Fracture Energy Influence on the Behavior of Steel Fiber Reinforced Concrete (SFRC) Beam Using Finite Element Method

Dr. Mehdi Ali Jawad Albayyati
Civil Engineering Department , Engineering College, Basrah University / Basrah
Email:Mehdialbayyati @yahoo.com.

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ABSTRACT

The principle aim of this research is concentrated on studying the effect of cracks and their propagations on the mechanical behavior of a steel fiber reinforced concrete (SFRC) beam. High strength concretes are being used more often due to their superior properties but these concretes have higher brittleness, which is a disadvantage. The idea of adding fibers to the concrete mixture to improve the mechanical response gives high strength steel fiber reinforced concrete (SFRC) with high toughness. The effect of steel fiber is introduced in the model by giving the concrete specified high value of fracture energy. Concrete cracking is divided into two major zones; the first one is the fracture zone (a combination of wide bridging zone effect and the cohesive microscopic cracking zone) which obeys a special law permitting the transmission of stress across the two faces of crack, this zone is considered as partially cracked concrete. When crack opening exceeds a specific value w_c , this zone is converted to a real crack (an open crack) and cannot transmit any stress across the two faces of a crack. Using the experimental data obtained from the flexural test on notched beam loaded under three-point bending, where fracture mode I is dominated. The response of the applied load-crack mouth opening displacement (CMOD) with using fracture energy calculated by the test and the cohesive stresses corresponding to their crack openings with different values of steel fiber respectively are drawn. The results show that the fracture zone for SFRC is wider than that occurs in plain concrete for many times. The contributions of plain concrete and steel fiber are being demonstrated.

تأثير طاقة التصدع على سلوكية العتبة الخرسانية المسلحة بألياف الفولاذ باستخدام طريقة العناصر المحددة

الخلاصة

يتركز الهدف الأساسي من هذا البحث على تحليل تأثير التصدعات وتطورها على السلوكية الميكانيكية لعتبة خرسانية مسلحة بألياف فولاذية. استخدمت غالبا الخرسانات عالية المقاومه

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لخواصها المتفوقة ولكنها تمتاز بالهشاشة العالية وهذا أحد عيوبها. إن فكرة إضافة ألياف إلى مزيج الخرسانة لتحسين الاستجابة الميكانيكية لغرض الحصول على خرسانة عالية المقاومة مسلحة بالألياف ذات متانة عالية. إن إدخال تأثير ألياف الفولاذ يتم من خلال إعطاء الخرسانة قيمة عالية محددة من طاقة التصدع. إن تصدع الخرسانة ينقسم إلى جزئين رئيسين, الجزء الأول منطقة التصدع (مركبة من منطقة واسعة من فعل التجسير ومنطقة التصدعات المجهرية التماسكية) وتخضع إلى علاقة تسمح بانتقال الأجهادات عبر وجهي الصدع ويمكن اعتبارها منطقة خرسانة متصدعة جزئيا. عندما تتجاوز فتحة التصدع, مقدار محدد w_c , تتحول هذه المنطقة إلى صدع حقيقي غير قابل لنقل الإجهاد عبر وجهي الصدع. باستخدام نتائج متحصل عليها من تجربة ألانحناء على عتبة مثلومة عند المنتصف ومحملة عند ثلاث نقاط, حيث يسيطر التصدع طور I. رسمت استجابة الحمل المسلط - أتساع فتحة أسفل الثلمة مع القيمة المحسوبة بالتجربة لطاقة التصدع وكذالك الاجهادات التماسكية مع فتحات التشققات. أظهرت النتائج بأن منطقة التصدع المجهرية التماسكية للخرسانة عند وجود الألياف تكون اكبر بكثير من تلك الحاصلة في الخرسانة العادية. تم إظهار مساهمة الخرسانة العادية وكذلك مساهمة ألياف الفولاذ.

INTRODUCTION

The idea of adding fibers to the concrete mixture to improve the mechanical response has been used since more than a century ago. It is from the early sixties that many researchers are trying to evaluate the mechanical properties of this material with the aim of systematizing its use. In the last three decades it has been done efforts in order to achieve a total or a partial substitute of conventional reinforcement on concrete. In this way, several discrete fibers were developed for concrete reinforcement, namely, steel, glass, synthetic and natural fibers [1], [2]. Steel fibers are the most used in concrete applications due to the following main reasons: economy, manufacture facilities, reinforcing effects and resistance to the environment aggressiveness. The industrial floors, the tunneling lines and the prefabrication are the main applications of steel fiber reinforced concrete (SFRC), where the conventional reinforcement is replaced by a given fiber content [3].

Nowadays fiber cementations materials are being used for a large variety of applications such as, bridge slab overlays, pavements and tunnel lining. The prediction of mechanical properties of those materials plays an important role in the selection of materials and the design of each application case. Cementations materials under tension show relatively brittle failure mechanism, which is critically influenced by crack localization. After a microcrack initiates, cementations matrix loses its tensile stress capacity. Due to the need of improvement in tensile capacity of typical cementations materials, fiber cementations materials have been developed. The addition of fibers inside cementations matrix results in the improvement of crack control capacity and tensile stress capacity. Several kinds of fibers can be introduced, and they can be broadly classified as metallic fibers, polymeric fibers, mineral fibers and naturally occurring fibers. Together with the development of properties of fiber cementations materials, the development of a method for predicting properties of those materials is essential. Fracture mechanics concept is considered as an effective tool in order to reproduce the failure mechanism and to predict the structural behavior type of cementations materials. Steel fibers reinforced (SFR) are the most used in concrete applications due to the following main reasons: economy, manufacture facilities, reinforcing effects and resistance to the environment aggressiveness.

Current structural design does not take into account the tensile strength, and is only based on elasticity and plasticity theories. It has been demonstrated that these conservative methods provide satisfactory results for design. However, most design equations are not based on physical principles but on empirical observations, and therefore need to be constantly calibrated for new materials and structural types. An ACI committee [4] advanced five strong arguments in support of the inclusion of the theory of fracture mechanics in the Codes. A brief overview of these arguments follows:-

(i) Energy requirement for crack growth.

A crack requires a certain amount of energy to grow. What is important for ensuring its integrity is how the cracks propagate under loading. This aspect can only be studied through an energy-based propagation criterion.

(ii) Objectivity of load and response calculations.

The finite element method has been applied to the analysis of cracked structures since the early 1960s. Cracking has been modeled either by neighboring elements using special discrete elements or by using smeared crack approach. In both the smeared and discrete models, cracks are assumed to form when the tensile strength is reached. Once this tensile strength is reached, the stress in the corresponding smeared and discrete element is allowed to drop instantaneously to zero, thus regarding concrete as a brittle material.

(iii) Lack of yield plateau.

In the limit or plastic analysis of a structure, the failure of various parts occurs in proportion to a single load parameter when sufficient plastic “hinges” have formed. A plateau in the load-response diagram characterizes such failure. When the yield plateau is absent, the failure is not plastic. It usually implies that the material is softening due to the fracture or other damage.

(iv) Absorbing energy capability and ductility.

If a concrete structure is loaded in tension or flexural and is recorded its response during the loading process right up to complete failure, a load-response diagram as shown in Figure (1) will be obtained. The area under the curve represents the energy absorbed during the loading process, if the energy lost is ignored in the loading grips and at the supports. The only part in which the energy is recoverable is the elastic one. The greater part of the energy is absorbed by the structure in the post-peak tension softening range, it determines the ductility of the structure; the greater energy is absorbed, more ductility is the response. The limit analysis does not take into account tension softening, so it cannot give us an indication of the energy absorbing capability of a concrete structure.

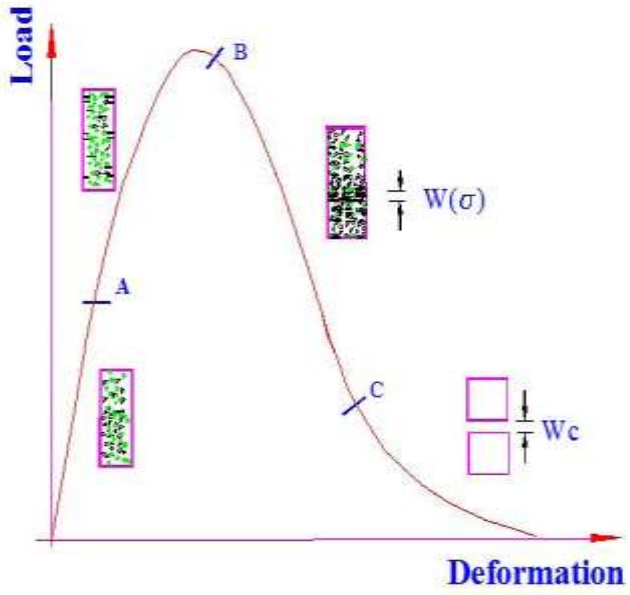


Figure (1) typical load-deformation response of a quasi-brittle Material in tension [5].

(v) Size effect.

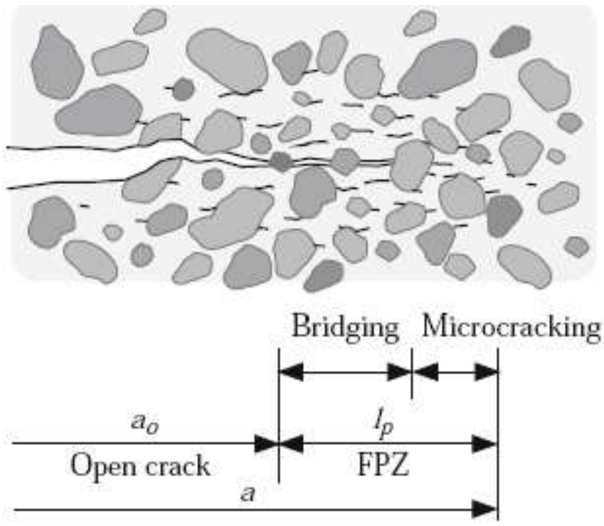
The most compelling argument to convince even the most skeptical designers is the so-called size effect. It is prevalent in many design situations but is generally ignored by the Codes.

Application fracture mechanics to plain structural concrete

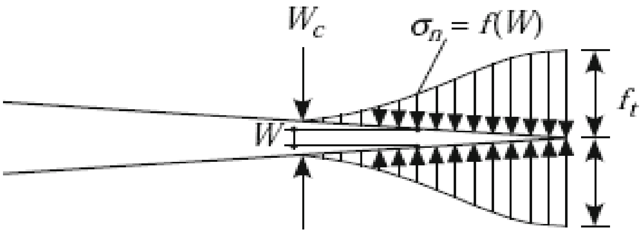
Cementitious materials such as concrete are quasi brittle material. The failure of a uniaxial tension specimen of these materials is characterized by a gradual reduction of tensile stress (so called tensile strain softening) due to the development of a single microcrack instead of sudden changes to non-traction cracks as shown in Figure (1). It is now clear that a fracture theory capable of describing the behavior of cementitious materials must include its material softening that takes place in the fracture process zone. The propagation of a crack due to the extension of the fracture process zone of cementitious matrix governs the failure of concrete structure under tensile loading. Therefore, the understanding of mechanisms along the fracture process zone is essential for the prediction of structural properties of concrete structures. Crack propagation and fracture of materials are primarily bound up with the behavior of the material in tension. The fracture of cementitious material such as concrete is governed by the existence of a fracture process zone. Crack bridging and fracture process zone of cementitious

materials are illustrated in Figure (2). The crack length a_o, is considered as the region where traction is free. Preceding this is the fracture process zone, which includes interlock, and coalescing microcracks. The transferred stress in the fracture process zone is so called "Bridging or Cohesive Stress". The bridging stress is mainly composed of aggregate bridging in plain concrete. The structural performance of cementitious material is strongly influenced by the crack bridging stress, which in turn depends on the bond behavior of aggregate/matrix. The relation that represents the bridging stress characteristics, which corresponds to the separation between crack surfaces, is referred to as "Bridging Stress Relation" or "Tension Softening Relation". This is considered as one of essential material properties of cementitious materials. Fracture mechanics has been introduced to describe tensile stress induced fracture behavior of plain, such as fracture and strength in bending and shear. Nonlinear fracture mechanics that consider a bridging stress that takes place in fracture process zone is applicable in describing the fracture process in ordinary concrete structures. The consideration of the fracture process zone leads to the capability in the prediction of performances of those concrete structures under flexure and tension.

The nonlinearity of plain concrete behavior can be illustrated in Figure (1). In an idealized load-deflection curve corresponding to unaxial tension [5], four different stages can be distinguished; the first consists of a linear response, the second stage is nonlinear leading to the peak load. The third and forth stages are characterized by an increase in the deformation with a decrease in the stress. Such response is called strain softening in tension, or simply tension beam softening to distinguish it from the strain softening in compression. These kinds of materials are called quasi-brittle materials. Recent studies on the fracture behavior of concrete reveal that some fracture characteristics differ from those normally observed in metallic materials. It is well known that microcracks exist in concrete even before it has been loaded. It is now clear that a fracture theory capable to include a description of the material softening taking place in the fracture process zone. Such a theory will necessarily be nonlinear but one must distinguish between the ductile-materials such as metals, from that applicable to quasi-brittle materials, such as rocks, concrete and ceramics. This is because in ductile materials the fracture process zone though small is surrounded by a large plastic zone, whereas in quasi-brittle material the fracture process zone practically occupies the entire zone of nonlinear deformation. In contrast, the nonlinear zone is absent in quasi-brittle materials. The above remarks are schematically illustrated in Figure (3).

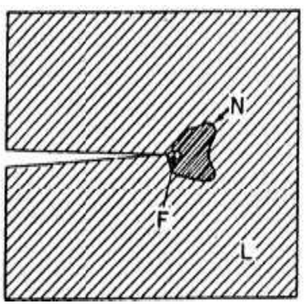


(a)

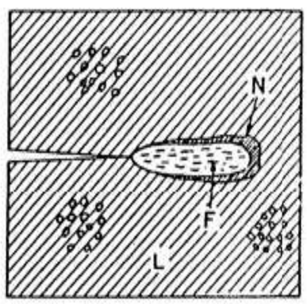


(b)

Figure (2) Fracture process zone and bridging stress distribution along a crack plane [6].



(a)



(b)

Figure (3) Fracture plastic zone a- Ductile-brittle (metals) b- Quasi-brittle (concrete) [7].

Fracture process zone and tension–softening phenomenon in Cementitious materials such as concrete and SFR concrete

Cementitious materials such as concrete and SFRC are heterogeneous materials that consists of aggregates and cement pastes bonded together at the interface, and the material is inherently weak in tension due to the limited bonding strength and various preexisting microcracks and flaws that form during hardening of the matrix. The tensile strength of concrete approximately ranges from 6 to 15 percent of its compressive strength. The increase on compression, tensile, shear and torsional strength is only marginal [8] for any content of fibers used in practice. Under external loading, a tension zone forms near the crack tip, in which complicated micro failure mechanisms take place. These fracture processes include microcracking, crack deflection, crack branching, crack coalescence, debonding of the aggregate from the matrix and bridging of fibers which are examples of inelastic toughening mechanisms that coexist with a crack when it propagates. In SFR concrete, the inelastic zone at the crack tip is extensively developed. Figure (4) schematically illustrates the formation of an inelastic zone in concrete, which is known as a fracture process zone (FPZ) that can be roughly divided into a microcracking zone of concrete and the bridging of fibers, along with two idealizations of the FPZ. It is known that bridging is a result of the fiber existence and it is an important toughening mechanism in SFR concrete. Within the damage zone the effective modulus of elasticity is reduced from that of the undamaged material E (initial) to a damaged material E (actual), if the process zone is modeled as a region of strain softening as shown in Figure (4 - b).

Hillerborg et al. (1976) [9] envisioned a fictitious crack method (FCM) in place of the physical FPZ and subjected it to closure tractions, as shown in figure (4-c). The closure stress associated with the bridging fibers and microcracks is a maximum at the tip of the FPZ and decreases to zero at the continuous crack tip where the crack opening reaches its critical value w_c, beyond which an open crack forms. Known as the tension-softening phenomenon, the relation between the closure stress and the crack opening with which the fracture energy of concrete is completely defined describes the local material behavior inside the FPZ when fracture takes place in SFR concrete.

Fracture energy G_F and tension-softening relation in plain and SFR concrete

As just discussed, fracture of plain or SFR concrete initiates in the FPZ ahead of an open crack through complicated micro-failure processes, and the fracture energy is consumed in overcoming the resistance of various toughening mechanisms to form an open crack at the end of the FPZ. The amount of fracture energy required to break a unit area of plain or SFR concrete is generally regarded as a material property (although it varies slightly with size) that determines the fracture behavior of the material through the fundamental relationship between the cohesive stress and the crack opening in the FPZ, which is known as the tension-softening law of concrete. Just like the constitutive relationship of a continuous material that stipulates the fundamental material behavior (whether it is elastic or inelastic), the tension-softening law with the fracture energy as its defining characteristic is the constitutive relationship for the material in the FPZ that describes the transitional material behavior from the continuous state to the discontinuous state, in other words, how the tensile stress decreases with the increasing discontinuity in the FPZ.

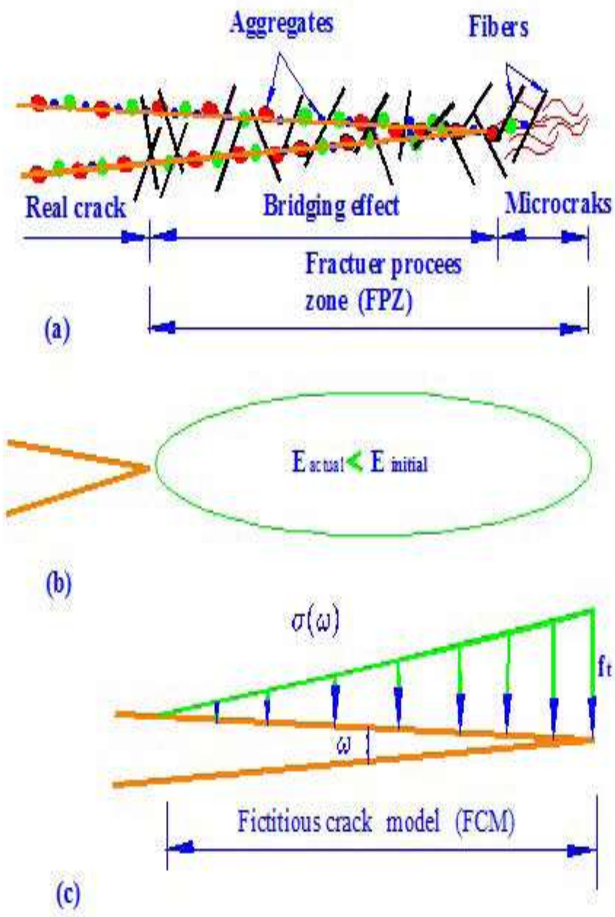


Figure (4) Concept of FPZ and tension-softening in concrete: (a) FPZ in front of an open crack, (b) reduced effective modulus of elasticity inside FPZ, and (c) tension-softening inside FPZ .

Fracture energy G_F .

The load-displacement relation is shown in Figure (5), the area enclosed by the response curve and the horizontal axis represents the work done by the external load to fracture the beam. Suppose that the crack growth is stable and the work done by the external load is spent solely in crack propagation. Based on the Griffith energy criterion, crack growth in an elastic body in the equilibrium state is a natural process of energy transfer between the strain energy of the body and the fracture energy required for creating a new crack surface so that a state of minimum potential energy is achieved for the system at a given load level. In the present case, the work is consumed in breaking the un-notched part of the beam cross-section, the ligament ahead of the notch. Denoting the work of the external load by W_F and the ligament area by A_{Lig} , the energy needed to create a crack of unit area, G_F according to the RILEM Technical Committee 50-FMC (Fracture Mechanics of Concrete) 1991 [10], is obtained as:

$$G_F = \frac{W_F}{A_{Lig}} = \frac{W_F}{(H - a_0)B} \qquad ...(1)$$

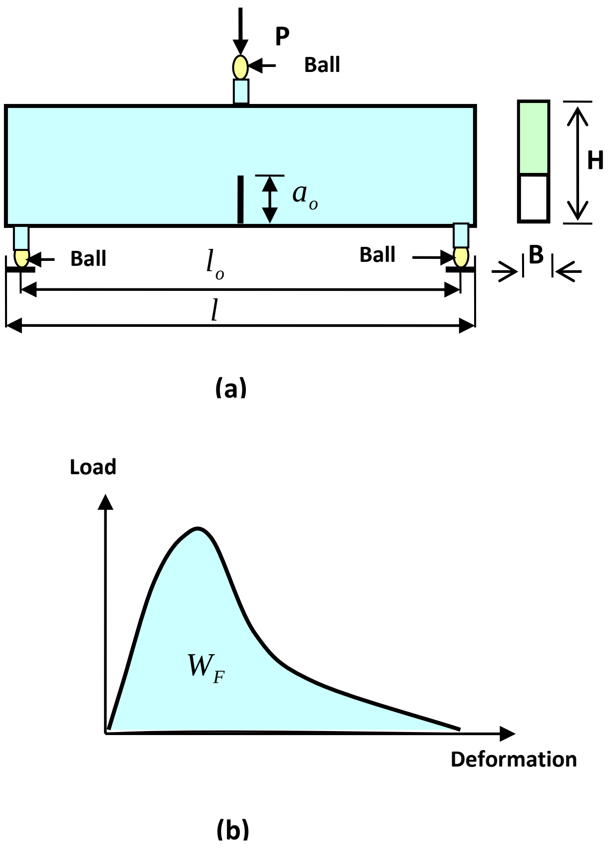


Figure (5) Determination of fracture energy G_F based on the RILEM method: (a) notched beam under three-point bending, and (b) load-deformation relations.

Obviously, this relationship can also be obtained from the stress-crack opening relations at the notch tip, where an open crack has just been created by fracturing the intact material there. Notice that the area enclosed by this tension-softening curve with the horizontal axis as shown in figure (6), is exactly the fracture energy that is:

$$G_F = \int_0^{w_c} \sigma_n(w)\, dw \qquad ...(2)$$

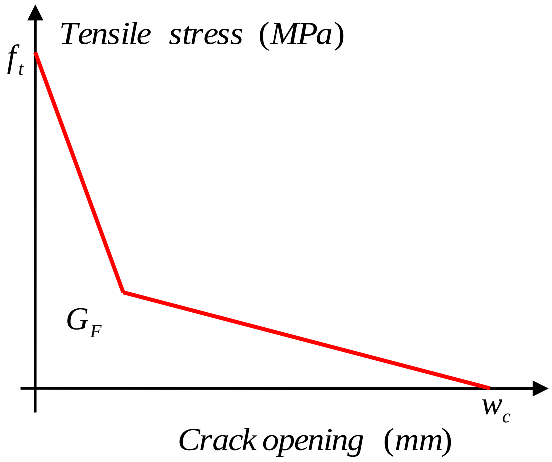


Figure (6) Relations between the tensile stress-crack openings.

Tension-Softening relation

As shown in Figure (6), the tension-softening relation of plain or SFR concrete possesses two distinctive features: the steep descending slope caused by the rapid loss of tensile strength in the initial stage of softening and a long tail with the increasing crack-opening displacement, illustrating the persistent stress-transferring capability of aggregate interlocking in the FPZ. Three shapes of post-peak constitutive laws are represented below: the Linear of Hillerborg et al. [9] (1976), the exponential of Jawad M. [11] (1989) and the bilinear of CEB-FIP Model Code [12] (1993). These relationships are implemented and depicted in Figure (7).

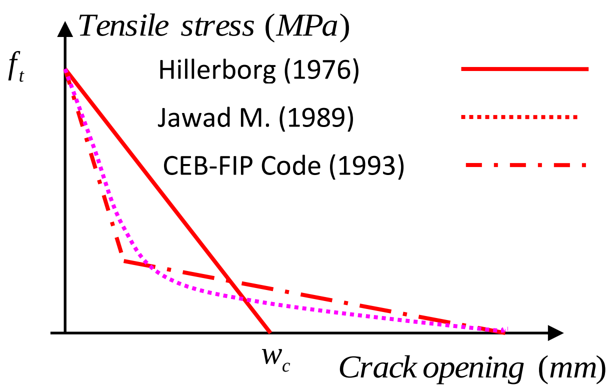


Figure (7) Proposed tension-softening models (a) by Hillerborg (1981) [9], (b) by Jawad M. (1989) [11] and CEB-FIP Code (1993) [12].

The relationship of the tension-softening used in this research takes the bilinear form of Rokugo et al. (1989) [13] as shown in Figure (8):

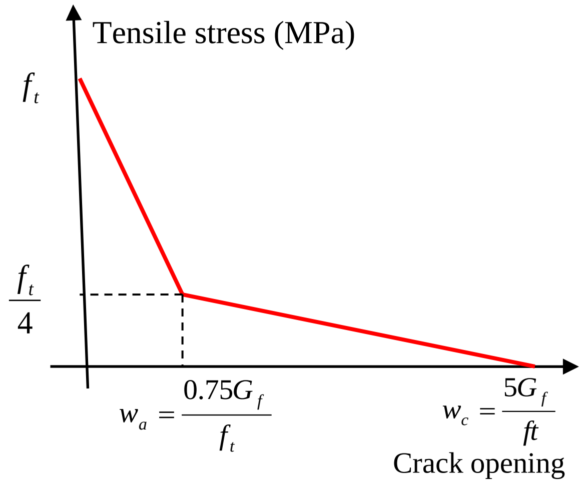


Figure (8) Proposed bilinear tension-softening model by Rokugo et al. (1989) [13].

The model program adopts a criterion states: when the concrete is cracked, there is a sudden loss in the tensile stress at a crack tip due to the immediate liberation energy. Taking this into account, the tensile stress at the tip of cohesive crack initiation equal to ($\alpha\ f_t$), where α takes values from 0.9 to 1.0.

APPLICATIONS AND RESULTS.

- Bending test of a notched beam mode I fracture test

The finite element program used in this application is prepared by the researcher to simulate the two-dimensional nonlinear behavior of plain and reinforced concrete beams by using the discrete cracking approach [11], [14]. This application is presented to gain an insight into the behavior of plain and SFR concrete under tensile stress with softening response. The experimental test is one of a series of testing carried out on three-point bend plain and SFR concrete specimens of a similar geometry with different steel fiber ratios by Carles F. B. [15].

The dimensions and loading conditions of the test are shown in Figure (9). The beam was modeled with mesh of (100) isoparametric quadrilateral of four nodes of uncracked concrete elements, concentrating the small size elements at a tip of notch beam and on the expected direction of the crack propagation as shown in Figure (10). The load-crack mouth opening displacements (CMOD) of experimental results for different ratios of steel fibers are shown in Figure (11).

The load-crack mouth opening displacement (CMOD) response of the plain concrete using the calculated fracture energy of the test comparing it with the model is shown in Figure (12). There is a very good agreement between the experimental data and the results of the model with the selection of G_F equal to 209 N/m. The first microcrack appears at a load level equal to approximately 7.19 kN and the ascending branch of the response curve far away from that point is slightly beginning to incline as the crack is propagated. Before the pre-peak of the response curve at load nearly to 16.44 kN, the microcrack is reached the elevation of 60 mm above a tip of notch beam and the stiffness of the beam is deteriorated, so that the inclination response curve becomes more flat and after that when the post-peak of the curve is exceeded, the rate of response curve slope is gradually dropped (load is decreased with crack opening displacement is increased) depending on the used value of fracture energy as it has a small value, the dropping rate is steeper till the divergence of numerical solution is occurred.

The influences of fibers steel reinforced SFR have been studied by using two fibers ratios of SFR 0.5% and 1.0% as shown in Figures (13), (14) respectively. There is a good agreement between the model and the experimental results, the response curves for both are approximately the same with the response curve of plain concrete (without SFR) till the pre-peak stage and after that there is a considerable increasing in the beam strength and ductility with soften hardening due to the SFR contribution. The concrete beam strength of 0.5% SFR is increased from 16.87 kN (for plain concrete) to more than 23.9 kN, means there is about 41% of strength load increment with more ductility is gained. The study of a concrete beam containing 1.0% SFR is increased from 23.9 kN (for 0.5% SFR) to more than 31.0 kN, means there is about 29.7% of strength load increment is gained but with high tensile stress is used. The using of SFR has increased the toughness and the ductility of the beam considerably, so, the high strength concrete can be improved his characteristic by adding the fibers inside cementious matrix results in the improvement of crack control capacity and tensile stress capacity and converting it from a brittle material to a ductile material which was one of its defect.

The program used in this research adopts the mathematic solution criterion of arc length method to follow the post-peak response of structure with constraint equation such that the level of loading is only just fulfill with one Gauss point reaches the tensile strength of concrete.

The contributions of concrete and fiber response curves with its overall 0.5% SFR concrete response curve are shown in Figure (15). The SFR contribution is insignificant at initial of loading till the response curve reaches the pre-peak stage, where sudden incremental is happened and continuing with soften hardening till it reaches maximum value and after that there is a large zone of yield plateau till the numerical divergence is occurred. The SFR gives more ductility to the concrete beam especially in post-peak stage, so that improves considerably its toughness and ductility converting it to a ductile material.

The following Figures from (16) to (18) show the relationships between tensile stress of cohesive crack and cohesive crack opening with different ratios of SFR before the numerical divergence solution is occurred. Figure (19) of plain concrete (0% SFR) shows that the maximum Gauss point opening displacement (at point A) with $w = 0.1315\ mm$ exceeds the opening displacement $w_a = 0.0488\ mm$ which separates the softening curve into two parts as mentioned early but even with this value the crack does not convert to a real crack because its opening still less than the critical opening displacement $w_c = 0.325\,mm$, where the fictitious crack is converted to a real crack and ceases of transmitting cohesive stress. Figures (20) and (21) with 0.5% SFR and 1.0% SFR respectively have large values of w_a varies from 1.346 mm for 0.5% SFR to 1,825 mm for 1% SFR and w_c

varies from 8.974 mm for 0.5% SFR to 12.164 mm for 1% SFR due the effect of fiber bridging, so that the maximum Gauss point opening displacement (at point A) with $w = 0.2202\ mm$ for 0.5% SFR and $w = 0.269\ mm$ for 1% SFRC lies in the first part of softening curve and the crack is lost some of his ability of transmitting cohesive stress across the two faces of it. This state leads to less energy liberation and increasing considerably the ductility of the structure.

CONCLUSIONS

1- The SFR effect inserted in the program by giving the concrete a large specified value of fracture energy G_f (calculated by the test) due to the high contribution of SFR in the post-peak stage under the bridging effect of fibers.

2- The over all energy liberation in post-peak stage is considerably less in the case of using SFRC than the plain concrete. This leads to improve the property especially of high strength concrete and convert it from a quasi-brittle material to a ductile material.

3- The strength of the beam is increased as the SFR ratio is increased due to the soften hardening but with decreasing rate and also the dropping rate of the descending softening curve is less.

4- The bridging effect of the SFRC plays a very important role such that the critical opening displacement w_c has a large opening displacement value so that, the crack is still transmitting high value of tensile stress across the two faces of it and is never converting to a real crack in this research.

5- The contribution of the fibers is most important at large values of crack opening while the contribution of concrete is really important at small vales of crack opening. This is because the most important transmission of stress across the crack is through the fibers beyond a certain crack width. The total response is the sum of the two contributions.

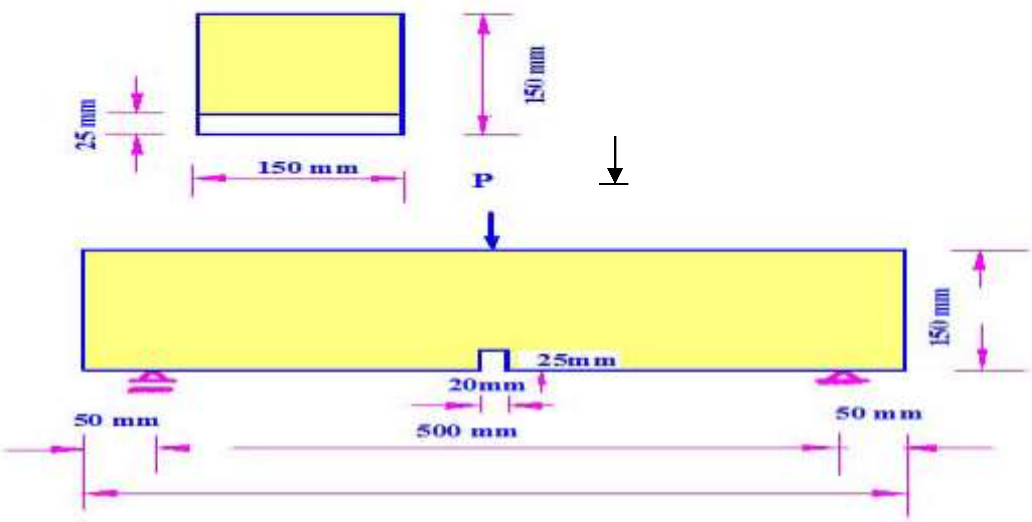


Figure (9) Test carried out by F. B, Carles.

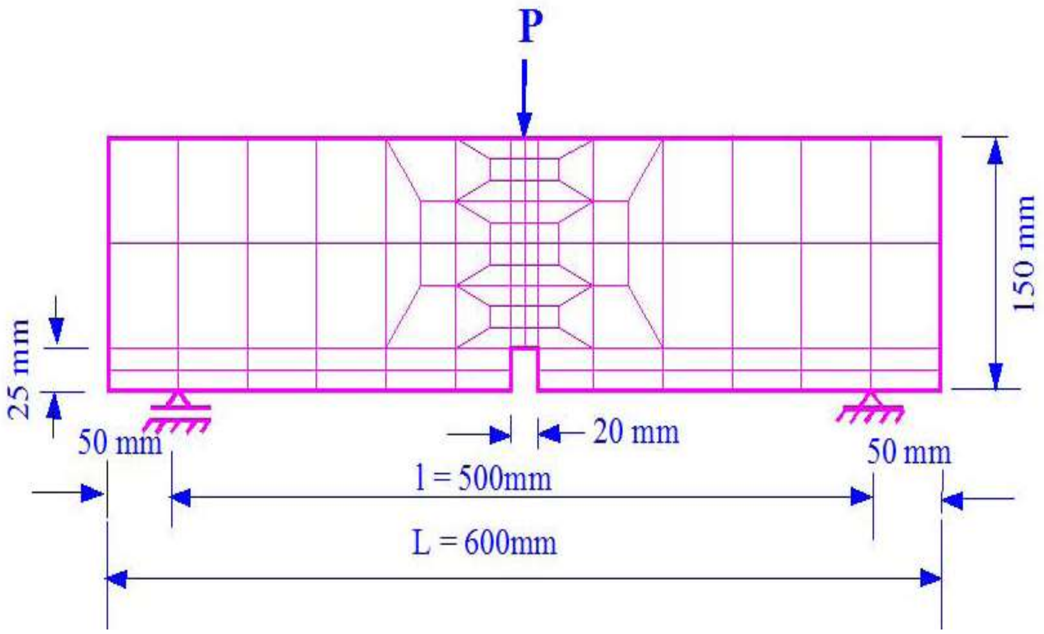


Figure (10) Finite element mesh of the beam.

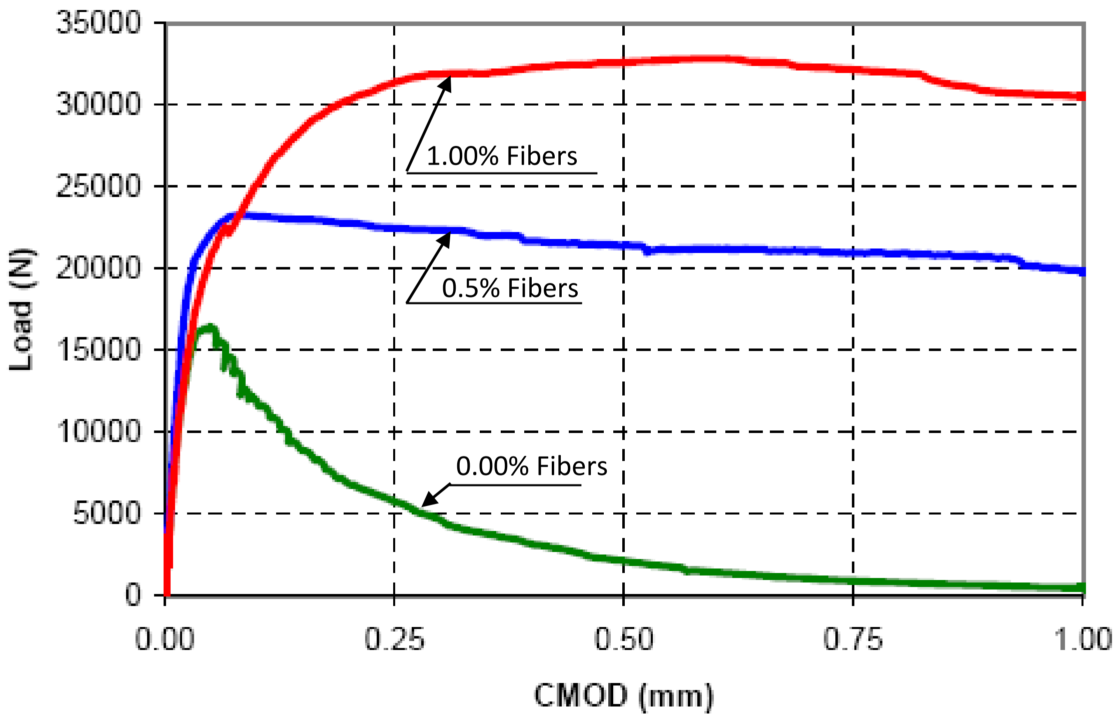


Figure (11) Load-CMOD (crack mouth opening displacement) responses for plain concrete, 0.50% and 1.00% SFR [15].

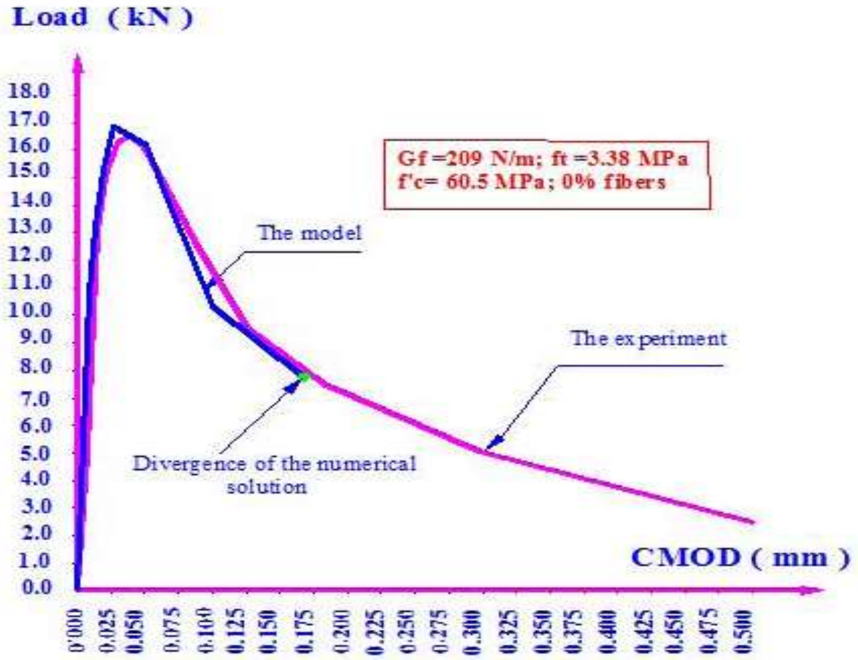


Figure (12) Load-CMOD response for plain concrete.

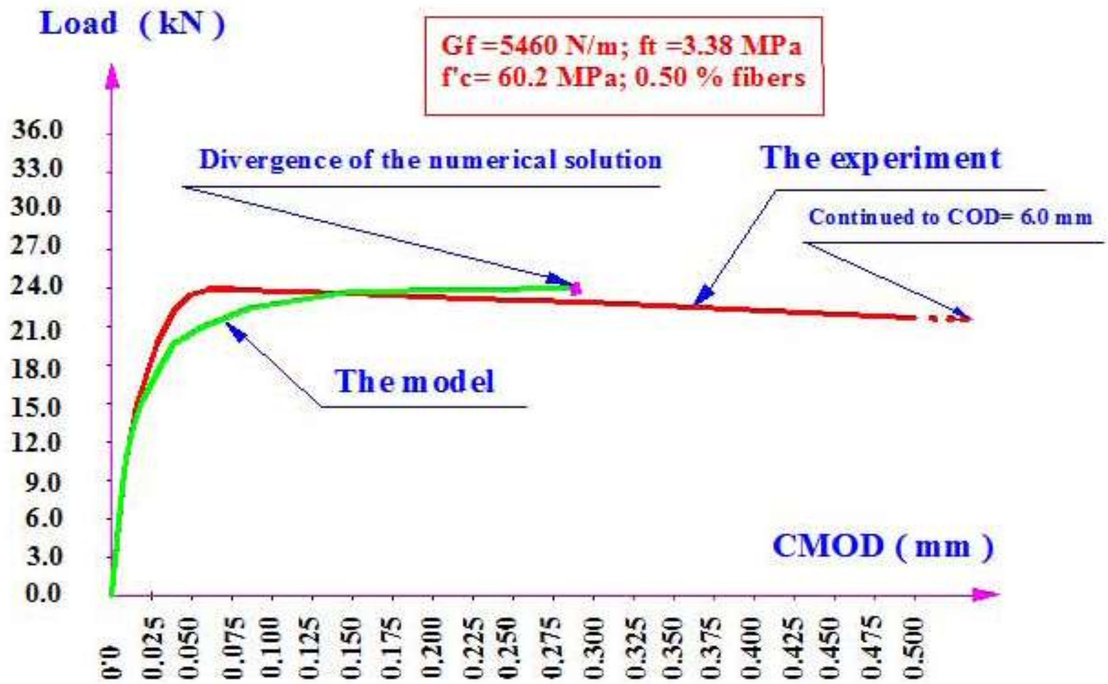


Figure (13) Load-CMOD response of 0.50% SFR.

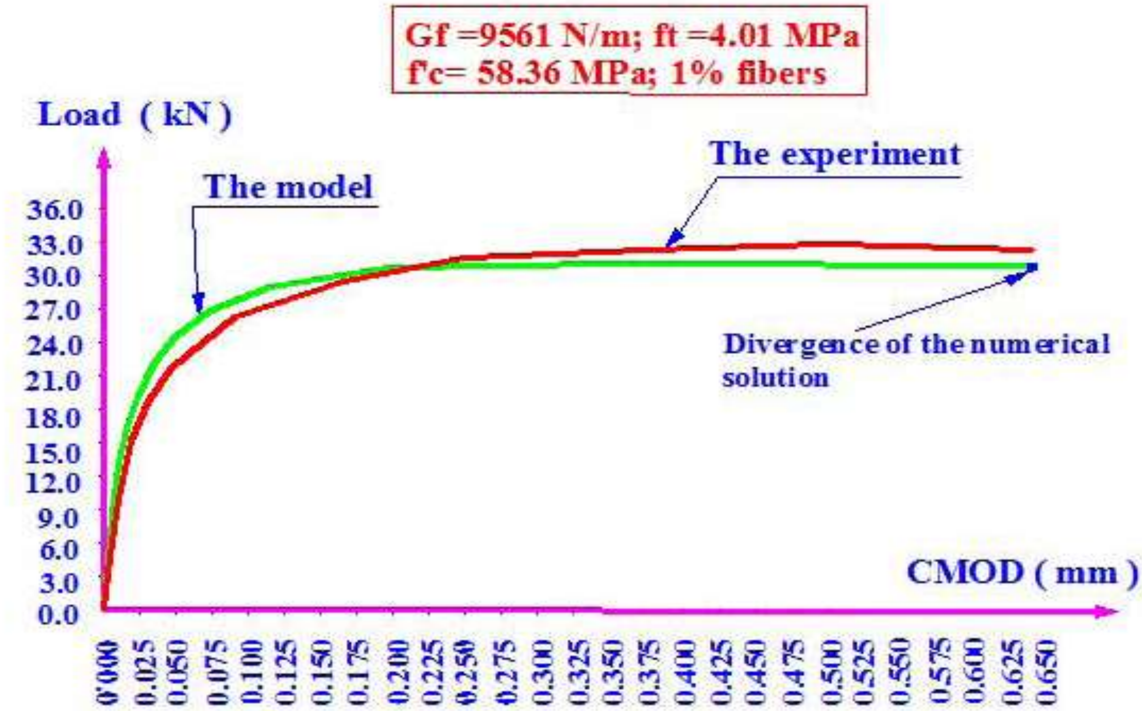


Figure (14) Load-CMOD response of 1.0% SFR.

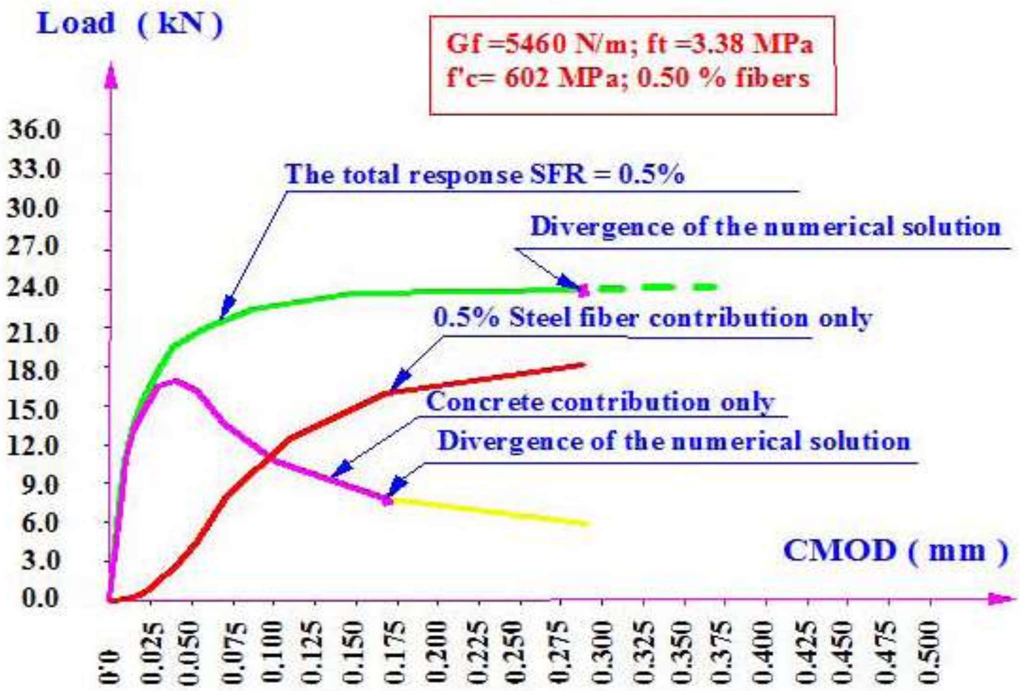


Figure (15) Contribution of plain concrete and steel fibers for 0.5% SFR.

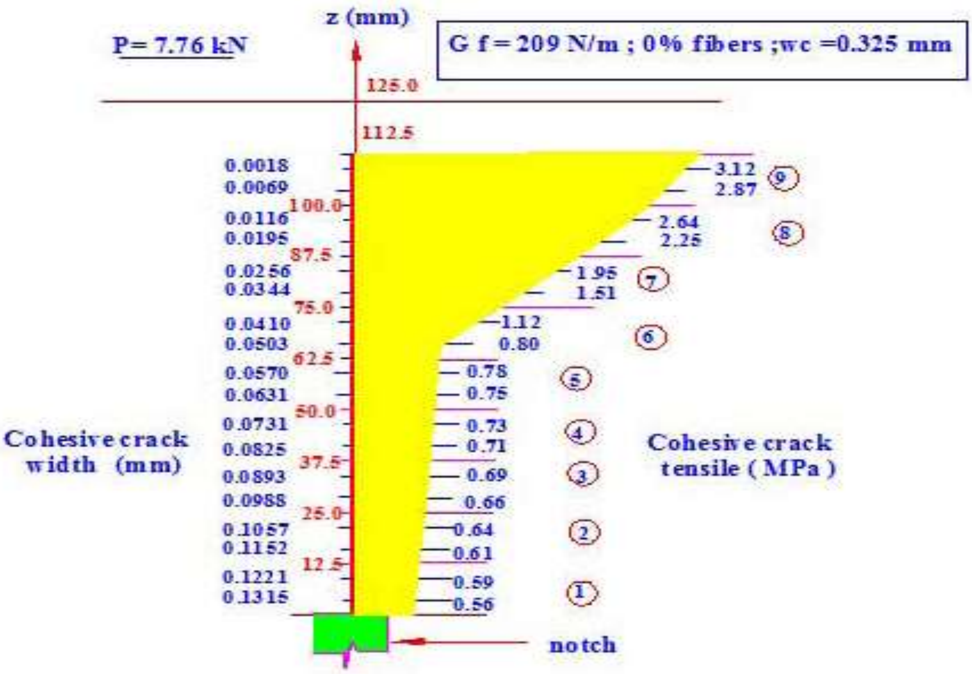


Figure (16) Crack cohesive stresses for plain concrete at load P= 7.76 kN.

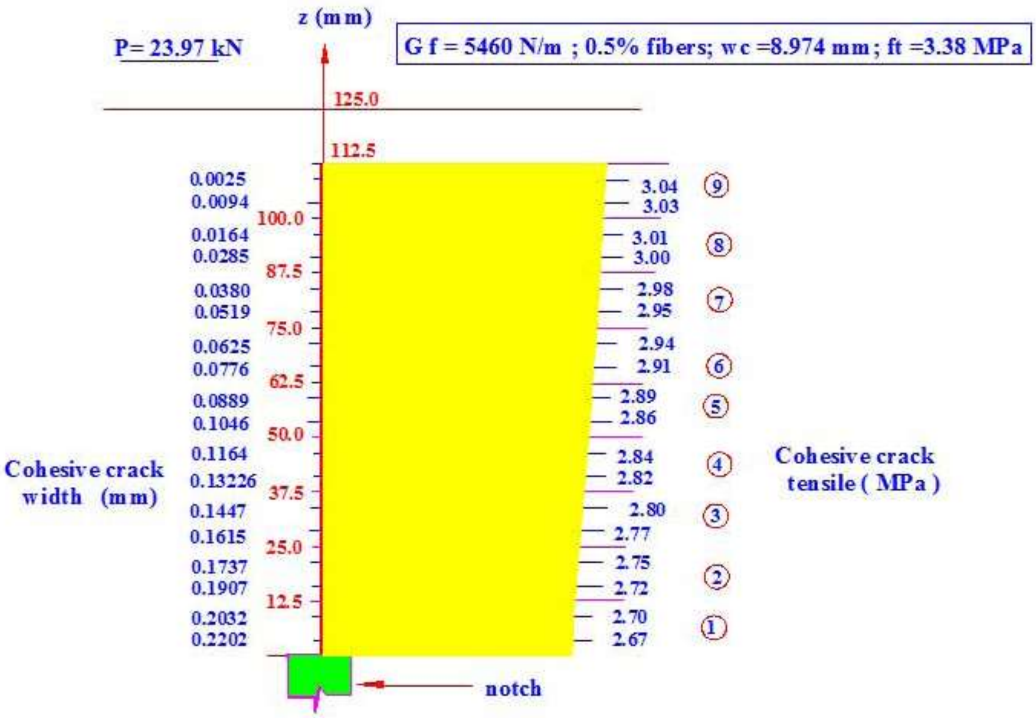


Figure (17) Crack cohesive stresses for 0.5% SFR at load P= 23.97 kN.

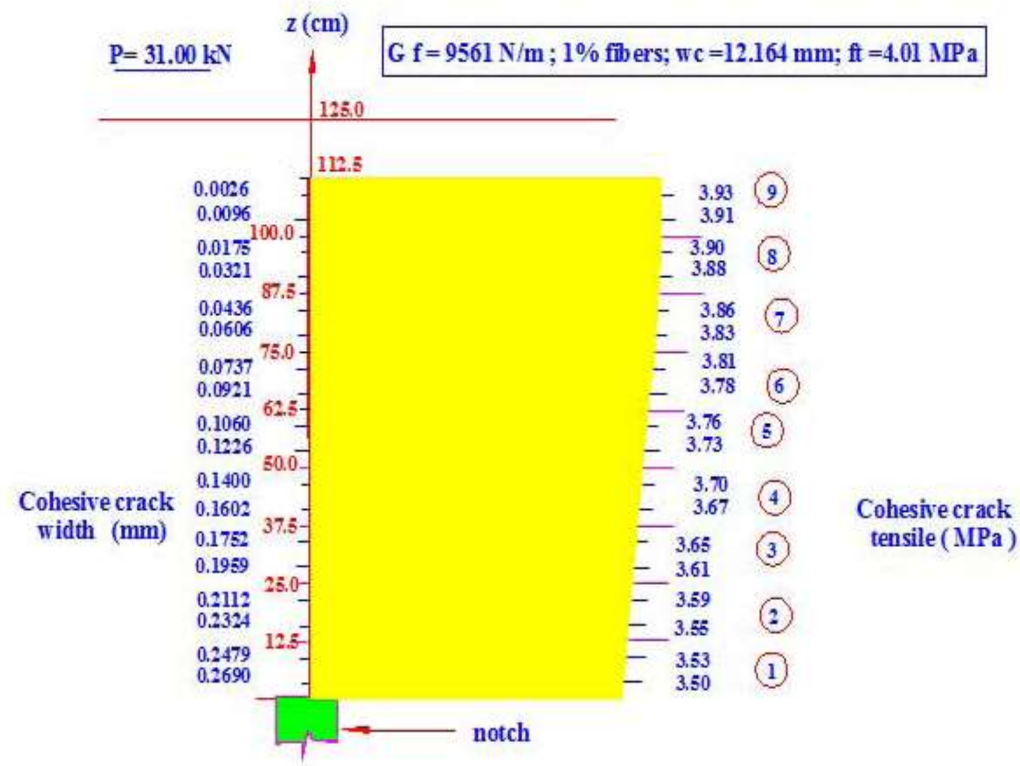


Figure (18) Crack cohesive stresses of 1% SFR at load P= 31.0 kN.

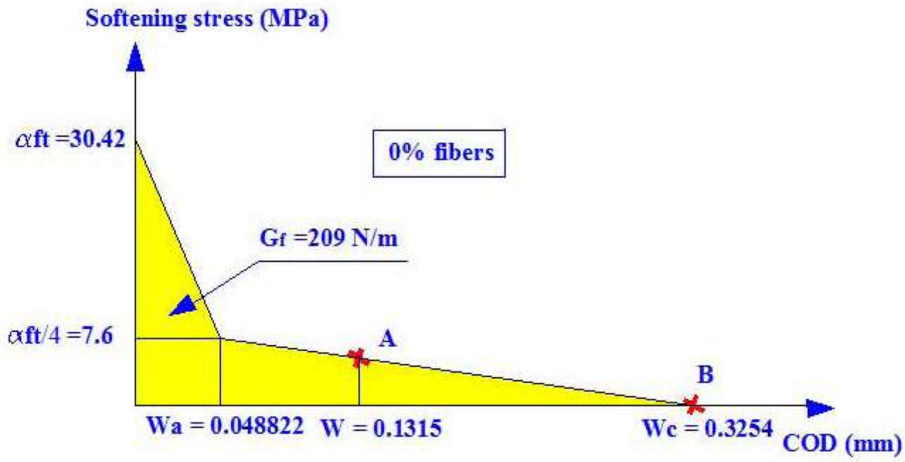


Figure (19) Maximum Gauss crack opening (point A) before the numerical divergence for plain concrete.

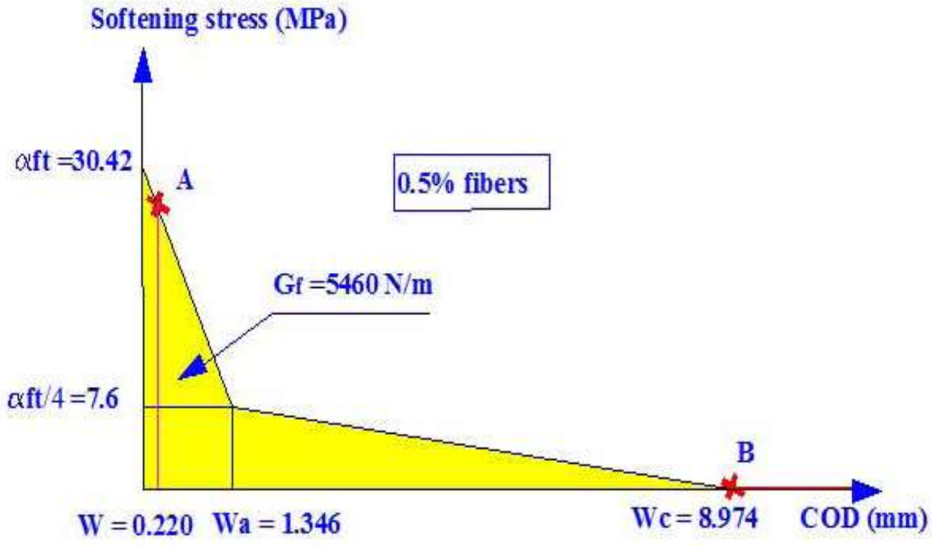


Figure (20) Maximum Gauss crack opening (point A) before the numerical divergence for 0.5% SFR.

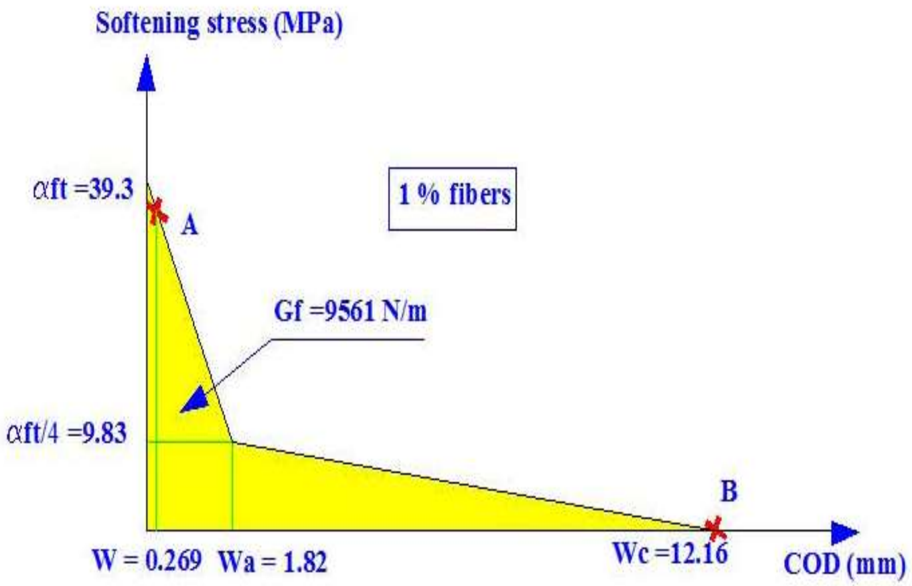


Figure (21) Maximum Gauss crack opening (point A) before the numerical divergence for 1% SFR.

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