*Kufa Journal of Engineering*
*Vol. 9, No. 3, July 2018, P.P. 112-127*
*Received 8 June 2017, accepted 10 October 2017*

# SUM-SINES EQUATION FOR ESTIMATING THE PERCENT OF SHADOW LENGTH OF TARGETS IN THE SATELLITE IMAGES

**Shurooq Mahdi Ali[1] and Emad Ali Al-Helaly[2]**

**[1] Department of Geology, Faculty of Science, University of Kufa, Email: Shurooq.alazzawi@uokufa.edu.iq**

**[2] Department of Civil Engineering, College of Engineering, University of Kufa, Email: emaadalhilaly@gmail.com**

http://dx.doi.org/10.30572/2018/kje/090309

## ABSTRACT

With the development of high resolution sensors in remote sensing satellites, the shadow phenomena has appeared clearly in satellite image. The shadow is a separated feature in the satellite image, some time it may be considered as a problem due to the loss of ray information at the shaded region, other may be considered as a criteria to the height of the body which has a shadow. The length of shadow depends on the height of the body, location on the earth surface, and the sun location in sky at the imaging time. The sun location is varying every hour during the day and every day during year. These varying is calculated by complex astronomic equations. In this article, we simplified these calculations to just on equation depend on one parameter, and examine this equation by field measurements. The suggested equation is sum-sine equation with an enough accuracy to be used in civil, ecological, gardens designing near the high buildings, or architectural purposes. The equation can be used to estimate the building height from the shadow length in the satellite image, as will as it may used to estimate shadow length from the known body height on the earth.

**KEYWORDS:** Remote Sensing; Satellite Image; Sun angles; Shadow; Sum-Sines Equation.

## 1. INTRODUCTION: LIGHT AND SHADOW

Light is the electromagnetic spectrum rays with the visible range coming naturally from the sun. The electromagnetic rays propagate in straight lines from the light source.

The opaque objects obscure light to reach area in the opposite side if the light source, so the shadow area still dark relatively and called the shadow. Therefore, the shadow is the darkness that caused by an object when it blocks the light from getting a corresponding surface (Satellite Imaging Corporation). It can be considered that the shadow occurs at the image when there are objects which obscure the light coming from the source partially or completely (Prasath and Haddad, 2006).

## 2. PREVIOUS WORKS

The phenomena of the shadow was appeared clearly with the increasing of resolution satellite sensors (IKONOS, QuickBird, WorldView, and other) after 2000, so many studies were written about shadow to remedy or use it as a function to know the properties of shaded body.

Cooper (1969) simplified the complex Fourier equation that calculate the angle of solar deviation (δ) to simple sine equation depends on single variable which is day number ($d_n$):

$$\delta = \pi \times 18023.45 \times \sin(2\pi \times 365284 + d_n \qquad 1$$

where, $d_n$ is the day number, $d_n$ is equals1 at the first of January and 32 at the first of February.

Lin and Nevatia (1998) adopted complex algorithms to estimate the three-dimensional model from the two-dimensional image by triangular equations [R. 2].

Using SPOT satellite images Shettigara and Sumerling (1998) calculated tree and buildings heights, and succeeded in calculating the height of the buildings accurately, but they were not high accuracy in the calculation of tree height results due to informal form in the trees [R. 15].

Muneer (2004) derived other simple sine equation for the solar deviation angle as below, Fig. 1:

$$\delta = \sin^{-1} \times \sin(23.450) \times \sin(2\pi \times d_n - 81365) \qquad 2$$

Won Seok, et al. (2007) suggested geometric method for estimate the building (height) from one satellite image separately, he depended some astronomic parameters in the remote sensing imaging system as solar angles, image scale, direction of the shadow, angle of satellite imaging.

Karantzalos and Paragios (2008) were adopted a metadata or ground information to find the hidden dimension (height) of the object in a simple way not mathematically.

Arévalo et al. (2008) determined the shadow in images of high resolution satellite Quick-Bird of resolution (0.6 meters), but said it is a way suitable for images satellites IKONOS and WorldView. In all the angle of the Sun was taken fixed, and Arévalo et al. (2008) supposed that the calculating of sun angle at over particular day along the year is very complex.

## 3. THE SHADOW GEOMETRY

The shadows are two dimensional forms that result from three dimensional opaque bodies, and depend on the geometrical properties of these bodies, and at the angle of incident rays of the sun.

### 3.1. Angle of the sun and its synchronization

Sun is the main source of all electromagnetic waves directly or indirectly, the sun appears moving in the sky along the day, and the shadow is changing according to the sun direction and orientation.

The angle of the sun is varying all the year and day hours because of the difference in the angle of the fall of radiation on the ground.

In addition, it is seemed moving in complex orbit on the sky from the observer on the Earth.

The incident angle of the sun light on the Earth's surface varies by distance from the equator because of the spherical shape of the planet as it is well known. The sun is at 21/22 of March perpendicular to the equator so that there are no shadows to the vertical structures there, but it will be inclined in areas northern or southern equator due the spherical shape of the planet (Al-Najim and Al-Naimy, 1981). Here we will discuss the relation between the sun angle and the scene on the earth, which affect the shadow length and direction.

### 3.2. The sun

The sun's diameter is about 1,390,000 kilometers and the average distance to the Earth's surface is about 149.5 million kilometers (Spencer, 1971). It generate various wavelengths of electromagnetic radiation from long waves of radios and short wavelength of cosmic rays .

Electromagnetism which coming from sun consists of approximately 46 % visible radiation and 46 % thermal radiation (National Center for Atmospheric Research).

The Earth is rounding at a biosphere orbit around the sun, though, the amount of rays intensity falling on the ground varies throughout the year. The earliest distance to the Earth is 147 million km at the beginning of January, and further distance is 152 million km at the beginning of July, Fig. 2 (Appleton, 1945).

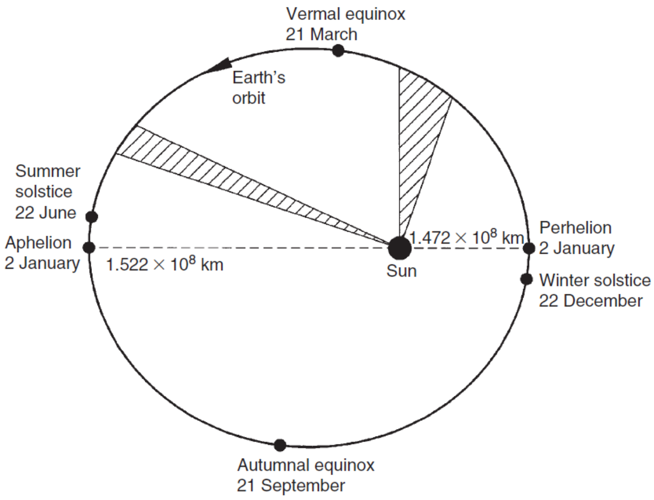


**Fig. 1. Earth's orbit around the sun (Muneer, 2004).**

Accurately the distance (Iqbal, 1983) between the sun and earth ($r_o$)= 1.496 $\times 10^8$ km, or in more accurate 149597890±500 km (Sun et al., 2013) and (r) is the sun-earth distance for every day in the year.

The day angle ($\Gamma$ : rad) is calculated from the equation below:

$$\Gamma = 2\pi(d_n - 1)/365 \qquad 3$$

Where $d_n$ is the day number, as it is mentioned.

The Earth's axis of rotation across the year is varying to be about ±23.5 degrees (Iqbal, 1983) during the year, which affects the sun incident angle and it's difference on the surface of the earth, thus affects the direction of the shadow and its extension.

The angle, position and elevation of the sun can be organized by astronomical equations (Appleton, 1945) in the following manner.

### 3.3. Sun location

**The angle of solar deviation (δ):** it is the angle between the equatorial plane (the plane of Earth revolution about the sun) and the line between the center of the Earth and the center of the sun. It is also termed declination angle, Fig. 3.

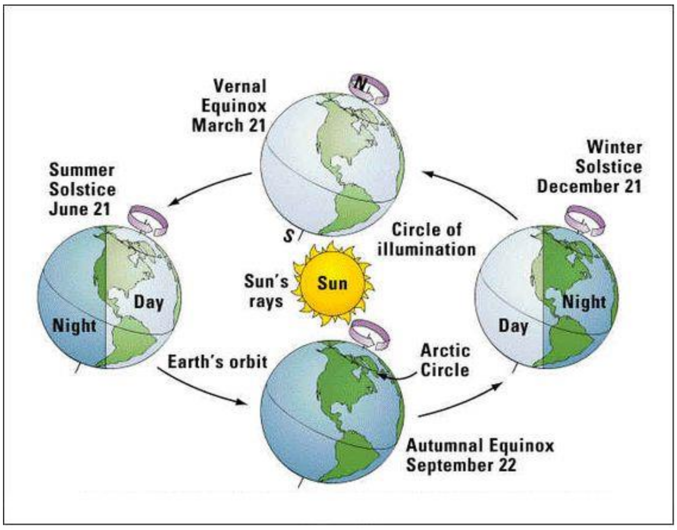


**Fig. 2. The changing in the Earth's axis rotation around itself during the year (Appleton, 1945)**

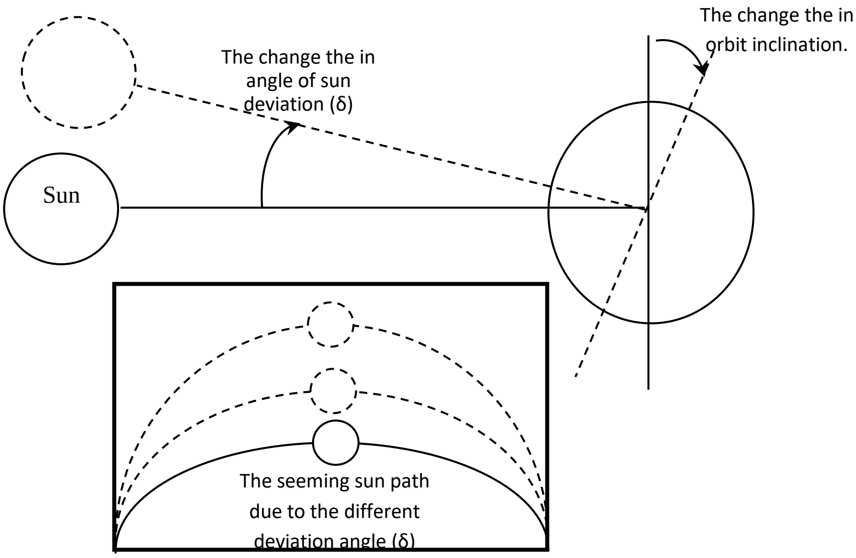


**Fig. 3. A sketch to the sun seeming deviation angle.**

The value of this angle ranges between +23.45 and -23.45, and the value of this angle varies day to day. At 20/21 March and 22/23 September it equals zero. It is calculated from the relationship:

$$\begin{aligned}\delta = (&0.006918 - 0.399912 \times \cos(\Gamma) + 0.070257 \times \sin(\Gamma) - 0.006758 \times \cos(2\Gamma) \\ &+ 0.000907 \times \sin(2\Gamma) - 0.002697 \times \cos(3\Gamma) + 0.00148 \times \sin(3\Gamma) \times (180/\pi)\end{aligned} \quad (4)$$

This equation is came from a Fourier formula, was developed by Spencer, (1971), and it is the most accurate equation. It is derived from the apparent sun path from the ground.

**The geographic latitude angle (ϕ):** this is the earth point coordinate, it is positive to the north of the Earth, and zero at the equator line.

The shadow of all objects is lengthened words toward the poles of the planet, even if they are in the same latitude, Fig. 4.

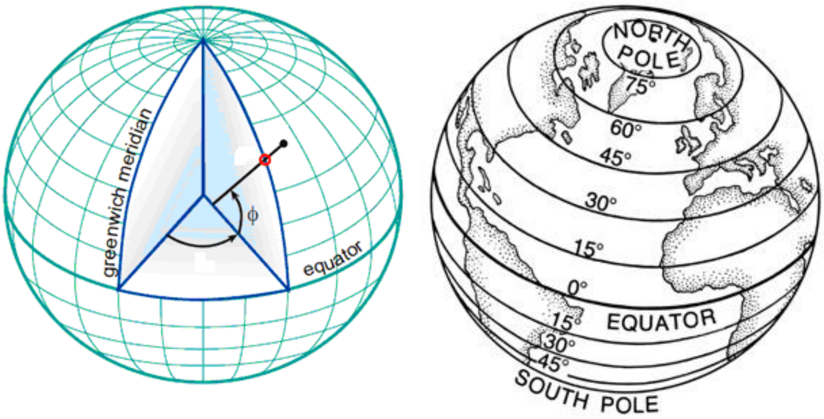


**Fig. 4. The geographic latitude angle (ϕ).**

**The hour angle (ω):** the hour angle measured from the zenith line to the horizon. It is zero at noon, and with positive value along the morning. At the sine shine, it equals 90$^{o}$, and it is distributed equally with the hours of the day half. The same graduating after noon but in negative values, Fig. 5 (Iqbal, 1983).

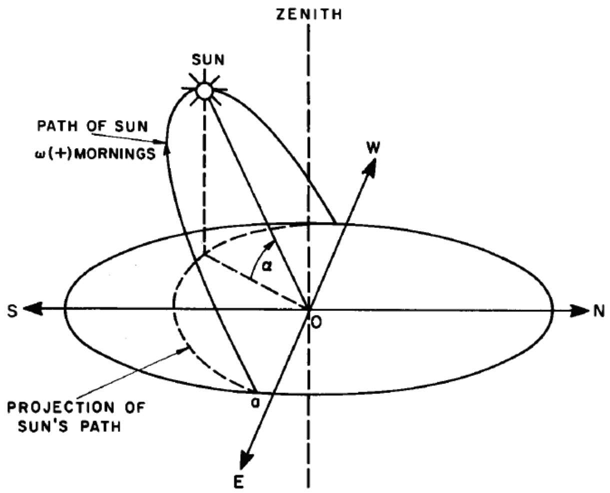


**Fig. 5. The hour angle (ω)**

**The angle of the solar rise (α):** is the angular height of the sun in the sky measured from the horizon (the horizon of the observation with the Earth is the circle obtained from the intersection of the sky of the observer with the Earth). This angle is equal to zero at sunrise and sunset, and 90 when the sun is directly above the observatory. It also may be named solar elevation angle, Fig. 6 (Iqbal, 1983).

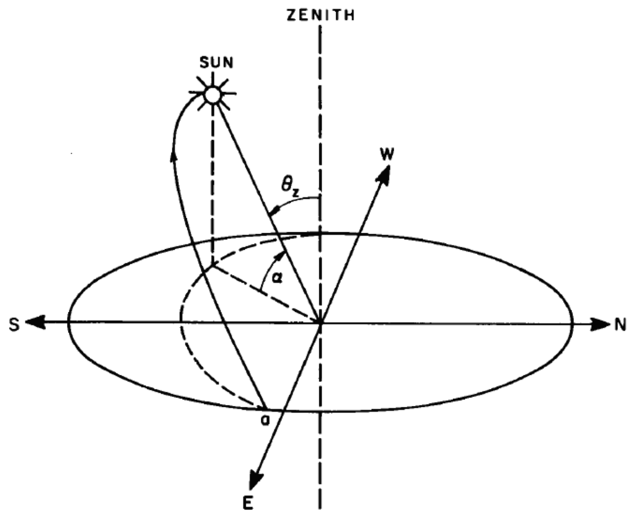


**Fig. 6. The angle of the solar rise (α)**

**The azimuth angle ($\theta_z$):** is the complementary angle of the angle of elevation (α), and measured from the roots, which is after the center of the sun degrees from the point of the head (Zenith), and roughly the angle of the head's head at sunrise or sunset is 90 degrees. It is also named (zenith angle), and can be calculated from the equation below (Iqbal, 1983):

$$\alpha = 90 - \theta_z \quad (5)$$

The elevation angle of the relationship is calculated by (Arévalo et al., 2008):

$$\cos(\theta_z) = \sin(\alpha) = \cos(\phi) \times \cos(\delta) \times \cos(\omega) + \sin(\phi) \times \sin(\delta) \quad (6)$$

Now, we can calculate every sun coordinate, or angles depending, and shadow on the number of days for everybody when its orientation and dimensions are known.

## 4. THE SELECTED BUILDING

The selected building is a (Qasr Al-Safer hotel and restaurant) in Najaf, City. The orientation of it is (32° 00' 02.49" N) and (44° 21' 32.88" E). or (32.000691 N 44.359133 E).

The image in Fig. 7 is a band composed QuickBird imagery by WGS 84 map projections. And the seen above is imaged at 09:43 a.m. according to the key information file which is attached with the imagery CD.

The building is 25.30 m height over the adjacent street level, on the crossing of two streets (Al-Rawan and Al- Ameer) as shown in Fig. 8, and there are no high buildings around it except at one side. These properties make easy to measure the length of the shadow from two sides. By the electronic and magnetic composes, we found the direction of building walls to measure the direction of shadow according to it, by this we need no repeat the field measurements of shadow according to the eastern direction, Fig. 9.


**Fig. 7. The orientation of Qasr Al-Safeer hotel and restaurant.**

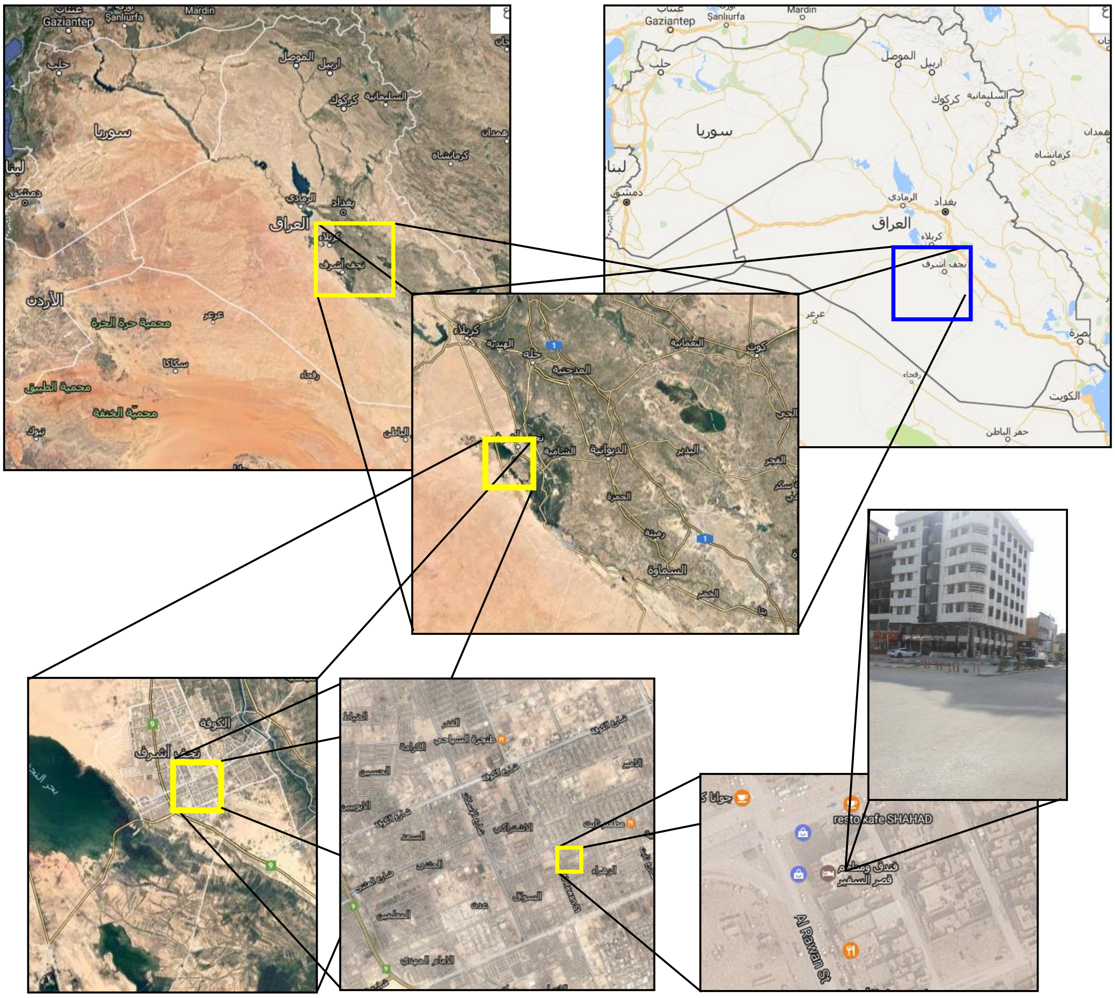


**Fig. 8. The location of the selected building.**

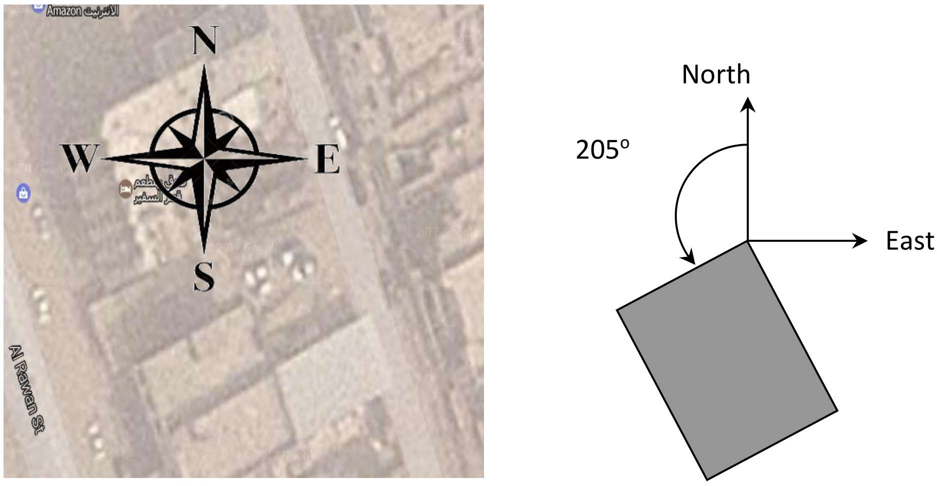


**Fig. 9. The direction of the walls of the selected building**

## 4. THE SUN ORIENTATIONS DURING THE YEAR

The sun orientation in the sky is calculated from astronomy equations during the years as presented.

For the Quick-Bird imaging time at 29/8/2006:

The day number ($d_n$) is 241.

$\Gamma = 2\pi(d_n-1)/365 = 4.131409517$ rad, from equation (3)

The deviation angle ($\delta$) from equation (4) is:

$\delta=(0.006918-0.399912\times\cos(4.13141)+0.070257\times\sin(4.13141)-0.006758\times\cos(2\times4.13141)+0.000907\times\sin(2\times4.13141)-0.002697\times\cos(3\times4.13141)+0.00148\times\sin(3\times4.13141)\times(180/\pi)$

$\delta = 0.168285459$ rad

$= 9.642046551$ degrees

The latitude angle ($\phi$) is + 32.000691 degrees. From the latitude circle

The hour (solar) angle ($\omega$) = 12:00- 09:43= 02:17 hours = 34.25 degrees

= 0.711 rad

The zenith angle ($\theta_z$) from eq. (6)

$\cos(\theta_z)=\sin(\alpha)= \cos(\phi)\times\cos(\delta)\times\cos(\omega)+\sin(\phi)\times\sin(\delta)$

$= \cos(32)\times\cos(9.642)\times\cos(34.25)+\sin(32)\times\sin(9.642)$

$= 0.848 \times 0.98587 \times 0.82658975 + 0.52992 \times 0.16749$

$= 0.7798$

$\theta_z = 38.7577$ degrees

The angle of solar rise (α)= 51.24 degrees from equation (5), From this angle we can calculate the shadow length:

tan (α)= building height /shadow width (measured from field or image at that date: 29/August), from Table 1 20.20 m.

$\tan(\alpha) = H / 20.20$ m

$H = 20.20 \times \tan(51.24227) = 25.16$ m

But from the real information the height of building is 25.30 m.

So, the errors in the field measurements is 0.006%.

Note that the shadow length from the astronomical equation is (20.31 m) from Table 2.

Now, if we calculate the shadow geometrically from the astronomic equation with known height.

tan (α)= 25.3/shadow length 20.31 m

shadow length= 25.3/tan (α)= 20.31 m

**Table 1. Illustrate the field measurements for the length and direction of building shadow.**

| Date | Shadow angle from the North (degrees) | shadow length (m) | Date | Shadow angle from the North (degrees) | shadow length (m) |
|---|---|---|---|---|---|
| 29-Aug | 51 | 20.20 | 12-Mar | 42 | 28.25 |
| 13-Sep | 47 | 23.05 | 27-Mar | 46 | 24.20 |
| 28-Sep | 43 | 26.85 | 11-Apr | 50 | 21.00 |
| 13-Oct | 38 | 31.45 | 26-Apr | 53 | 18.65 |
| 28-Oct | 34 | 36.80 | 11-May | 55 | 17.00 |
| 12-Nov | 30 | 42.70 | 26-May | 57 | 16.00 |
| 27-Nov | 27 | 48.15 | 10-Jun | 58 | 15.50 |

| Date | Shadow angle from the North (degrees) | shadow length (m) | Date | Shadow angle from the North (degrees) | shadow length (m) |
|---|---|---|---|---|---|
| 12-Dec | 26 | 51.85 | 25-Jun | 58 | 15.35 |
| 27-Dec | 26 | 52.50 | 10-Jul | 58 | 15.60 |
| 11-Jan | 27 | 49.80 | 25-Jul | 57 | 16.30 |
| 26-Jan | 29 | 44.80 | 9-Aug | 55 | 17.60 |
| 10-Feb | 33 | 38.90 | 24-Aug | 52 | 19.50 |
| 25-Feb | 37 | 33.20 | | | |

**Table 2. Shadow length and direction by the astronomy equations (rounded values to two digits).**

| date | day number | day angle | delineation angle | latitude angle | hour angle | azimuth angle | solar rise angle | shadow length (m) |
|---|---|---|---|---|---|---|---|---|
| 29-Aug | 241 | 4.13 | 0.17 | 32 | 34.25 | 0.68 | 51.25 | 20.31 |
| 13-Sep | 256 | 4.39 | 0.07 | 32 | 34.25 | 0.74 | 47.49 | 23.19 |
| 28-Sep | 271 | 4.65 | -0.03 | 32 | 34.25 | 0.82 | 43.23 | 26.91 |
| 13-Oct | 286 | 4.91 | -0.13 | 32 | 34.25 | 0.89 | 38.77 | 31.50 |
| 28-Oct | 301 | 5.16 | -0.22 | 32 | 34.25 | 0.97 | 34.43 | 36.90 |
| 12-Nov | 316 | 5.42 | -0.31 | 32 | 34.25 | 1.04 | 30.61 | 42.76 |
| 27-Nov | 331 | 5.68 | -0.37 | 32 | 34.25 | 1.09 | 27.68 | 48.23 |
| 12-Dec | 346 | 5.94 | -0.40 | 32 | 34.25 | 1.12 | 25.98 | 51.93 |
| 27-Dec | 361 | 6.20 | -0.41 | 32 | 34.25 | 1.12 | 25.70 | 52.57 |
| 11-Jan | 11 | 0.17 | -0.38 | 32 | 34.25 | 1.10 | 26.90 | 49.88 |
| 26-Jan | 26 | 0.43 | -0.33 | 32 | 34.25 | 1.06 | 29.42 | 44.86 |
| 10-Feb | 41 | 0.69 | -0.26 | 32 | 34.25 | 0.99 | 33.00 | 38.96 |
| 25-Feb | 56 | 0.95 | -0.16 | 32 | 34.25 | 0.92 | 37.25 | 33.28 |

| date | day number | day angle | delineation angle | latitude angle | hour angle | azimuth angle | solar rise angle | shadow length (m) |
|---|---|---|---|---|---|---|---|---|
| 12-Mar | 71 | 1.20 | -0.06 | 32 | 34.25 | 0.84 | 41.77 | 28.33 |
| 27-Mar | 86 | 1.46 | 0.04 | 32 | 34.25 | 0.76 | 46.19 | 24.27 |
| 11-Apr | 101 | 1.72 | 0.14 | 32 | 34.25 | 0.69 | 50.18 | 21.09 |
| 26-Apr | 116 | 1.98 | 0.23 | 32 | 34.25 | 0.64 | 53.49 | 18.72 |
| 11-May | 131 | 2.24 | 0.31 | 32 | 34.25 | 0.59 | 55.97 | 17.08 |
| 26-May | 146 | 2.50 | 0.37 | 32 | 34.25 | 0.57 | 57.60 | 16.06 |
| 10-Jun | 161 | 2.75 | 0.40 | 32 | 34.25 | 0.55 | 58.45 | 15.54 |
| 25-Jun | 176 | 3.01 | 0.41 | 32 | 34.25 | 0.55 | 58.64 | 15.42 |
| 10-Jul | 191 | 3.27 | 0.39 | 32 | 34.25 | 0.56 | 58.19 | 15.69 |
| 25-Jul | 206 | 3.53 | 0.35 | 32 | 34.25 | 0.57 | 57.06 | 16.39 |
| 9-Aug | 221 | 3.79 | 0.28 | 32 | 34.25 | 0.61 | 55.12 | 17.64 |
| 24-Aug | 236 | 4.05 | 0.20 | 32 | 34.25 | 0.66 | 52.35 | 19.52 |

## 5. THE FIELDS MEASUREMENTS

A scale tape and simple instruments achieved the field measurements.

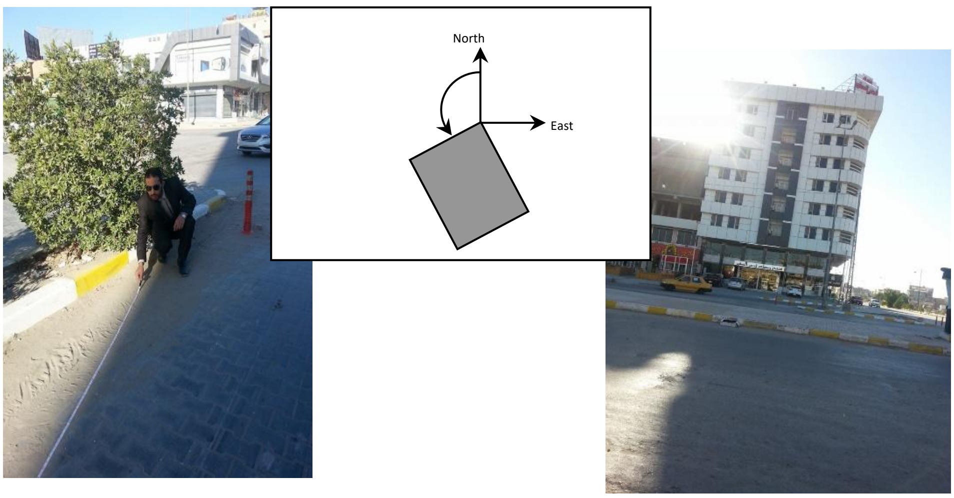


**Fig. 10. Field measurements of shadow length and direction.**

The results are illustrated in the table below:

## 6. GRAPHICAL REPRESENTATION

We need represent the results graphically by Fig. 11 and Fig. 12. Fig. 11 represents the shadow length of the chosen building, and Fig. 12 is more general, because it represent the percent of shadow with the real height of any building.

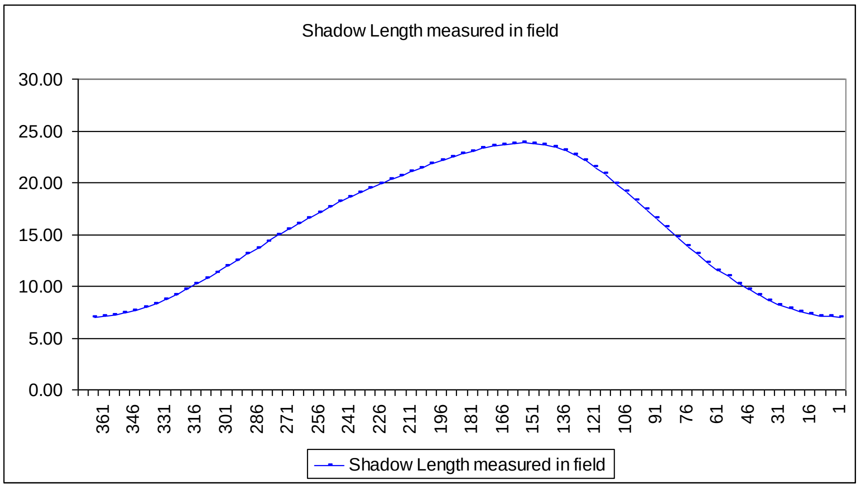


**Fig. 11. The change of shadow length during a year.**

We depend the numbers that resulted from the astronomic equations for more accuracy possible.

The two curves are similar to the sine distribution but there is a skew to the left, this need some modification and similar equation take into account this skew.

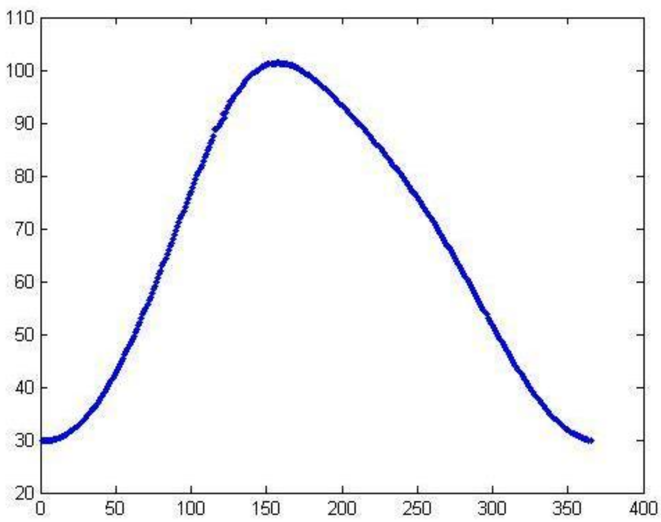


**Fig. 12 The percent of shadow length vs. day number along the year.**

## 7. THE SUGGESTED SUM-SINE EQUATION

The proper equation form to describe the percent shadow length curve in Fig. 12 is suggested by MATLAP / Curve fitting tool, as below:

Shadow length percent = $a_1 \times \sin(b_1 \times d_n + c_1) + a_2 \times \sin(b_2 \times d_n + c_2)$ in radians

$a_1 = 92.39$ , $b_1 = 0.007249$ , $c_1 = 0.2752$ , $a_2 = 9.29$ , $b_2 = 0.02703$ , $c_2 = 3.438$

Or say:

$$=92.39 \times \sin(0.007249 \times d_n + 0.2752) + 9.29 \times \sin(0.02703 \times d_n + 3.438) \qquad 7$$

R-square: 0.9916. Adjusted R-square: 0.9915

Now, examine it with 29/August or any shadow length

Shadow length percent $=92.39 \times \sin(0.007249 \times 241+0.2752) + 9.29 \times \sin(0.02703 \times 241+3.438)$

$= 92.39 \times \sin(2.0022209) + 9.29 \times \sin(9.95223)$

$= 83.92439342 - 4.675966394$

$= 79.25\ \%$

The shadow length calculated from the astronomical equation is 20.31 m

The resulted real height = 20.31/0.7925=25.628 m. It is very close to the real height which is (25.30 m)

By testing some other day numbers it is cleared that we can depend this accurate sum-sine equation to find the percent of shadow of any point in our Najaf city. But we can also find the Sum-Sine equation for any other city for getting the shadow percent of any known height building when it is needed to know the shadow extension on the street, architectural aims, or any other civil purposes.

The sum-sine equation provide practical calculation for any engineer or designer, and can be achieved by a simple hand calculator or a programmed Excel sheet instead of the astronomy equations.

This equation can be used in to ways, either for calculating the height of building from the shadow length that measured from the satellite image at the certain imaging time, or for calculating the shadow length from the known height of any building in our region.

## 8. CONCLUSION

It is recommend to use the suggested equation to calculate the percentage of shadow for any building in any time of the year from the satellite image. Also, it is possible to estimate the shadow percent for any formal building at any day in the year at the imaging time. However, there are some local parameter it is recommended to re-derived the equation for the regions to test the differences in use.

## 9. REFERENCES

Al-Najim, F. A. and Al-Naimy, H. M. (1981). Space Physics, Meteorology. Ministry of Higher Eduacation and Scientific Research, Baghdad, pp. 61 – 65.

Appleton, E. V. (1945). Nature Magazine 3966:535.

Arévalo, V., González, J., Valdes, J., & Ambrosio, G. (2006). Detecting shadows in QuickBird satellite images. In ISPRS Commission VII Mid-term Symposium" Remote Sensing: From Pixels to Processes", Enschede, the Netherlands (pp. 8-11).

Cooper, P.I (1969). The Absorption of Radiation in Solar Stills. Solar Energy, 12(3), 333 - 346.

Iqbal, M (1983). An Introduction to Solar Radiation. Academic Press.

Karantzalos, K., & Paragios, N. (2008). Automatic model-based building detection from single panchromatic high resolution images. ISPRS Archives.

Lin, C. and Nevatia, R. (1998). Building Detection and Description from a Single Intensity Image. Computer Vision and Image Understanding, 72(2), 101-121.

Muneer, T. (2007). Solar radiation and daylight models. Routledge.

NASA Report. Scale Model of Sun and Earth. [online] Available at https://sunearthday.nasa.gov .

National Center for Atmospheric Research (1996). Satellite Coverages and Orbits. [online] Available at https://www.rap.ucar.edu/~djohnson/satellite/coverage.html .

Prasath, V. B. S and Haddad, O. (2006). Radar Shadow Detection in SAR Images Using DEM and Projections. IEEE Applied Imagery and Pattern Recognition Workshop (AIPR).

Satellite Image Corporation. TripleSat Satellite Image of Dubai, UAE. [online] Available at https://www.satimagingcorp.com/gallery/triplesat/triplesat-satellite-image-burj-al-arab-hotel-dubai-uae/

Seok, W., Yoon, S. C. G., Jang, J. S., Oh, S. H., & Jung, S. K. (2007). Development of 3D Model Reconstruction and Terrain Registration Technology Using Satelliate Images. A Previous Project of Virtual Reality Laboratory Jan.

Shettigara, V. K. and Sumerling, G. M. (1998). Height Determination of Extended Objects Using Shadows in SPOT Images. Photogrammetric Engineering & Remote Sensing, 64(1), 35 - 44.

Spencer, J. W. (1971). Fourier series representation of the position of the sun. Search, 2(5), 172-172.

Sun, J., Jiao, W., Wu, H., & Shi, C. (Eds.). (2013). China Satellite Navigation Conference (CSNC) 2013 Proceedings. Lecture Notes in Electrical Engineering.