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Study of Properties of ^{146,148}Nd Isotopes by Using IBM-1 Model

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Abstract:

The spectra, B(E2), B(E2) branching ratios and Potential energy surface are studied in the Interacting Boson Model (IBM-1). It is found that^{146,148}Nd are in the transition region U(5)-O(6). The values of B(E2) are increased and quadrupole moment are decreased when the neutron number increased, the branching ratios showed the isotopes in transition region U(5)-O(6). The potential energy surface (P.E.S) shows that ^{146,148}Nd deformed and conductance of U(5)-O(6) and have vibrational mfeature.

Keywords: spectra, B(E2), potential energy surface, ^{146,148}Nd, nuclear structure.

الخلاصة:

مستويات الطاقة الطيفية، رباعي القطب الكهربائي، نسب التفرع وطاقة جهد السطح تمت دراستها باستخدام نموذج البوزونات المتفاعلة الاول (IBM-1). وقد وجد ان ^{146,148}Nd في المنطقة الانتقالية U(5)-O(6). ان قيم رباعي القطب الكهربائي B(E2) تزداد وعزم رباعي القطب(Q) يقل مع زيادة عدد النيوترونات وان نسب التفرع تثبت ان النظائر ضمن المنطقة الانتقالية المذكورة انفا. اما بالنسبة لطاقة جهد السطح (P.E.S) تظهر ان النظائر ^{146,148} Nd مشوهة وتتصرف بشكل O(6)- U(5)و تمتلك الصفة الاهتزازية.

الكلمات المفتاحية: الاطياف، B(E2)، طاقة جهد السطح، ^{146,148}Nd، التركيب النووي.

1-Introduction:

In nuclei, there are two collections of particles: protons, and neutrons. Each group is separately divided over certain energy state subjected to the restrictions of the Pauli Exclusion Principle. Nuclei have excited states, and nucleons can be removed from, or added to, nuclei.a lot datum about nuclear structure can be gained by studying these phenomena [Meyerhof,1967]. The theory of nuclear structure vary markedly from the theory of atomic structure. The atomic electrostatic force holding the atom is central, while that in the nucleus is not. The dynamics of many particles system are more complicated in the nucleus than that in the particles atom [Harvey,1969]. Arima and Iachello in 1974 suggested the nuclear model to describe the collective nuclear structure for medium and heavy nuclei called interacting boson approximation model (IBA or IBM) [Abdul Hussein,2009]. This model used to study the low lying collective states in even-even nuclei through a combination of nucleons resulting from coupling two fermion outside the closed shell with angular momentum L=0 and is called s boson and coupled with angular momentum L=2 and is called d boson [Arima and Iachello,1978] [Arima and Iachello,1979].

2- Interacting Boson Model (IBM-1)

The Hamiltonian of the IBM-1 is written as multipole expansion mutual into equation of various boson _boson interactions [Casten,1990]:

$$\hat{H} = \varepsilon \hat{n}_d + a_0 \hat{P}.\hat{P} + a_1 \hat{L}.\hat{L} + a_2 \hat{Q}.\hat{Q} + a_3 \hat{T}_3.\hat{T}_3 + a_4 \hat{T}_4.\hat{T}_4 \qquad (1)$$

Where ε is the energy of d_ bosons and $\varepsilon = \varepsilon_d - \varepsilon_s$, $\varepsilon_s = 0$, therefore $\varepsilon = \varepsilon_d$, while the parameters $(a_0, a_1, a_2, a_3, a_4)$ represents the interaction strength for paring, angular momentum, quadrupole momentum, octupole and hexadecapole between bosons respectively. Also $\hat{n}_d$ operator produces the number of *d* bosons, $\hat{P}$ stand for the paring operator, $\hat{L}$ represents the angular momentum operator, $\hat{Q}$, is the quadrupole operator, $\hat{T}_3$ and $\hat{T}_4$ stand for the octupole and hexadecapole

operators. For U(5)-O(6) the nuclei have properties between vibrational limit and γ_ unstable limit in the transition region and the Hamiltonian is [Martin,1998]:

$$\hat{H}^{(I+III)} = \varepsilon \hat{n}_d + a_0 \hat{P}.\hat{P} + a_1 \hat{L}.\hat{L} + a_2 \hat{T}_2.\hat{T}_2 \qquad (2)$$

Characteristics of this limit being depends on the ratio ($\varepsilon \hat{n}_d / a_0$).

The vibrational limit performed by sub-group U(5). Hamiltonian function can be known as follows [Arima and Iachello,1981].

$$\hat{H}^{(I)} = \varepsilon \hat{n}_d + a_1 \hat{L}.\hat{L} + a_2 \hat{T}_2 + a_4 \hat{T}_4.\hat{T}_4 \qquad (3)$$

the electric quadrupole transition operators can be written as [8]

$$\hat{T}^{(E2)}_m = \alpha_2 \left[d^\dagger \tilde{s} + s^\dagger \tilde{d} \right]^{(2)}_m + \beta_2 \left[d^\dagger \tilde{d} \right]^{(2)}_m \qquad (4)$$

Where α_2 and β_2 are two parameters used for fitting the experimental results.

The quadruple momentum can be documentary as follow [Raman et al, 1989]:

$$Q_{2_1^+} = \beta_2 \sqrt{\frac{16\pi}{5}} \sqrt{\frac{2}{7}} \qquad (5)$$

The potential energy surface ($E(N,\beta,\gamma)$) allows a final shape to the nucleus that agrees to the function of Hamiltonian, as shown in Eq.(5) [Casten and Warner, 1988]

$$V(N,\beta,\gamma) = \frac{N\varepsilon_d \beta^2}{(1+\beta^2)} + \frac{N(N+1)}{(1+\beta^2)^2}(a_1\beta^4 + a_2\beta^3 \cos 3\gamma + a_3\beta^2 + a_4) \qquad (6)$$

β : is a measure of the total deformation of nucleus(0-2.4). γ: is the amount of deviation from the focus symmetry and correlates with the nucleus.

3- Results and Discussion:

3.1 Energy Levels

The examination of the experimental energy levels for the nuclei ^{146,148}Nd shows that they belong to U(5)-O(6) by the energy ratios $\frac{E_{4_1^+}}{E_{2_1^+}}$, $\frac{E_{6_1^+}}{E_{2_1^+}}$, $\frac{E_{8_1^+}}{E_{2_1^+}}$ has been calculated theoretically for the even-even ^{146,148}Nd isotopes and compared with their corresponding experimental values taken from ref. [ENSDF, 2010] and with the typical values for each limit [Arima and Iachello,1987] [Casten and Warner, 1988]. The parameters so obtained are given in table (1). The Figure (1-3) represent the energy ratios for the isotopes and Figure (4,5) represent the energy levels for each isotopes.

Table (1) The Hamiltonian parameters used in the IBM-code (PHINT) for ^{146,148}Nd isotopes

A	N	EPS (MeV)	$\hat{P}.\hat{P}$ (MeV)	$\hat{L}.\hat{L}$ (MeV)	$\hat{Q}.\hat{Q}$ (MeV)	$\hat{T}_2.\hat{T}_2$ (MeV)	$\hat{T}_4.\hat{T}_4$ (MeV)	CHI	SO6
^{146}Nd	7	0.0249	0.1048	0.0092	-0.0000	0.2799	0.0000	-1.3228	1.0000
^{148}Nd	8	0.0248	0.1048	0.0092	-0.0000	0.1799	0.0000	-1.3228	1.0000

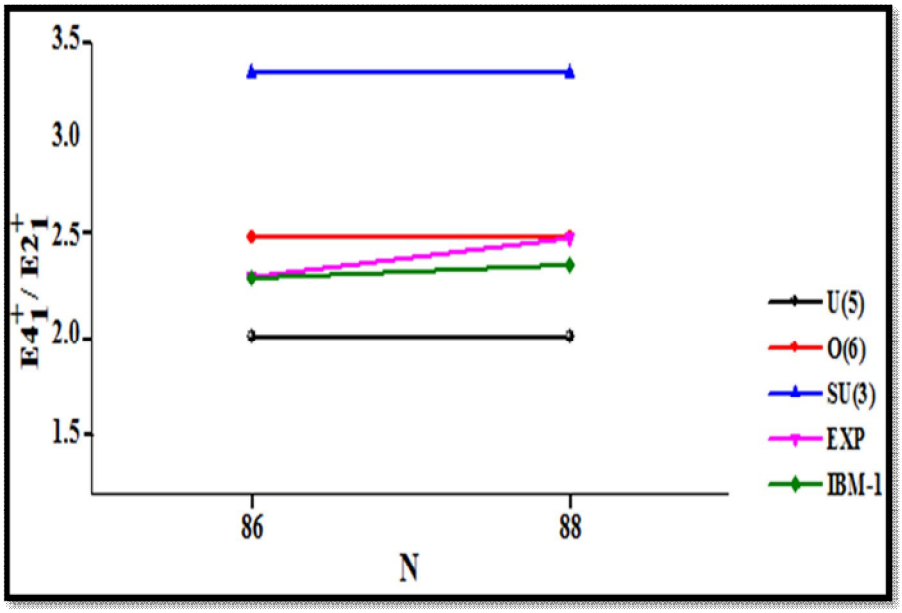
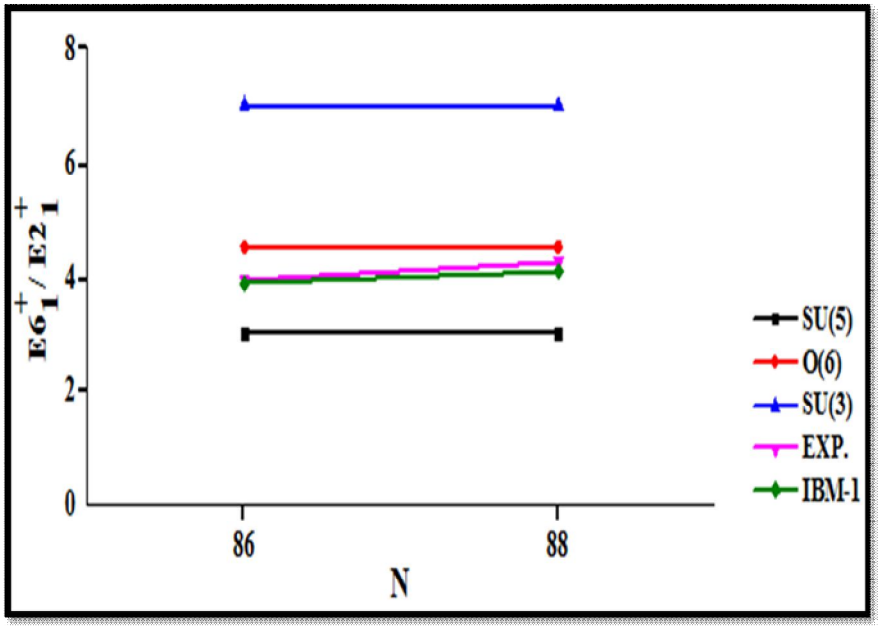
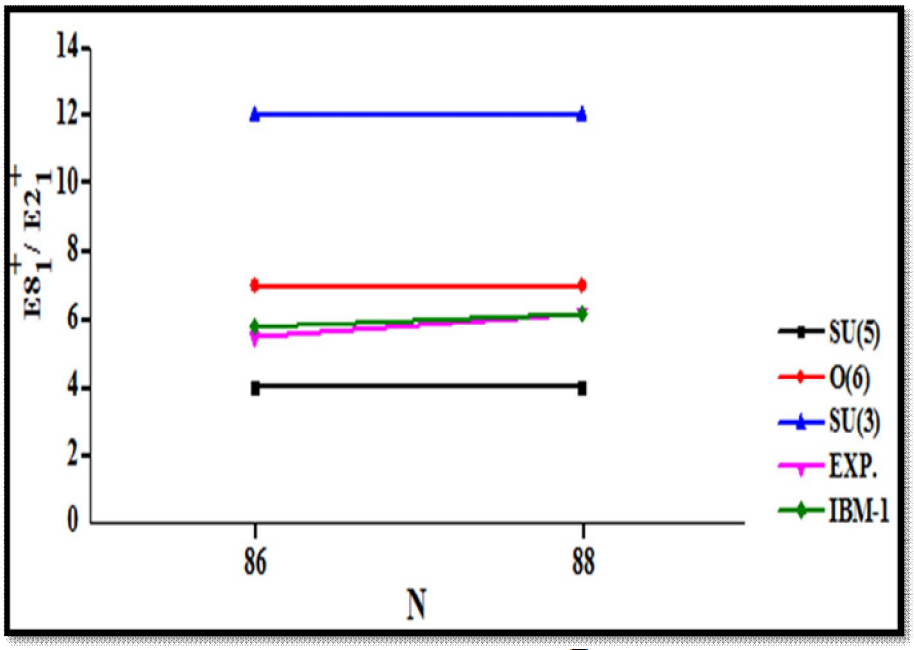


Figure (1) A Comparison of $\frac{E_{4_1^+}}{E_{2_1^+}}$ Theoretically and Experimentally with the typical values for each limit



Theoretically and Experimentally with the typical $\frac{E_{6_1^+}}{E_{2_1^+}}$ **Figure (2)** A Comparison of values for each limit



Theoretically and Experimentally with the typical $\frac{E_{8_1^+}}{E_{2_1^+}}$ **Figure (3)** A Comparison of values for each limit

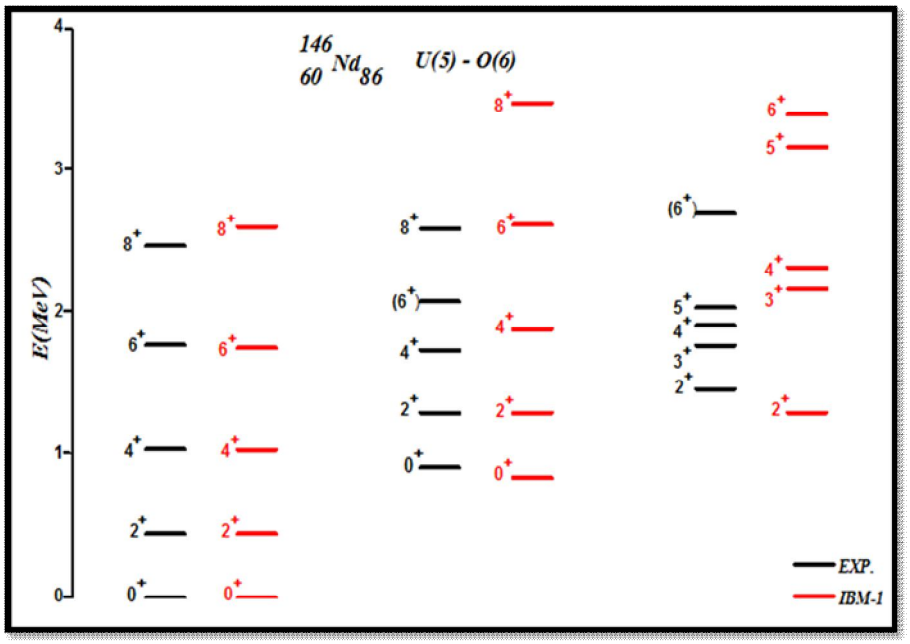
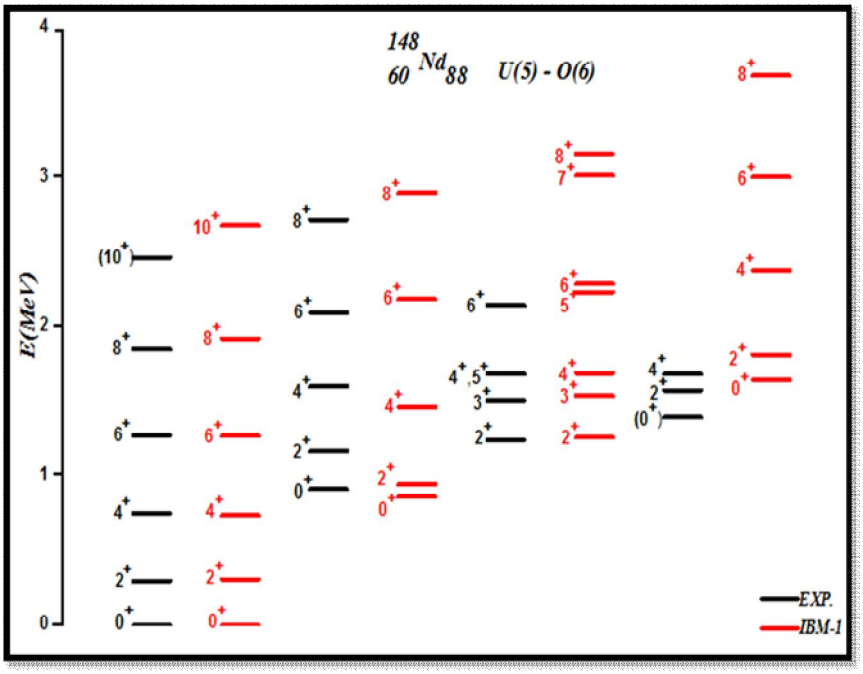


Figure (4) A Comparison of IBM-1 calculations with the experimental data for ^{146}Nd isotope



Figure(5) A Comparison of IBM-1 calculations with the experimental data for ^{148}Nd isotope

3.2 Reduced Transition Probabilities B(E2) and Quadrupl moment $Q_{2_1^+}$

More information can be obtained by studying the reduced transition probabilities B(E2).The (IBMT-code) are employed (α_2 , β_2). The parameters (E2SD) and (E2DD) used in the present calculations are determined by normalizing the calculated values to the experimentally known ones and displayed in (Table 2).

Table (2) The experimental values of B(E2) and the coefficients(E2SD, E2DD) for ^{146} , ^{148}Nd used in the present work.

A	$B(E2: 2_1^+ \rightarrow 0_1^+)$ **e^{2}b^2**	**E2SD (e b)**	**E2DD (e b)**
^{146}Nd	0.1338	0.082998	- 0.045235
^{148}Nd	0.2693	0.150439	- 0.0010

A comparison between the experimental [ENSDF,2010] and calculated $B(E2;2_1^+ \rightarrow 0_1^+)$ are shown in Figure (6) and shows that the results are quite well for the isotopes under study.

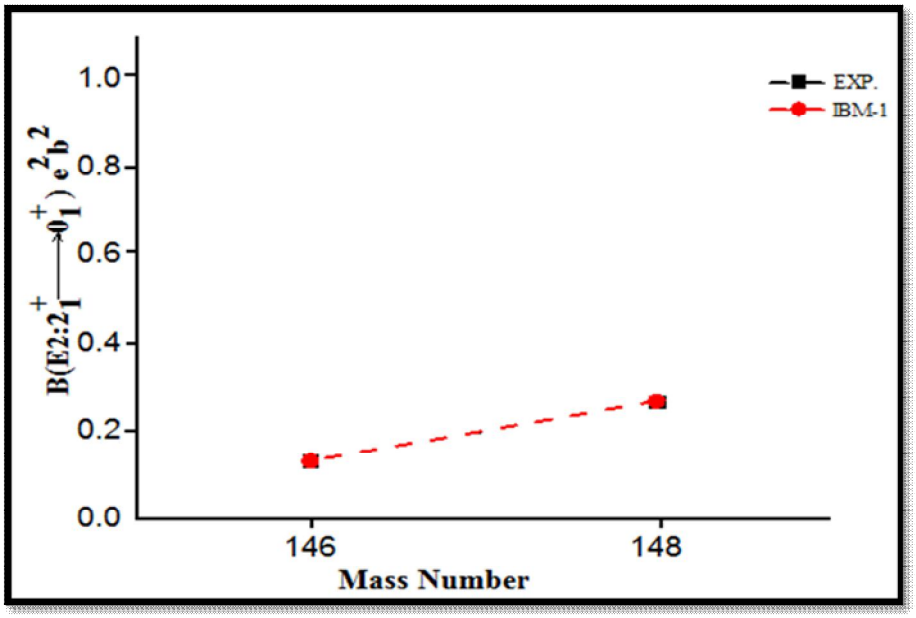


Figure (6) A Comparison of the Experimental data [ENSDF, 2010] and Calculated $B(E2;2_1^+ \rightarrow 0_1^+)$ for ^{146,148}Nd Isotopes

The quadrupole moment (Q) is an important feature for nuclei and is known as the deviation from the spherical charge distribution inside the nucleus and from the qudrupole moment we can set if the nucleus is spherical, distorted oblate or prolate shapes [Muwaffeq,2007]. The values of $Q2_1^+$ so obtained are given in table (3) and Figure (7) represent the comparison between experimental and theoretical values of $Q2_1^+$.

(e b) $Q2_1^+$ **Table (3)** A Comparison between Experimental and Theoretical values of for^{146,148}Nd Isotopes

Isotopes	$Q2_1^+$ **(e b)**	
	Exp.	**Theo.**
^{146}Nd	-0.7664	-0.9998
^{148}Nd	-1.46 [Reich,2012]	-1.4176

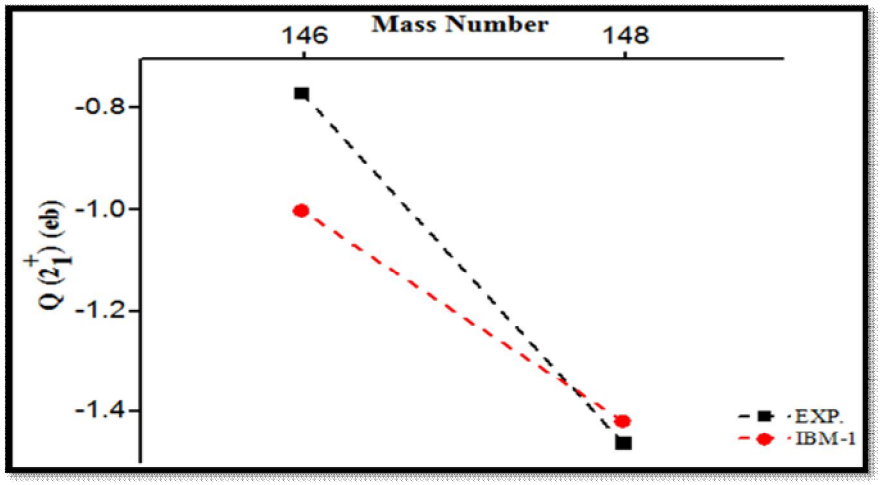


Figure (7) A Comparison Between the Experimental Quadrupole Moment Q(e b) Taken from Ref. [ENSDF, 2010] and the Calculation from Present Work for ^{146,148}Nd Isotopes

3.3 The B(E2) Branching Ratios

The B(E2) branching ratios (R, R' and R^{Th}) are calculated in the present work for ^{146,148}Nd and defined as the ratio between two reduced electric qudarupole transitions. The importance of study the branching ratios is to study the shape of the nucleus and it's dynamical symmetries and to determine which dynamical symmetries. The branching ratios (R, R' and R^{Th}) defined as [Iachello,2001, Cejnar P.,2010]:

$$R = \frac{B(E2;4_1^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)} \tag{7}$$

$$R' = \frac{B(E2;2_2^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)} \tag{8}$$

$$R'' = \frac{B(E2;0_2^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)} \tag{9}$$

The branching ratios (R, R' and R^{Th}) for the three dynamical limits are defined as [Iachello,2001, Cejnar P.,2010]

$$R = \frac{10}{7}\frac{(N-1)(2N+5)}{N(2N+3)} \approx 1.4,\ R' = R'' = 0 \rightarrow \mathrm{SU(3)} \tag{10}$$

$$R = R' = \frac{10}{7}\frac{(N-1)(N+5)}{N(N+4)} \approx 1.4,\ R'' = 0 \rightarrow \mathrm{O(6)} \tag{11}$$

$$R = R' = R'' = 2\frac{(N-1)}{N} = 2 \rightarrow \mathrm{U(5)} \tag{12}$$

The calculated branching ratios and their equivalent experimental values are presented in Table (4). The comparison experimental and calculated branching ratios and the typical values for the three limits are shown in Figure (8).

Table (4) A Comparison between experimental[ENSDF, 2010] and theoretical B(E2) branching ratios for ^{146,148}Nd isotopes

B(E2) Ratios	^{146}Nd		^{148}Nd	
	Exp.	Theo.	Exp.	Theo.
$R = \frac{B(E2;4_1^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)}$	1.47	1.31	1.62	1.29
$R' = \frac{B(E2;2_2^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)}$	-----	0.00	-----	0.03
$R'' = \frac{B(E2;0_2^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)}$	-----	0.0029	0.5389	0.0056

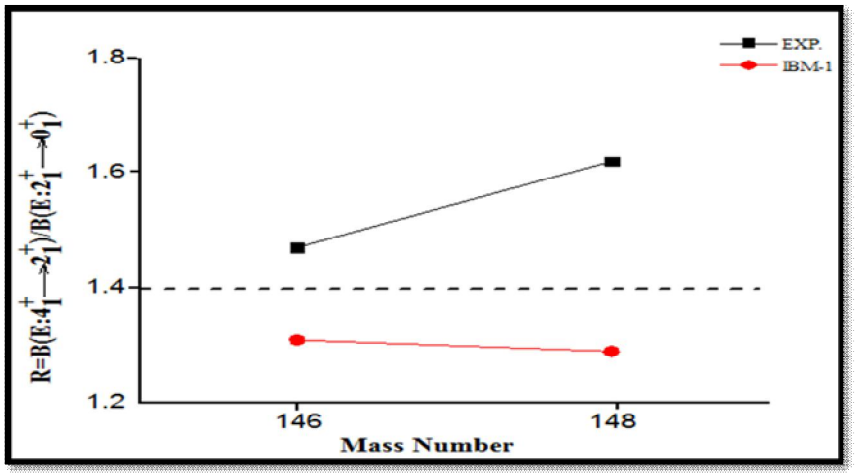


Figure (8) A Comparison between the Experimental and Calculated B(E2) Branching Ratios for even-even ^{146,148}Nd isotopes with the typical values of U(5) and O(6) limits

3.4 Potential Energy Surface (P.E.S)

The P.E.S.FOR program is used to calculate the potential energy surface $E(N,\beta,\gamma)$. the contour plots in the (γ-β) plane resulting from $E(N,\beta,\gamma)$ are shown for ^{146,148}Nd isotopes. For most of the considered Nd nuclei the mapped IBM energy surfaces are triaxial shape. Triaxial shape is associated with intermediate values ($0< \gamma < 60$). The triaxial deformation helps to understand the prolate to oblate shape transition that occurs in the considered Nd isotopes. The parameters so obtained are given in table (5).

Table (5) The Hamiltonian parameters used in the IBMP-code (PHINT) for ^{146,148}Nd isotopes

A	N	EPS (MeV)	EPD (MeV)	A_1 (MeV)	A_2 (MeV)	A_3 (MeV)	A_4 (MeV)
^{146}Nd	7	0.0000	0.4720	0.0260	0.0000	0.0530	0.0000
^{148}Nd	8	0.0000	0.3320	0.0260	0.0000	0.0530	0.0000

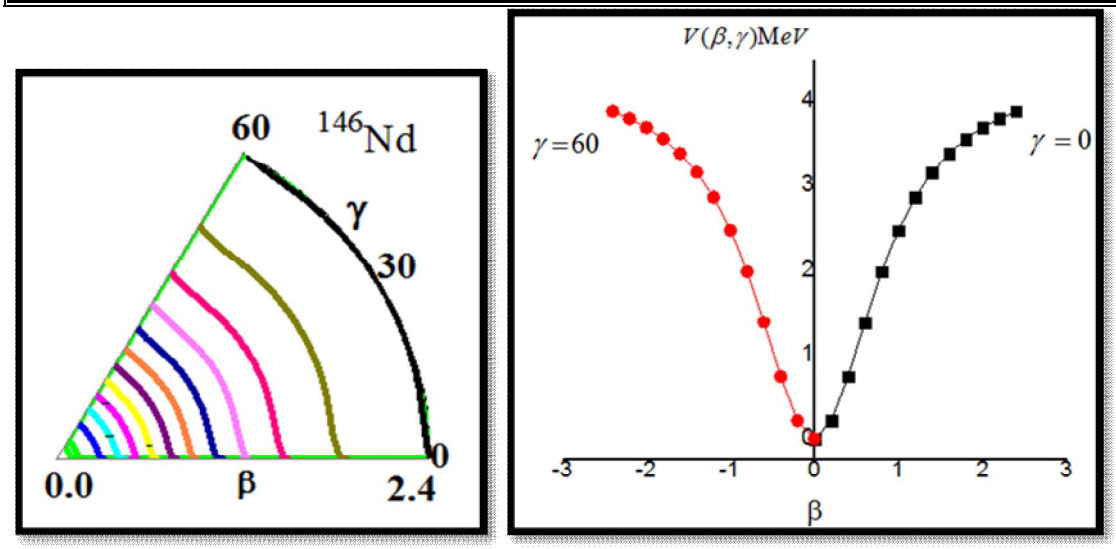


Figure (9) The potential energy surface in γ-β plane for ^{146}Nd isotope

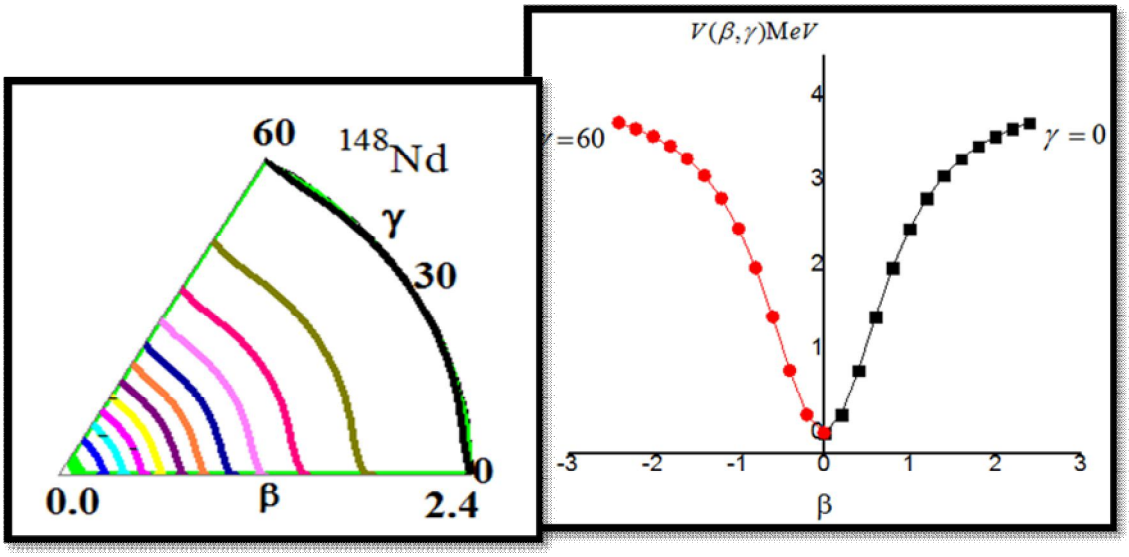


Figure (10) The potential energy surface in γ-β plane for ^{148}Nd isotope

4- Conclusions

1. The even-even^{146,148} Nd isotopes have (60) protons and (86,88) neutrons. The core is taken at major closed shell (82) for protons and neutrons. Therefore, the number of bosons were determined for^{146,148} Nd are 7 and ٨ respectively.
2. The studied structure bands of nuclei ^{46,148}Nd looks as distorted. For **^{146}Nd** in the transitional region U(5)-O(6) as have been pointed out in refs. [Scholten, 1980; Maino and Ventura, 1983] this nucleus is so vibrational. In calculation, this state is nicely reproduced and its consistent with the vibrational pattern. The higher spin states in the ground band up to (10^+) are well reproduced. The present calculation for**^{148}Nd** in the transitional region U(5)-O(6) gives a very good reproduction of the ground-state band. The quasi-beta and quasi-gamma are reasonably well reproduced. There are two (0^+) states at about (1 MeV), one at (0.917MeV) and the other at about (0.97 MeV). The state (0.917MeV) is taken as 0_β and the state at (0.97 MeV) is treated as an intruder state and is not included as pointed in Ref.[Gupta, 1995].
3. There is no fitting in some energy positions of the other bands because of mixed symmetry states for the some excited energy levels, studying in IBM-2.
4. The ratios of the reduced transition probabilities R, R' and R'' have been found in agreement both experimentally and theoretically also with in consistence with their ideal corresponding limits**.**
5. The axially symmetric for the isotope ^{146,148}Nd shows in Figure [9-10] represents the behaviors of the potential energy. For^{148}Nd, there is an agreement with the behavior of the O(6) limit compared with the typical triaxial symmetric. For ^{146}Nd, the deformation is not permanently, it was just vibrational surface around the spherical shape results from phonons interaction.
6. The values of B(E2) and quadrupole moment increases when the neutron number increases, the branching ratios shows that the isotopes in the transition region U(5)-O(6).

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