Sliding Mode Controller For Nonlinear System Based on Genetic Algorithm

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ABSTRACT

Sliding Mode Controller (SMC) is a simple and effective method to recognize a robust controller for nonlinear system. It is a strong mathematical tool which gives a nonlinear robust controller with acceptable performance. The chattering phenomenon is the major drawback that sliding mode control suffers from. This phenomenon causes a zigzag motion along the sliding surface. In this work to design SMC, the Saturation function (i.e., boundary layer) has been used instead of the Sign function that was used in classical sliding mode controller in order to reduce the chattering phenomena which are appearing in the sliding mode phase. The genetic algorithms have also been proposed in this work for the parameter selection method of Sliding Mode Controller and the results showed a high speed of the system state for reaching the sliding surface during the reaching phase and a chattering reduction during the sliding phase. A pendulum system has been used for testing the designed sliding mode controller. The simulation results showed good validity of the suggested method. Matlab programming and simulink were adopted for the simulation results.

Keywords: Sliding mode control, sliding surface, hitting time, chattering, genetic algorithms.

مسيطرات النمط الأنزلاقي للأنظمة اللا خطية بالأعتماد على الخوارزمية الجينية

الخلاصة:

تعتبر طريقة أستخدام مسيطرات النمط الأنزلاقي بسيطة وفعالة للحصول على مسيطر متين وجيد للتعامل مع الأنظمة اللاخطية, وهي طريقة رياضية فعالة تعطي مسيطر متين لا خطي يعطي نتائج ذات خصائص ومواصفات جيدة ومقبولة. وتعتبر ظاهرة التذبذب هي أكثر صفة سيئة تعاني منها أنظمة النمط الأنزلاقي. وتسبب هذه الظاهرة حركة متعرجة على أمتداد السطح الأنزلاقي. في هذا البحث وعند تصميم مسيطرات النمط الأنزلاقي تم أستخدام دالة الأشباع بدلا من دالة الأشارة المستخدمة في أنظمة السيطرة الأنزلاقية التقليدية من أجل تقليل ظاهرة التذبذب التي تظهر في مرحلة الأنزلاق. كذلك تم أقتراح الخوارزمية الجينية في هذا البحث من أجل أختيار العناصر المكونة لمسيطر النمط الأنزلاقي و أظهرت النتائج سرعة عالية لحالة النظام للوصول الى السطح الأنزلاقي خلال مرحلة الوصول وكذلك تقليل في ظاهرة التذبذب في مرحلة الأنزلاق. تم أستخدام نظام البندول لأختبار مسيطر النمط الأنزلاقي الذي تم تصميمه. أن التنفيذ من خلال المحاكاة بالحاسوب أظهر نتائج جيدة وأيجابية للطريقة

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المقترحة في هذا البحث. لقد تم أستخدام البرمجة بلغة ماتلاب لأيجاد نتائج المحاكاة بأستخدام بالحاسوب.

INTRODUCTION

In various nonlinear control system issues, sliding mode control method is recently a popular method since it gives a nonlinear robust controller with good performance. Sliding mode control methodology was first proposed in the 1950 but this controller has been analyzed by many researchers in recent years. The Sliding Mode Controller (SMC) is a special case of Variable Structure Controller (VSC) which provides an algorithm for designing a system in such a way that the controlled system will be completely insensitive to the external disturbances and parameters uncertainty [1].

The great advantage of the sliding mode control is that when the system enters the sliding mode, it will be insensitive to the external disturbance and parameters variations, so if the hitting time is reduced, the parameters uncertainty of the system can be attenuated and the time taken by the system to be closely with the desired dynamic behavior can also be reduced. Therefore, reduction of the hitting time is very important requirement in designing the slide mode control [1].

In practical applications, classical sliding mode controllers are characterized by high switching frequency of control which results in undesirable properties called chattering phenomena. This chattering phenomenon is appears in the sliding mode phase which is characterized by the states continuously crossing the sliding surface, rather than stay on it. The chattering phenomenon is a severe problem in the classical sliding mode control. The stability of the controlled system is affected by the chattering phenomenon in the switching plane; so reducing this chattering is a very important matter. In [2] the boundary layer switching function was used instead of the Sign function, which was used in classical sliding mode controller, and as a result the chattering phenomenon is decreased.

Finally, the system performance is sensitive to sliding surface slope coefficient, λ. Thus, determination of an optimum value of λ is an important problem. For this reason the method that proposed in [3] is suggested a method for obtaining the optimum value of sliding surface slope coefficient. Finally, the parameter selection method of the sliding mode controller using genetic algorithm was proposed in [4] in order to find the optimal values of the SMC parameters.

In this present work, the genetic algorithm is used to obtain the optimal parameters of the sliding mode controller, which used the boundary layer, so the contribution of this work is to solve together and at the same time the three problems discussed above (i.e., the hitting time reduction, the chattering attenuation and obtaining the optimum value of the sliding surface slope coefficient λ) where the previous methods which are discussed above are solving these problems separately not together. The validity of the proposed method has been proved by the simulation results in section VII.

II. Classical Sliding Mode Control

The sliding mode controller method (SMC) is a very powerful nonlinear controller which has been used in recent years by many researchers. SMC can be used to ensure the stability of the controller. The Sliding mode controller consists of two phases; reaching phase and sliding phase. In reaching phase, the sliding mode controller drives the state trajectory toward a sliding surface in the state space. In sliding phase, the sliding mode controller forcing the state trajectory to stay on this sliding surface and to slide on this surface until reaching the origin [1 and 5].

The sliding mode control law is consisting of two main parts;

$$u = u_{eq} + u_{dis} \quad \ldots \quad (1)$$

Where, u_{eq} is the equivalent part of SMC which is compensated the nominal system dynamics and u_{dis} is the discontinuous part of the SMC.

The u_{dis} is computed as bellow;

$u_{dis} = k.\ sign(s)$; Where k is constant with positive value. .. (2)

By replacing the formulation (2) in (1) the control output can be written as;

$$u = u_{eq} + k.\ sign(s) \quad ..(3)$$

The sliding surface is described as;

$s(x,t) = \lambda x + \dot{x} = 0$; Where λ is constant with positive value. … (4)

For $x_1 = x$ and $x_2 = \dot{x}$

$$s = \lambda x_1 + x_2 \qquad \lambda > 0 \quad \ldots (5)$$

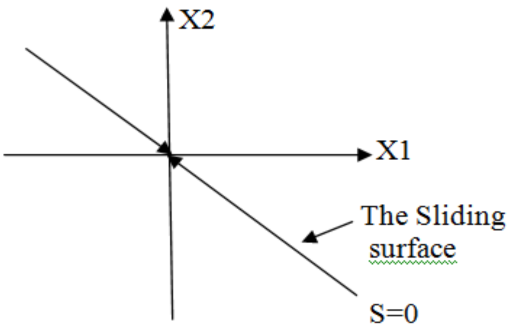


Figure (1): The Sliding Surface in the state space.

The function $sign(s)$ is defined as bellow;

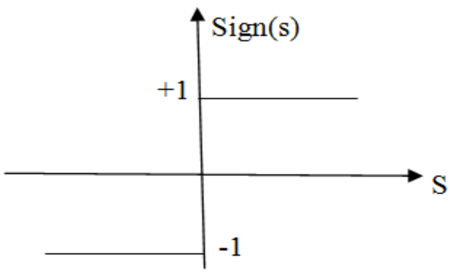


Figure (2): The sign(s) function.

$$sign(s) = \begin{cases} +1 & s > 0 \\ -1 & s < 0 \\ 0 & s = 0 \end{cases} \quad \ldots(6)$$

The main object is to keep $s(x,t)$ close to zero when the state of the system trajectory is outside the sliding surface in the phase plane.

The chattering phenomenon is one of the most important drawbacks that the sliding mode control suffers from, which is caused a zigzag motion in output along the sliding surface results from the discontinuous part. Chattering phenomenon is undesirable property since it excites the un-modeled dynamics of the mechanical systems.

III. Sliding Mode Control with boundary Layer method

The boundary layer method $sat(s)$ has been used instead of the $sign(s)$ function in order to reduce the chattering phenomenon.

The $sat(s)$ function is computed as bellow [3];

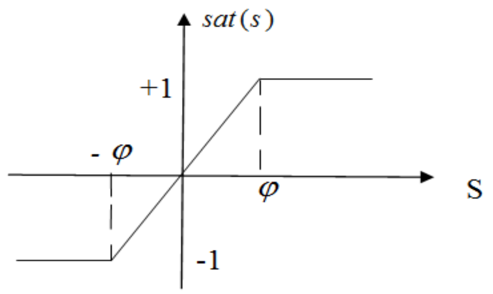


Figure (3): The sat(s) function.

$$sat(s/\varphi) = \begin{cases} +1 & (s/\varphi > 0) \\ -1 & (s/\varphi < 0) \\ s/\varphi & (-1 < s/\varphi < 1) \end{cases} \quad \ldots(7)$$

$$u_{sat} = k.\ sat(s/\varphi) \quad \ldots(8)$$

By replacing the formulation (8) in (1) the control output can be written as;

$$u = u_{eq} + u_{sat} \quad \ldots(9)$$

$$u = u_{eq} + k.sat(s/\varphi) \quad \ldots(10)$$

$$u = \begin{cases} u_{eq} + k.sign(s) & ,|s| \geq \varphi \\ u_{eq} + k.s/\varphi & ,|s| < \varphi \end{cases} \quad \ldots(11)$$

The reduction of chattering is an important issue in the sliding mode controller that has been wildly used in the recent years. The proper choice of the width φ of the boundary layer of sliding surface in equation (11) can suppress chattering in sliding surface very well [3].

The hitting time that the system state hits the sliding surface from initial state is an important issue that affected the performance of SMC. If the system state is able to hit the sliding surface faster, the stability and performance of the system is more guaranteed. So a suitable choice of a parameter *K* in equation (11) can be reduced the hitting time [1].

Finally, the sliding surface slope coefficient λ is one of the very important challenging factors in classical sliding mode controller, so a proper choice of λ in equation (5) gives a high performance response.

For the above reasons, the genetic algorithms method is used in this work to chose the appropriate gain parameters (φ, K and λ) in order to have a good and better performance.

IV. Problem Description

Consider the Pendulum equation which is shown in figure (4) below:

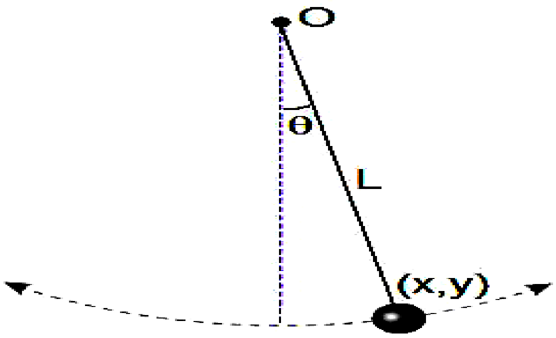


Figure (4): the Pendulum.

$$\ddot{\theta} = -a\sin\theta - b\dot{\theta} + cT \quad \ldots(12)$$

Where:

θ: is the angle caused by the rod with the vertical axis.

T: is the torque applied at the end point of the pendulum. The torque T is considered as the control input u applied at the endpoint of the pendulum.

Let us suppose that we want the pendulum to be stabilize at the angle $\theta = \theta_f$, where θ_f is the equilibrium point and it is considered as the desired value, where θ is considered as the controlled variable (i.e., the output).

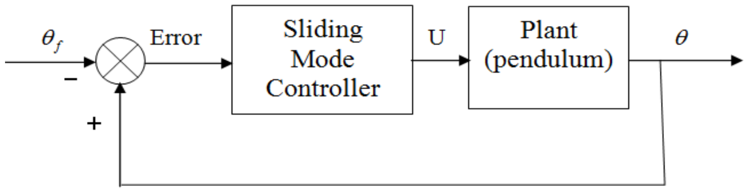


Figure (5): the pendulum system with the Sliding Mode controller.

Then the error and its derivatives are defined as:

$$e = \theta - \theta_f \quad \text{…(13)}$$

and $\dot{e} = \dot{\theta}$ (since $\dot{\theta}_f = 0$ because θ_f is constant). … (14)

Let x_1 *and* x_2 are the state variables of the system;

Let $x_1 = e$ *and* $x_2 = \dot{e}$

$\therefore\ x_1 = \theta - \theta_f$, and $x_2 = \dot{\theta}$ …(15)

And also let the control variable $u = T$. …(16)

Equation (12) can be written as:

$$\dot{x}_1 = x_2 = \dot{\theta}$$

$$\dot{x}_2 = -a \sin(x_1 + \theta_f) - b\,x_2 + c\,u = \ddot{\theta} \quad \text{… (17)}$$

The aim of the control design is to translate the pendulum to the desired angle θ_f and maintain it at this angle.

$$s = \lambda\,x_1 + x_2 \qquad \lambda > 0 \quad \text{…(18)}$$

$$\dot{s} = -a \sin(x_1 + \theta_f) - b\,x_2 + c\,u + \lambda\,x_2 \quad \text{…(19)}$$

$$u_{eq} = \frac{1}{c}[a \sin(x_1 + \theta_f) + (b - \lambda)\,x_2] \quad \text{… (20)}$$

For classical SLM controller, we will add equations (14) and (2) to find the overall control law;

$$u = \frac{1}{c}[a \sin(x_1 + \theta_f) + (b - \lambda)\,x_2] - k.\ sign(s) \quad \text{..(21)}$$

For Sliding Mode Control with boundary Layer method**,** we can add equations (14) and (8) to find the overall control law;

$$u = \frac{1}{c}[a \sin(x_1 + \theta_f) + (b - \lambda)\,x_2] - k.\ sat(s/\phi) \quad \text{… (22)}$$

For $k = 2,\ \lambda = 1,\ a = 10,\ b = 1,\ c = 10,$ equations (15) and (16) will be as bellow;

1: For classical SLM controller: Equation (15) will be as bellow;

$$u = a \sin(x_1 + \theta_f) - 0.2\,sign(s) \quad \text{…(23)}$$

2: For Sliding Mode Control with boundary Layer method equation (16) will be as bellow;

$$u = a \sin(x_1 + \theta_f) - 0.2\,sat(s/\varphi) \quad \text{.. (24)}$$

And the sliding surface for both above cases will be as:

$$s = x_1 + x_2 \qquad \dots (25)$$

Where x_1 and x_2 are the system states.

V. **Genetic Algorithm GA:**

The Genetic Algorithm GA that will be used in this work can be described as bellow:

Step 1: Given the plant G(s) of Pendulum and Sliding Mode Controller U(s), implement the simulation of close loop control system by using the Runge Kutta method.

Step 2: Choose the appropriate Simulation step size and Observation time.

Step 3: Choose the upper and lower bounds for the SMC gain parameters (φ , K and λ).

Step 4: Choose the type of chromosome code of SMC gain parameters (φ , K and λ).

In this work, the real coded decimal number is used.

Step 5: Choose the type of selection method and the type of crossover operator.

Step 6: Select the GA parameters, such as the Population size, the Maximum number of generation, the Crossover probability and the Mutation probability.

Step 7: Set randomly the initial value of GA population x_i (i=1, 2, ……, n).

Where x_i represent each individual, n is the number of the Population size.

Step 8: Choose the Population generation g=1.

Step 9: Compute the ISE or ITSE or IAE or ITAE for each individual x_i of the GA.

Where ISE is Integral Square Error, ITSE is Integral time Square Error, IAE is Integral Absolute Error and ITAE is Integral time Absolute Error.

Step 10: Calculate the Darwinian fitness value.

$$\text{Fitness} = \frac{1}{ISE + 0.001} \text{ or } \frac{1}{ITSE + 0.001} \text{ or } \frac{1}{IAE + 0.001} \text{ or } \frac{1}{ITAE + 0.001}$$

Step 11: Select the individuals using the selection method in step 5, then apply the Crossover and Mutation process to the individual.

Step 12: If g equals the maximum number of generation, Stop, otherwise set g=g+1, and go to step 9. Where g = the number of generation of GA.

Step 13: Finally choose the individual which has the maximum fitness, and this individual is the best one from all individuals in the GA population.

VI. Parameters Selection Method using Genetic Algorithms

The simple Genetic Algorithms describe above can be used with predefined fitness function in order to find the optimal value of the SMC gain parameters (φ , K and λ) in order to obtain a better performance and improve some disadvantage in classical sliding mode controller. By using these optimal value of the SMC gain parameters (φ , K and λ) the chattering and hitting time will be reduced.

The flow chart of Genetic Algorithm is shown bellow in figure (6).

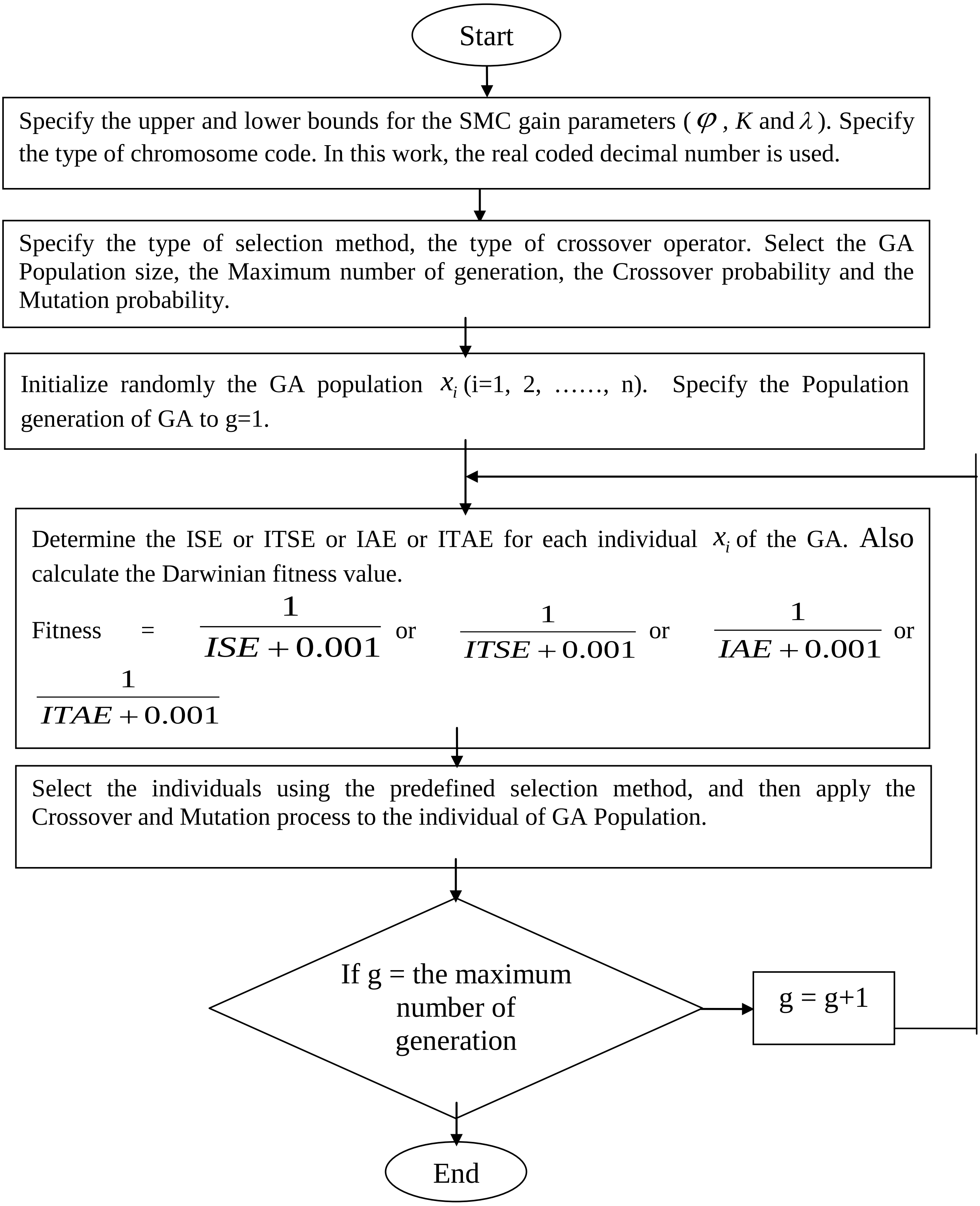


Figure (6): The flow chart of Genetic Algorithm

VII. The Simulations and the Results

The results for all the following **three cases are tested with the following input:**

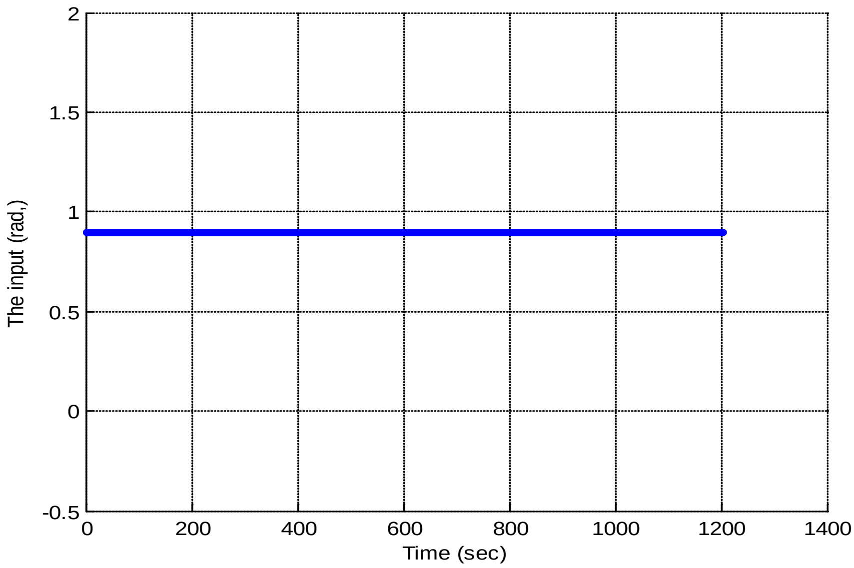
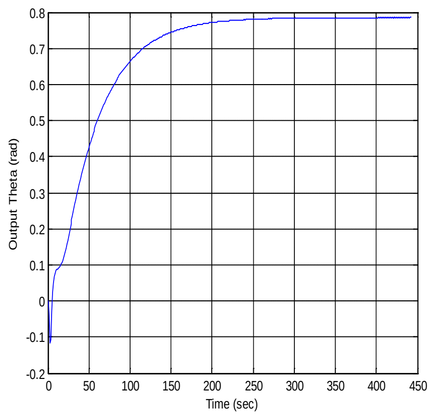


Figure (7): The plot of the input θ_f with time.

Case 1: The results when using equation (23) for classical SLM controller with the initial conditions as: $x_1 = pi/4,\ x_2 = pi/4$ and $\theta_f = pi/4$ are shown in figures (8, 9, 10, 11, 12 and 13).



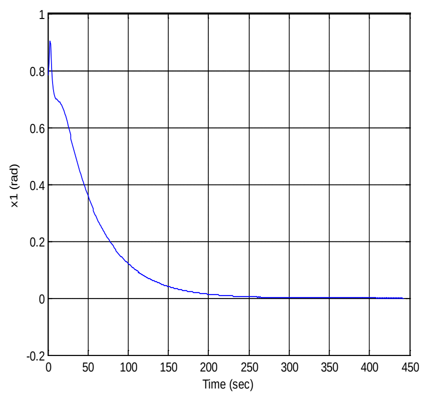


Figure (8): The plot of the output θ with time. Figure (9): The plot of the error x_1 with time.

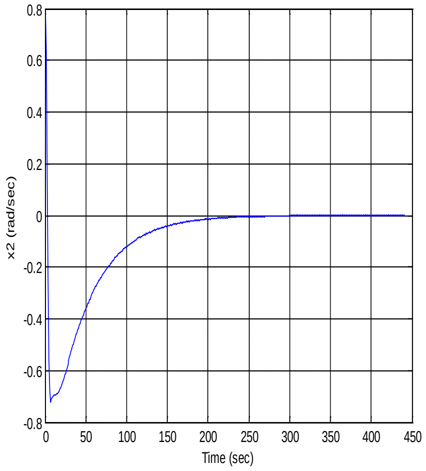


Figure (10): The plot of of x_2 with time.

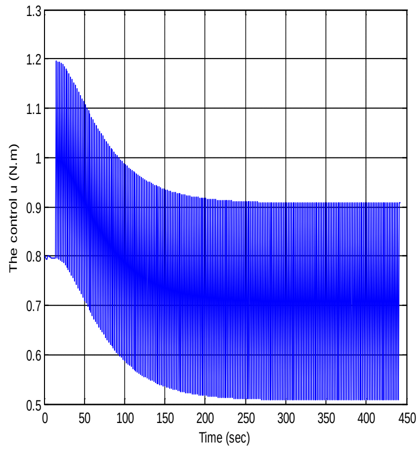


Figure (11): The plot of the control action u with time.

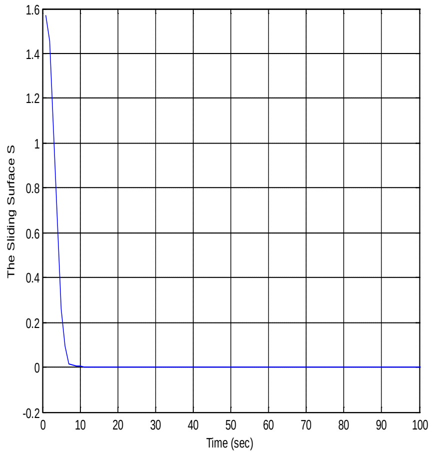


Figure (12): The plot of the Sliding Surface S with time.

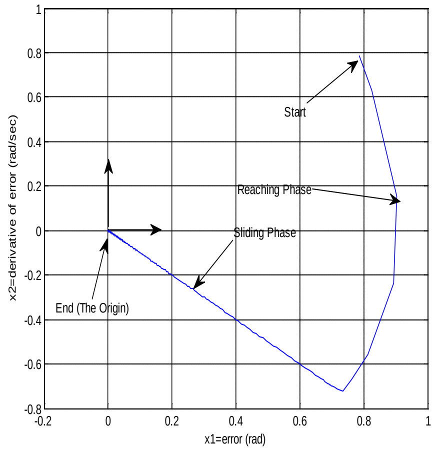


Figure (13): The plot of the phase plane of x_2 against x_1.

Case 2: The results when using equation (24) for SLM controller with the boundary layer with $\varphi = 1$ and with the initial conditions as: $x_1 = pi/4,\ x_2 = pi/4$ and $\theta_f = pi/4$ are shown in figures (14, 15, 16, 17, 18 and 19).

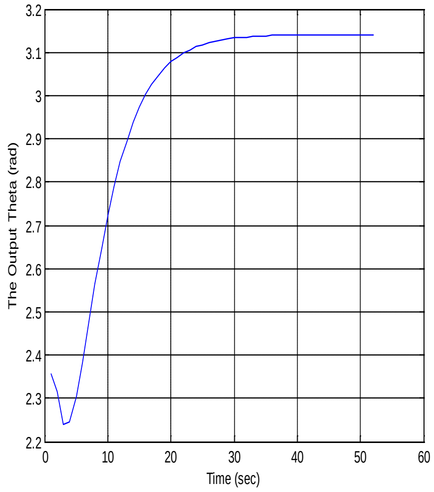


Figure (14): The plot of the output θ with time.

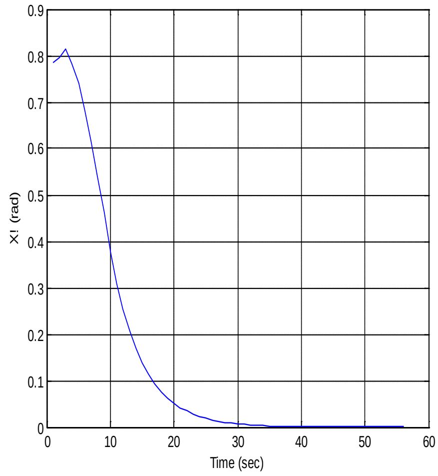


Figure (15): The plot of the error x_1 with time.

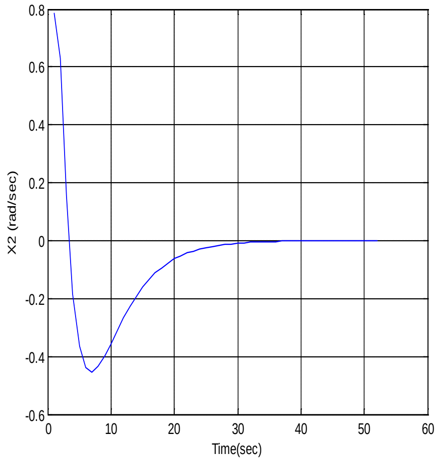


Figure (16): The plot of the derivative of error x_2 with time.

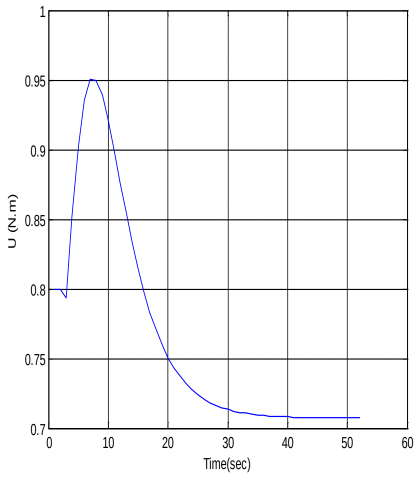


Figure (17): The plot of the control action u with time.

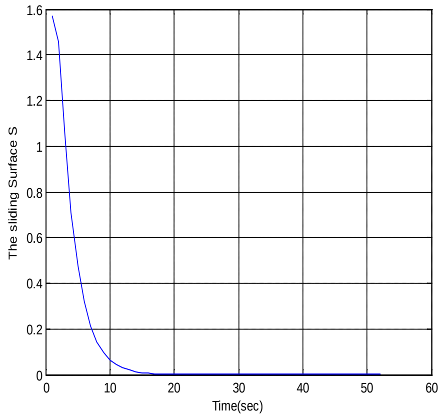


Figure (18): The plot of the Sliding Surface S with time.

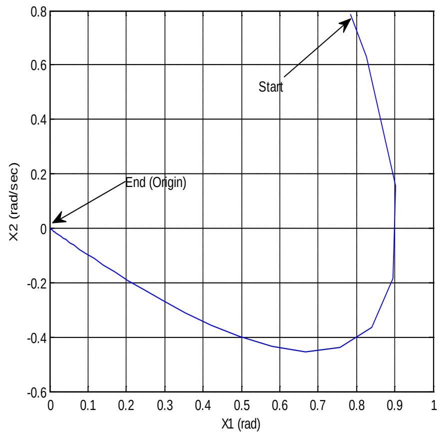


Figure (19): The plot of the phase plane of x_2 against x_1.

Case 3: The results of SLM controller with boundary layer using the Genetic Algorithm for purpose of choosing the proper gain parameters (φ, K and λ) are shown in figures (20, 21, 22, 23, 24 and 25), with the initial conditions as: $x_1 = pi/4,\ x_2 = pi/4,\ \theta_f = pi/4$ and the gain parameters which are obtained by using the Genetic Algorithm as: $\varphi = 0.75,\ k = 3.5\ and\ \lambda = 1.4$
The fitness function of the genetic algorithms is considered as:

$$Fitness = \frac{1}{Error + 0.001} = \frac{1}{x_1 + 0.001} = \frac{1}{(\theta - \theta_f) + 0.001}$$

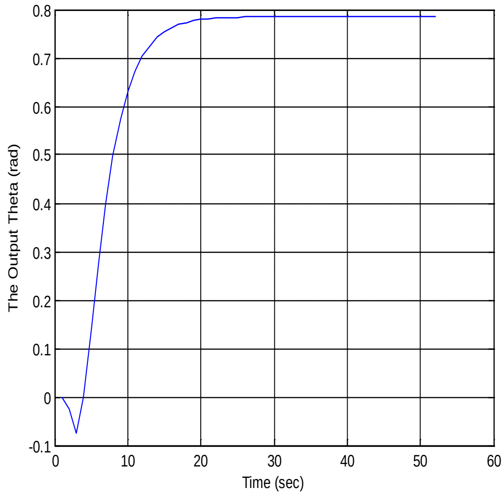


Figure (20): The plot of the output θ with time.

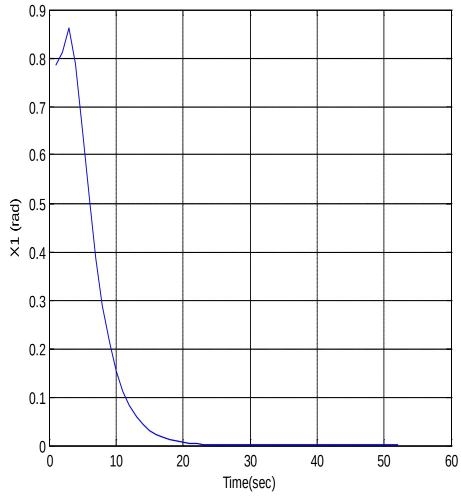


Figure (21): The plot of the error x_1 with time.

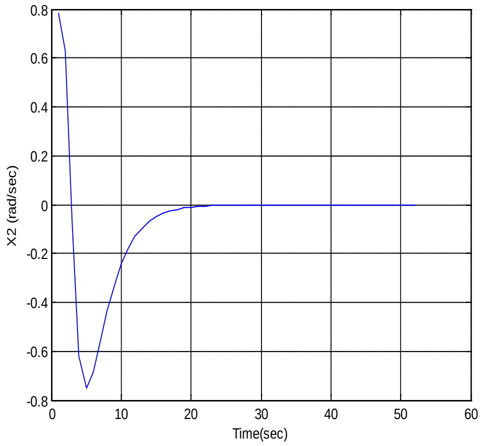


Figure (22): The plot of of x_2 with time.

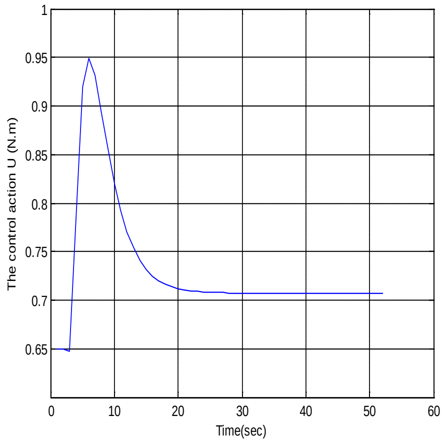


Figure (23): The plot of the control action u with time.

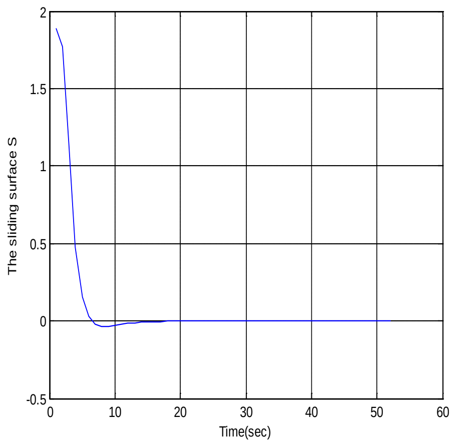


Figure (24): The plot of the Sliding Surface S with time.

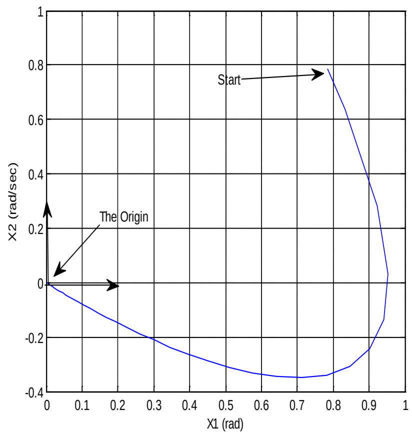


Figure (25): The plot of the phase plane of x_2 against x_1.

VII. Discussion and Conclusion

A parameter selection method based on genetic algorithms is proposed in this work.

This proposed method can efficiently choose the suitable gain parameters based on a predefined fitness function in order to achieve better performance.

The contribution of this proposed method can be summarized as that the controlled systems can simultaneously deal with multiple objective functions at the same time such as hitting time reduction, chattering attenuation and suitable choice of sliding surface slope coefficient λ, so that multiple objective functions can be considered, and as a result a good and high performance response can be obtained.

The results showed the validity of the proposed method because:

1: The error x_1 and the derivative of error x_2 go to the origin in the final trajectory of the phase plane (see figures 13, 19 and 25). This means that the system is asymptotically stable. This result can be considered as one advantage of using the sliding mode controller.

2: The chattering phenomenon is reduced as it is clear in figure (23) compared with figures (11 and 17).

3: The proposed method can speed up the system response in the reaching phase and as a result the hitting time will be reduced. Reducing the hitting time is very important. Because this means that the system will reach the sliding surface quickly and in short time. This can be considered as a great advantage of the sliding mode controller since when the system reached the sliding surface it will be insensitive to the external disturbance and parameters variations. The reduction in hitting time can be seen clearly in figure (24) compared with figures (12 and 18).

From the simulation results we can notice clearly that by using the proposed method, we can have a fast reaching velocity to the sliding surface during the reaching phase (i.e., reducing in hitting time) and then during the sliding phase the system state will stay in the sliding surface with small chattering and slide until reaching the origin.

We can also see clearly that the results in the case of using SMC with the boundary layer is better than the results of classical sliding mode controller while on other hand, the results using the Genetic algorithm for SMC with boundary layer is the best one comparing with the previous two cases.

This is seen clearly when we measured θ , x_1, x_2, u and S as discussed bellow;

1: In the case of classical sliding mode controller, the output θ reached the steady state value in 260 sec, the error x_1 reached the zero value in 260 sec, the derivate of error x_2 reached the zero value in 220 sec, the control action u reached the value of 1 in 15 sec and at this time the system suffers from the chattering phenomenon which is undesirable properties. The sliding surface S reached the zero value in 0.8 sec. Finally in figure (13) we can see clearly that the system state is approximately hits the sliding surface vertically.

2: In the case of using SMC with the boundary layer, we see that θ reached the steady state value in 30 sec, x_1 reached the zero value in 30 sec, x_2 reached the zero value in 32 sec, while in using of using the Genetic Algorithm for the SLM controller with boundary layer x_1 reached the zero value in 20 sec and x_2 reached the zero value in 22 sec, u reached the value of 0.71 in 38 sec and we can notice that this case can suppress the chattering phenomenon, and finally the sliding surface S reached the zero value in 15 sec. Finally in figure (19) we can see clearly that the system state is approximately hitting the sliding surface in arc shape.

3: In the case of the SLM controller with boundary layer using the Genetic Algorithm in order to chose the proper gain parameters for (φ , K and λ), we see that θ reached the steady state value in 22 sec, x_1 reached the zero value in 22 sec, x_2 reached the zero value in 22 sec, u reached the value of 0.71 in 24 sec and we can see that in this case chattering phenomenon was completely eliminated, and finally the sliding surface S reached the zero value in 12 sec. Finally in figure (25) we can see clearly that the system state is approximately hitting the sliding surface in arc shape which is closely to the circle.

From the above figures we can notice clearly that in case 1 the output θ reached the steady state value in 220 sec while in case 3 the output θ reached the steady state value in 22 sec which means that we improve the system response approximately about ten times.

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