Some Fixed Point Theorems for Single Valued Self Contractive Mapping in Complete Cone Metric Space by Using Altering Function "Ms"

Teaching assistant . Tamara Shehab Ahmed
Ibn-Al-Haitham College of Education
University of Baghdad

Abstract

In this paper, we will prove some fixed point theorems for single valued self contractive mapping satisfying special conditions in complete cone metric space by using a vector valued altering function (Ms-function) with the assumption that the Cone is not normal. Our results generalize some recent results.

Key Words: Cone metric space, Altering function "Ms", normal Cone, non-normal Cone, fixed point.

1 - Introduction

It is well known that the classical contraction mapping principle of Banach is a fundamental result in fixed point theory, so several types of generalization contraction mappings on metric space have appeared. One such method of generalization is altering the distances. Delbosco [1] and Skof [2] have established fixed point theorems for self mapping of complete metric spaces by altering the distances between the points with the use of a positive real valued function.

Haung and Zhang [3] introduced the concept of Cone metric space by replacing the set of real numbers by an ordered Banach space by using the normality of a Cone and obtained some fixed point results, Rezapour and Hamlbarani [4] generalized some results of [3] by omitting the assumption of normality of a Cone in the results, which is a milestone in developing fixed point theory in Cone metric space.

Recently, Asad and Soleimani [5] proved some fixed results on Cone metric space by using altering distance function and the (ID) property of partially ordered Cone metric space, on the other hand wise, S.K.Malhotra, S.Shukla and R.Sen [6] were give some new results by introducing a vector valued altering function (Malhotra- Shukla altering function) denoted by "Ms" function in a complete Cone metric space this function take it's domain and codomain is a subset of real Banach space which is called by a Cone, also this function satisfied certain properties, and in the results of [6], it was assumed that the Cone is normal, this "Ms" function become the generalization of altering distance function in view of Cone used in place of positive real numbers as well as the constraints used for self mapping of Cone metric spaces.

Now, the main purpose of this paper is to generalize some results of [4] by using the 'Ms" function in a complete Cone metric space with out using the assumption that the Cone is normal, also our results are a generalization of the results of [6] by omitting the normality of the Cone in the results.

2 – Preliminaries

Through this paper, we denote always E by a real Banach space, R is the set of real numbers and N is the set of positive integer numbers.

Definition 2.1: [3]

Let P be a subset of E, P is called a Cone if and only if:

(a) P is closed, non empty and $P \neq \{0\}$.

(b) $a, b \in R$, $a, b \geq 0$, $x, y \in P \Rightarrow ax + by \in P$.

(c) $x \in P$ and $-x \in P \Rightarrow x = 0$.

Given a Cone $P \subseteq E$, we define a partial ordering "≤" with respect to P by $x \leq y$ if and only if $y - x \in P$, we shall write $x < y$ if $x \leq y$ but $x \neq y$ and $x \ll y$ if $y - x \in \text{int } P$, where int P is the interior of P.

The Cone P is called normal if there exists a number $k > 0$ such that for all $x, y \in E$, $0 \leq x \leq y$ implies $\|x\| \leq k\|y\|$, the least positive number satisfying the previous inequality is then called the normal constant of P.

Example (2.2): [4]

Let $E = C_R([0,1])$ with supremum norm and $P = \{f \in E: f \geq 0\}$, then P is a Cone with normal constant of $k = 1$.

Remark (2.3):

There are Cones are not normal, the following example show that:

Example (2.4): [4]

Let $E = C_R^2([0,1])$ with the norm $\|f\| = \|f\|_\infty + \|f'\|_\infty$ and consider the Cone $P = \{f \in E: f \geq 0\}$, for each $k \geq 1$, put $f(x) = x$, $g(x) = x^{2k}$, $0 \leq g \leq f$, P is not normal Cone for the details see example (2.3) of [4].

Proposition (2.5): [6]

Let P be any Cone in E, if for $a \in P$ and $a \leq a\,k$, for some $k \in [0,1)$ then $a = 0$.

Proposition (2.6): [6]

Let P be any Cone in E, if for $a \in E$ and $a \ll c$, for all $c \in \text{int } P$ then $a = 0$.

Proposition (2.7): [6]

Let P any Cone in E, if a, b $\in$ E and a $\ll$ b and b $\ll$ c then a $\ll$ c and if a $\leq$ b and b $\ll$ c then a $\ll$ c.

Proposition (2.8): [3]

Let X be non-empty set, suppose the mapping d: $X \times X \longrightarrow E$ satisfying for all x, y in X the following axioms:

(d_1) $0 \le d(x,y)$ and $d(x,y) = 0$ if and only if $x = y$.

(d_2) $d(x,y) = d(y,x)$ (symmetry).

(d_3) $d(x,y) \le d(x,z) + d(z,y)$ (triangular inequality).

Then d is called a Cone metric on X and (X,d) is called a Cone metric space, this definition is more general than that a metric space.

Example (2.9): [3]

Let $E = R^2$, $P = \{(x,y) \in E: x, y \ge 0\} \subset R^2$, if $X = R$, d: $X \times X \longrightarrow E$ defined by:$d(x,y) = (|x - y|, \alpha|x - y|)$, where $\alpha \ge 0$ is a constant then (X,d) is a Cone metric space.

Definition (2.10): [3]

Let (X,d) be a Cone metric space, let $\langle x_n \rangle$ be a sequence in X and $x \in X$:

(a) If for every $c \in E$ with $0 \ll c$, there is $n_0 \in N$ such that for all $n > n_0$, $d(x_n,x) \ll c$, then the sequence $\langle x_n \rangle$ converge to x, for some x in X, we denote this by $x_n \to x$ as $n \to \infty$ (or) $\lim_{n\to\infty} x_n = x$ (or) $\lim_{n\to\infty} d(x_n,x) = 0$.

(b) If for every $c \in E$ with $0 \ll c$, there is $n_0 \in N$ such that for all $n, m > n_0$, $d(x_n,x_m) \ll c$, then the sequence $\langle x_n \rangle$ is called a Cauchy sequence in X, we denote this by $\lim_{n,m\to\infty} d(x_n,x_m) = 0$.

(c) (X,d) is called a complete Cone metric space, if every Cauchy sequence in X is a convergant sequence in X.

Definition (2.11): [6]

A subadditive function is a function f: $P \longrightarrow P$ with the following property:

For all $x, y \in P$, $f(x + y) \le f(x) + f(y)$, P is any Cone.

Example (2.12):

Let $E = R$, $P = \{x \in E: x \ge 0\} \subset E$, define $\|x\| = |x|$ for all $x \in E$.

So, $\|x\| = |x|$ for all $x \in P$.

Now, since $\|x + y\| = |x + y| \le |x| + |y| = \|x\| + \|y\|$ for all $x, y \in P$.

So, $\|\cdot\|: P \longrightarrow P$ is a subadditive function on P.

Definition (2.13): [6]

If Y be any partially ordered set with relation "$\le$" and $\psi: Y \longrightarrow Y$, we say that ψ is non-decreasing if and only if $\psi(x) \le \psi(y)$ for all $x \le y$.

Definition (2.14): [6]

Let (X,d) be a Cone metric space, T: $X \longrightarrow X$. Then:

(a) T is said to be continuous if $\lim_{n\to\infty} x_n = x$ implies that $\lim_{n\to\infty} Tx_n = Tx$.

(b) T is said to be subsequentially convergent, if we have for every sequence $<y_n>$ in X and $T(y_n)$ is convergent implies that $<y_n>$ has a convergant subsequence.

(c) T is said to be sequentially convergent if for every sequence $<y_n>$ in X and $T(y_n)$ is convergent that implies $<y_n>$ is also convergent.

Definition (2.15): [6]

Let $\psi: P \longrightarrow P$ be a vector valued altering function on P, P is any Cone, ψ is called "Ms" altering function if:

(a) ψ is non-decreasing, subadditive, continuous and sequentially convergent.

(b) $\psi(a) = 0$ if and only if $a = 0$.

Example (2.16): [6]

Let (X,d) be a Cone metric space as in example (2.9), we define $\psi: P \longrightarrow P$ by $\psi[(x,y)] = k(x,y)$ and $k > 0$, then ψ is "Ms"-altering function on P.

3- Main Result

First, we will recall the following contractive condition on the mapping T which were appeared in theorem (2.3), (2.6), (2.7), (2.8) as respectively of [4] under the assumption that (X,d) is a complete Cone metric space with non-normal Cone P and T be a single valued self mapping on X, for all x, y in X, T satisfied :

(i) $d(T(x),T(y)) \leq k\, d(x,y)$, $k \in [0,1)$ is a constant.

(ii) $d(T(x),T(y)) \leq k[d(T(x),x) + d(T(y),y)]$, $k \in [0, \frac{1}{2})$ is a constant.

(iii) $d(T(x),T(y)) \leq k[d(T(x),y) + d(x,T(y)]$, $k \in [0, \frac{1}{2})$ is a constant.

(iv) $d(T(x),T(y)) \leq kd(x,y) + \ell d(y,T(x))$, $k, \ell \in [0,1)$is a constant with $k+\ell <1$.

Now, in the this section we will generalize the above results of [4] into following theorem, where the first part of it's proof is similar to the first part of proof of the theorem (3.1) in [6].

Theorem (3.1):

Let (X,d) be a complete Cone metric space, suppose $T: X \longrightarrow X$ be a mapping and $\psi: P \longrightarrow P$ be "Ms"-altering function with non normal Cone P and satisfying for all x, y in X:

$$\psi[d(T(x),T(y))] \leq a_1\, \psi[d(x,y)] + a_2\, \psi[d(T(x),x)] + a_3\, \psi[d(T(y),y)] + a_4\, \psi[d(T(x),y)] + a_5\, \psi[d(T(y),x)] \quad \ldots(3.1.1)$$

where $a_1, a_2, a_3, a_4, a_5 \in [0,1)$ is constant and satisfying $\sum_{i=1}^{5} a_i < 1$, then T has a unique fixed point for each $x \in X$, the iterative sequence $<T^n(x)>_{n \geq 1}$ converges to the fixed point.

Proof:

For each $x_0 \in X$ and $n \geq 1$, set $x_1 = T(x_0) \ldots x_{n+1} = T^{n+1}(x_0) = T(x_n)$

First we will show that $<x_n>$ is a Cauchy sequence.

Taking $x = x_n$, $y = x_{n-1}$ in equation 3.1.1, we get

$$\psi[d(T(x_n),T(x_{n-1}))] \leq a_1\, \psi[d(x_n,x_{n-1})] + a_2\, \psi[d(T(x_n),x_n)] + a_3\, \psi[d(T(x_{n-1}),x_{n-1})] + a_4\, \psi[d(T(x_n),x_{n-1})] + a_5\, \psi[d(T(x_{n-1}),x_n)]$$

$$\psi[d(x_{n+1},x_n)] \leq a_1\, \psi[d(x_n,x_{n-1})] + a_2\, \psi[d(x_{n+1},x_n)] + a_3\, \psi[d(x_n,x_{n-1})] + a_4\, \psi[d(x_{n+1},x_{n-1})] + a_5\, \psi[d(x_n,x_n)]$$

But by triangular inequality we have:

$$d(x_{n+1},x_{n-1}) \leq d(x_{n+1},x_n) + d(x_n,x_{n-1})$$

Also, we have ψ non-decreasing and subadditive function. So we get:

$$\psi[d(x_{n+1},x_{n-1})] \leq \psi[d(x_{n+1},x_n)] + \psi[d(x_n,x_{n-1})]$$

Thus,

$$\psi[d(x_{n+1},x_n)] \leq a_1\, \psi[d(x_n,x_{n-1})] + a_2\, \psi[d(x_{n+1},x_n)] + a_3\, \psi[d(x_n,x_{n-1})] + a_4\, \psi[d(x_{n+1},x_n)] + a_4\, \psi[d(x_n,x_{n-1})]$$

So,

$$\psi[d(x_{n+1},x_n)] \leq (a_1 + a_3 + a_4)\, \psi[d(x_n,x_{n-1})] + (a_2 + a_4)\, \psi[d(x_{n+1},x_n)] \qquad \ldots(3.1.2)$$

Now, in the same way, taking $y = x_n$, $x = x_{n-1}$ in equation (3.1.1) and using symmetry of inequality in x, y, so we have:

$$\psi[d(x_n,x_{n+1})] \leq (a_1 + a_2 + a_5)\, \psi[d(x_n,x_{n-1})] + (a_3 + a_5)\, \psi[d(x_n,x_{n+1})] \qquad \ldots(3.1.3)$$

Now, if we combine equations (3.1.2) and (3.1.3), we get:

$$2\, \psi[d(x_{n+1},x_n)] \leq (2a_1 + a_2 + a_3 + a_4 + a_5)\, \psi[d(x_n,x_{n-1})] + (a_2 + a_3 + a_4 + a_5)\, \psi[d(x_{n+1},x_n)]$$

So

$$\psi[d(x_{n+1},x_n)] \leq \frac{2a_1 + a_2 + a_3 + a_4 + a_5}{2 - a_2 - a_3 - a_4 - a_5}\, \psi[d(x_n,x_{n-1})]$$

Take $\alpha = \frac{2a_1 + a_2 + a_3 + a_4 + a_5}{2 - a_2 - a_3 - a_4 - a_5} < 1$, so, $\alpha < 1$ that is

$$\psi[d(x_{n+1},x_n)] \leq \alpha\, \psi[d(x_n,x_{n-1})].$$

So, if we continue in the same way we obtain that:

$$\psi[d(x_{n+1},x_n)] \leq \alpha^n\, \psi[d(x_1,x_0)],\ \alpha < 1,\ n \geq 1 \qquad \ldots(3.1.4)$$

Now, let C be given, where $0 \ll C$, choose $\delta > 0$, such that $C + N_\delta(0) \subseteq P$, where $N_\delta(0) = \{y \in E: \|y\| < \delta\}$, also choose $N_1 \in N$ such that α^n

$\psi[d(x_1,x_0)] \in N_\delta(0)$ for all $n \geq N_1$, then $\alpha^n \psi[d(x_1,x_0)] \ll C$, for all $n \geq N_1$, hence by proposition (2.7) we have $\psi[d(x_{n+1},x_n)] \ll C$, so by proposition (2.6) if we take that $\psi[d(x_{n+1},x_n)] = 0$ and by property of ψ that is ($\psi(a) = 0 \Leftrightarrow a = 0$, for $a \in P$), so we get that $d(x_{n+1},x_n) = 0$ and then $x_{n+1} = x_n$ which is trivial case.
Hence, suppose that the limit of $\psi[d(x_{n+1},x_n)]$ as $n \to \infty$ is exist, therefore we have:

$$\lim_{n\to\infty} \psi[d(x_{n+1},x_n)] \ll \lim_{n\to\infty} C$$

$$\lim_{n\to\infty} \psi[d(x_{n+1},x_n)] \ll C$$

So, $\lim_{n\to\infty} \psi[d(x_{n+1},x_n)] \ll \frac{C}{m}$, for all $m \geq 1$ then

$\frac{C}{m} - \lim_{n\to\infty} \psi[d(x_{n+1},x_n)] \in P$, for all $m \geq 1$. Since $\frac{C}{m} \to 0$ (as $m \to \infty$) and P is closed, so

$(-\lim_{n\to\infty} \psi[d(x_{n+1},x_n)]) \in P$, but we have $\lim_{n\to\infty} \psi[d(x_{n+1},x_n)] \in P$, therefore by definition (2.1), (c) we have:

$\lim_{n\to\infty} \psi[d(x_{n+1},x_n)] = 0$, but ψ is sequentely convergent hence $d(x_{n+1},x_n)$ is also convergent and ψ is continuous, so $\psi[\lim_{n\to\infty} d(x_{n+1},x_n)] = 0$ and then we get $\lim_{n\to\infty} d(x_{n+1},x_n) = 0$. Therefore, for every $C \in E$ with $0 \ll C$, there exists $n_0 \in N$ such that $d(x_{n+1},x_n) \ll C$ for all $n > n_0$. Hence by triangular inequality we have:

$$d(x_{n+2},x_n) \leq d(x_{n+2},x_{n+1}) + d(x_{n+1},x_n)$$
$$\ll \frac{C}{2} + \frac{C}{2} = C \quad \text{for all } n > n_0$$

Similarly by induction we have $d(x_m,x_n) \ll C$, for all $m > n > n_0$. Therefore $\langle x_n \rangle$ is a Cauchy sequence, by comletness of X, it must be convergent in X, hence $\lim_{n\to\infty} x_n = z$ for some z in X.

Now, to show that z is a fixed point of T, by triangular inequality we have:

$$d(T(z),z) \leq d(T(z), x_{n+1}) + d(x_{n+1},z)$$

But ψ is non-decreasing and subadditive function on P, so we have:

$$\psi[d(T(z),z)] \leq \psi[d(T(z),x_{n+1})] + \psi[d(x_{n+1},z)]$$

By equation (3.1.1) we have

$$\psi[d(T(z),x_{n+1})] = \psi[d(T(z),T(x_n))]$$
$$\leq a_1\psi[d(z,x_n)] + a_2\psi[d(T(z),z)] + a_3\psi[d(T(x_n),x_n)] + a_4\psi[d(T(z),x_n)] + a_5\psi[d(T(x_n),z)]$$

$\psi[d(T(z),x_{n+1})] \leq a_1\psi[d(z,x_n)] + a_2\psi[d(T(z),z)] + a_3\psi[d(x_{n+1},x_n)] + a_4\psi[d(T(z),x_n)] + a_5\psi[d(x_{n+1},z)]$

But by triangular inequality and by non-decreasing and subadditiving of ψ we have:

$\psi[d(T(z),x_{n+1})] \leq a_1\psi[d(z,x_n)] + a_2\psi[d(T(z),z)] + a_3\psi[d(x_{n+1},z)] + a_3\psi[d(z,x_n)] + a_4\psi[d(T(z),z)] + a_4\psi[d(z,x_n)] + a_5\psi[d(x_{n+1},z)]$

Hence

$\psi[d(T(z),z)] \leq a_1\psi[d(z,x_n)] + a_2\psi[d(T(z),z)] + a_3\psi[d(x_{n+1},z)] + a_3\psi[d(z,x_n)] + a_4\psi[d(T(z),z)] + a_4\psi[d(z,x_n)] + a_5\psi[d(x_{n+1},z)] + \psi[d(x_{n+1},z)]$

That implies:

$(1 - a_2 - a_4)\ \psi[d(T(z),z)] \leq (a_1 + a_3 + a_4)\ \psi[d(z,x_n)] + (a_3 + a_5 + 1)\ \psi[d(x_{n+1},z)]$…(3.1.5)

Now, by taking limit to both sides of equation (3.1.5) as $n \to \infty$ and by the countinuing of ψ we obtain:

$$\psi \lim_{n\to\infty} [d(T(z),z)] \leq \frac{a_1 + a_3 + a_4}{1 - a_2 - a_4}\ \psi \lim_{n\to\infty} [d(z,x_n)] + \frac{a_3 + a_5 + 1}{1 - a_2 - a_4}\ \psi \lim_{n\to\infty} [d(z,x_n)]$$

That implies by the property of ψ $[(\psi(a) = 0 \Leftrightarrow a = 0]$

$\psi[d(T(z),z)] \leq 0 \Leftrightarrow \psi[d(T(z),z)] \leq \psi(0)$

But ψ is non-decreasing, so we obtain that $d(T(z),z) \leq 0$ and then $d(T(z),z) = 0$ and $T(z) = z$. Therefore, z is a fixed point of T.

Now, to show that z is the unique fixed point of T, suppose w is another fixed point of T, so T(w) = w. Taking x = z, y = w in equation (3.1.1), so we get:

$\psi[d(T(z),T(w))] \leq a_1\psi[d(z,w)] + a_2\psi[d(T(z),z)] + a_3\psi[d(T(w),w)] + a_4\psi[d(T(z),w)] + a_5\psi[d(T(w),z)]$

$\psi[d(z,w)] \leq a_1\psi[d(z,w)] + a_2\psi[d(z,z)] + a_3\psi[d(w,w)] + a_4\psi[d(z,w)] + a_5\psi[d(w,z)]$

$= (a_1 + a_4 + a_5)\ \psi[d(z,w)]$

Since $a_1 + a_4 + a_5 < 1$, so by proposition (2.5) we have

$\psi[d(z,w)] = 0$ and $d(z,w) = 0 \Leftrightarrow z = w$.

Therefore T has a unique fixed point.

As a consequence of therem (3.1), we have the following corollaries:

Corollary (1):

If X, ψ, P as in theorem (3.1) and $T:X \longrightarrow X$ satisfies for all x, y in X: $d(T(x),T(y)) \leq a_1\, d(x,y)$, $a_1 \in [0,1)$ is a constant, then we obtain theorem (2.3) of [4].

Proof:

Taking $a_2 = a_3 = a_4 = a_5 = 0$ in equation (3.1.1) and $\psi(a) = a$ for all $a \in P$, we get the required result.

Corollary (2):

If X, ψ, P as in theorem (3.1) and $T:X \longrightarrow X$ satisfies for all x, y in X:

$d(T(x),T(y)) \leq a_2\ [d(T(x),x) + d(T(y),y)]$, $a_2 \in [0,\frac{1}{2})$ is a constant, then we obtain theorem (2.6) of [4].

Proof:

Taking $a_1 = a_4 = a_5 = 0$ and $a_3 = a_2$ in equation (3.1.1) and $\psi(a) = a$ for all $a \in P$, we get the required result.

Corollary (3):

If X, ψ, P as in theorem (3.1) and $T:X \longrightarrow X$ satisfies for all x, y in X:

$d(T(x),T(y)) \leq a_4\ [d(T(x),y) + d(x,T(y))]$, $a_4 \in [0,\frac{1}{2})$ is a constant, then we obtain theorem (2.7) of [4].

Proof:

Taking $a_1 = a_2 = a_3 = 0$ and $a_5 = a_4$ in equation (3.1.1) and $\psi(a) = a$ for all $a \in P$, we get the required result.

Corollary (4):

If X, ψ, P as in theorem (3.1) and $T:X \longrightarrow X$ satisfies for all x, y in X:

$d(T(x),T(y)) \leq a_1 d(x,y) + a_4 d(y,T(x))$, $a_1, a_4 \in [0,1)$ is a constant with $a_1 + a_4 < 1$, then we obtain theorem (2.8) of [4].

Proof:

Taking $a_2 = a_3 = a_5 = 0$ in equation (3.1.1) and $\psi(a) = a$ for all $a \in P$, we get the required result.

Corollary (5):

If X, ψ, P as in theorem (3.1) and $T:X \longrightarrow X$ satisfies:

$d(T(x),T(y)) \leq a_1 d(x,y) + a_2 d(T(x),x) + a_3 d(T(y),y) + a_4 d(T(x),y) + a_5 d(T(y),x)$, where $a_1, a_2, a_3, a_4, a_5 \in [0,1)$ is a constant with $\sum_{i=1}^{5} a_i < 1$. Then T has a unique fixed point.

Proof:

Taking $\psi(a) = a$ for all $a \in P$, we get the required result.

Corollary (6):

If X, ψ, P as in theorem (3.1) and $T:X \longrightarrow X$ satisfies:

$\psi[d(T(x),T(y))] \leq a_1 \psi[d(x,y)]$ where $a_1 \in [0,1)$ is a constant, then T has a unique fixed point.

Proof:

Taking $a_2 = a_3 = a_4 = a_5 = 0$ in equation (3.1.1), we get the required result.

Corollary (7):

If X, ψ, P as in theorem (3.1) and $T:X \longrightarrow X$ satisfies:

$\psi[d(T(x),T(y))] \leq a_2 \{\psi[d(T(x),x)] + \psi[d(T(y),y)]\}$, where $a_2 \in [0,\frac{1}{2})$ is a constant, then T has a unique fixed point.

Proof:

Taking $a_1 = a_4 = a_5 = 0$ and $a_3 = a_2$ in equation (3.1.1), we get the required result.

Corollary (8):

If X, ψ, P as in theorem (3.1) and $T:X \longrightarrow X$ satisfies:

$\psi[d(T(x),T(y))] \leq a_4 \{\psi[d(T(x),y)] + \psi[d(T(y),x)]\}$, where $a_4 \in [0,\frac{1}{2})$ is a constant, then T has a unique fixed point.

Proof:

Taking $a_1 = a_2 = a_3 = 0$ and $a_5 = a_4$ in equation (3.1.1), we get the required result.

Corollary (9):

If X, ψ, P as in theorem (3.1) and $T:X \longrightarrow X$ satisfies:

$\psi[d(T(x),T(y))] \leq a_1\psi[d(x,y) + a_2\psi[d(T(x),x)] + a_3\psi[d(T(y),y)]$, where $a_1, a_2, a_3 \in [0,1)$ is a constant with $a_1 + a_2 + a_3 < 1$, then T has a unique fixed point.

Proof:

Taking $a_4 = a_5 = 0$ in equation (3.1.1), we get the required result.

Corollary (10):

If X, ψ, P as in theorem (3.1) and $T:X \longrightarrow X$ satisfies:

$\psi[d(T(x),T(y))] \leq a_1\psi[d(x,y)] + a_4\psi[d(T(x),y)] + a_5\psi[d(T(y),x)]$, where $a_1, a_4, a_5 \in [0,1)$ is a constant with $a_1 + a_4 + a_5 < 1$. Then T has a unique fixed point.

Proof:

Taking $a_2 = a_3 = 0$ in equation (3.1.1), we get the required result.

References

1. D.Delbosco, Un'estension di.un teorema sulpun to fisso dis.Reich, Rend. Sem.Mat., Universe. Politean, Torino, 35 (1976-77), 233-238
2. F.Skof, Teorema di punti fisso per applicazioni neglispazi metrici, atti.Accad.Sci.Torino, 111(1997), 323-329.
3. L.G.Haung, X.Zhang, Cone Metric Space and Fixed Point Theorems of Contractive Mappings, J.Math.Anal.Appl., 332(2007), 1468-1476.
4. SH.Rezapour, R.Hamlbarani, Some Notes on the Paper Cone Metric Spaces and Fixed Point Theorems of Contractive Mappings, J.Math.Anal.Appl., 345 (2008), 719-724.

5. Mehdi Asadi, Hossein Soleimani, Fixed Point Theorems of Generalized Contraction in Partially Ordered Cone Metric Spaces, Arxive, 1102, 4019 VI[MathFA] (2011).
6. S.K.Malhotra, S.Shukla; R.Sen; Fixed Point Theorems in Cone Metric Spaces by Altering Distances, Vol.6, (2011), No.54, 2665-2671.

بعض مبرهنات النقطة الصامدة لتطبيقات انكماشية ذاتية وحيدة القيمة في فضاءات كون المترية الكاملة باستخدام دالة “Ms” الابدالية

م. م. تماره شهاب أحمد

قسم الرياضيات – كلية التربية– ابن الهيثم – جامعة بغداد

المستخلص

في هذا البحث، تم برهان بعض مبرهنات النقطة الصامدة لتطبيقات انكماشية ذاتية وحيدة القيمة تحقق شروطاً خاصة في فضاءات كون المترية الكاملة باستخدام دالة متجهية ابدالية سميت بـ دالة (Ms-function) مع افتراض ان الكون غير اعتيادية. نتائجنا كانت تعميم لنتائج آخرين.