examples:

2.6. Example:

A function in example (2.3) is compact, but not S_β-compact.

2.7. Example:

Let $X = (0,1)$ with topology $T = \left\{\emptyset, X, G = \left(0, 1 - \frac{1}{n}\right)\right\}, n = 2, 3, \ldots$. A constant function f from a space X is a S_β-compact , but not its compact.

Recall that every S-compact function is compact **[Al-Sheikhly H. 2003]**, the converse is not true as the following example shows:

2.8. Example:

Let $X = \mathrm{I} \cup \{x\}$ with I uncountable set and $x \notin \mathrm{I}$, let $T = \{\emptyset, X, \{x\}\}$ be a topology on X , a constant function from X into itself is compact but not S-compact , because X is not S-compact space , since $\{ \{x, x_i\} : i \in \mathrm{I} \}$ is a S-covering of X but has no finite subcover .

2.9. Theorem:

Let $f: X \to Y$ be a S_β-compact function and A be a clopen subset of X , then $f_{|A}: A \to Y$ is also S_β -compact.

Proof:

Let K be a compact subset of Y, then $f^{-1}(K)$ is S_β-compact set in X, since A clopen in X . Then by Theorem (1.4.iii) , $A \cap f^{-1}(K)$ is a S_β-compact set in A, but $A \cap f^{-1}(K) = f_{|A}^{-1}(K)$, then $f_{|A}$ is S_β- compact.

2.10. Remark:

A composition of two S_β-compact functions not necessary S_β-compact.

2.11. Theorem:

Let $f: X \to Y$ and $g: Y \to Z$ be two functions, then:

i. If f and g are S_β-compact and Y be a locally indiscrete, then gof is S_β-compact.

ii. If f and g are S_β-compact and Y be a T_1-space, then gof is S_β-compact.

iii. If g compact and f be a S_β-compact, then gof is S_β-compact.

iv. If g be a S-compact and f be a S_β-compact, then gof is S_β-compact.

2.12. Definition:

A function $f: X \to Y$ is said to be S_β-coercive if for every S_β-compact subset B of Y there is S_β-compact subset A of X such that $f(X/A) \subseteq (Y/B)$.

2.13. Example:

i. The identity function for any space is S_β-coercive.

ii. Let $X = \{1, 2, 3\}, Y = \{4, 5\}, T_X = \{\emptyset, X, \{3\}\}$, $T_Y = T_{ind}$ and $f: X \to Y$ be a function which defined by $f(1) = f(2) = 4$, $(3) = 5$, then f is S_β-coercive .

2.14. Theorem:

A S_β-compact function from S_β-compact space is S_β-coercive.

Proof:

Let $f: X \to Y$ be a S_β-compact function and let B be a S_β-compact subset of Y . Since X be a S_β-compact. Then$f(X/X) = \emptyset \subseteq f(Y/B)$, thus f is S_β-coercive.

2.15. Theorem:

A restriction S_β-coercive function on clopen subset is S_β-coercive.

Proof:

Let $f: X \to Y$ be a S_β-coercive function and F be a clopen subset of X , to show that $f_{|F}: F \to Y$ is a S_β-coercive function, let B be a S_β-compact subset of Y . Then there is a S_β -compact subset A of X such that $f(X/A) \subseteq (Y/B)$. Since F be a clopen subset of X, then by Theorem (1.4.iii) , $F \cap A$ is $S\beta$-compact subset of F. Since $f_{/F}(F \cap A) = f(F/A)$ and $F/A \subseteq X/A$ $\Rightarrow f(F/A) \subseteq f(X/A) \Rightarrow f_{/F}(F/F \cap A) \subseteq Y/B$, hence $f_{/F}: F \to Y$ is S_β -coercive function.

2.16. Theorem:

A composition of two S_β-coercive functions is S_β-coercive.

Proof: Clear.

Recall that the S_β-irresolute image of S_β-compact is S_β-compact **[Kalaf K. 2012]**.

2.17. Theorem:

Let $f: X \to Y$ and $g: Y \to Z$ be two functions such that:

i. If gof is S_β-coercive with g is S_β-irresolute and one to one, then f is S_β-coercive .

ii. If gof is S_β-coercive with f is S_β-irresolute and onto, then g is S_β-coercive.

Proof:

i. Let B be a S_β-compact subset of Y, then $g(B)$ is a S_β-compact subset of Z . Since gof be a S_β-coercive , then there is a S_β-compact subset A of X such that $gof\,(X/A) \subseteq Z/\,g(B)$, then:

$f(X/A) = g^{-1}(\,gof\,(X/A)) \subseteq g^{-1}(Z/g(B)) = g^{-1}(Z \cap (g(B))^c) = g^{-1}(Z) \cap g^{-1}(g\,(B^c))) =$ Y/B ,thus f is S_β-coercive function.

ii. Let C be a S_β-compact subset of Z . Since gof is S_β-coercive , then there is a S_β-compact subset A of X such that $gof(X/A) \subseteq Z/C$, thus $g(\,f(A^c)\,) \subseteq Z/C$, since f onto we get $g\,((\,f(A))^c)) \subseteq Z/C$. Then $f(A)$ is S_β-compact subset of Y . Thus g is S_β-coercive function.

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