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THE USE OF REDUCE DIFFERENTIAL TRANSFORM METHOD FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS

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Abstract

In this paper, we apply Reduced Differential Transform Method (RDTM) for solving partial differential equations (Heat equation with external force) , many numerical application are shown for implementing this method. The results show that this method is very effective and simple .

keyword: Heat equation , Reduced Differential Transform Method (RDTM)

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1. Introduction

Many physical problems can be described by mathematical models that involve partial differential equations. A mathematical model is a simplified description of physical reality expressed in mathematical terms. Thus , the investigation of the exact or approximation solution helps us to understand the means of these mathematical models. Several numerical methods were developed for solving partial differential equations with variable coefficients such us He's Polynomials[1], the homotopy perturbation method[2], homotopy analysis method [3] and the modified variational iteration method [4].

The main aim of this paper is to apply the reduced differential transform method (RDTM)[5-6] to obtain the exact solution for heat equations with external force of the form

$$u_t = u_{xx} + F(x,t)$$

with the initial condition

$$u(x,0) = f(x)$$

The proving that the (RDTM) is very efficient, suitable, quite accurate and simple to such types of equations.

2. Analysis of the Method

The basic definitions of reduced differential transform method are introduced as follows:

Definition 2.1

[5] If the function is analytic and differentiated continuously with respect to time and space in the domain of interest, then let

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right]_{t=0} \quad \dots (1)$$

where the t-dimensional spectrum function $U_k(x)$ is the transformed function. In this paper, the lowercase represent the original function while the uppercase stand for the transformed function.

Definition 2.2 [5-6] The differential inverse transform of $U_k(x)$ is defined as follows:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^k \quad \dots (2)$$

Then combining equation (1) and (2) we write

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right]_{t=0} t^k \quad \dots (3)$$

From the above definitions, it can be found that the concept of the reduced differential transform is derived from the power series expansion. The fundamental mathematical operations performed by RDTM can be readily obtained and are listed in Table 1.

Table 1: Reduced differential transform

Functional Form	Transformed Form
$u(x,t)$	$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right]_{t=0}$
$w(x,t) = u(x,t) \pm v(x,t)$	$W_k(x) = U_k(x) \pm V_k(x)$
$w(x,t) = au(x,t)$	$W_k(x) = aU_k(x)$ a is a constant
$w(x,t) = x^m t^n$	$W_k(x) = x^m \delta(k-n), \delta(k) = \begin{cases} 1 & ,k=0 \\ 0 & ,k \neq 0 \end{cases}$
$w(x,t) = x^m t^n u(x,t)$	$W_k(x) = x^m U_{k-n}(x)$
$w(x,t) = u(x,t)v(x,t)$	$W_k(x) = \sum_{r=0}^{k} V_r U_{k-r}(x) = \sum_{r=0}^{k} U_r V_{k-r}(x)$
$w(x,t) = \frac{\partial^r}{\partial t^r} u(x,t)$	$W_k(x) = (k+1) \dots (k+r) U_{k+r}(x)$
$w(x,t) = \frac{\partial}{\partial x} u(x,t)$	$W_k(x) = \frac{\partial}{\partial x} U_k(x)$
$w(x,t) = \frac{\partial^2}{\partial x^2} u(x,t)$	$W_k(x) = \frac{\partial^2}{\partial x^2} U_k(x)$

3. Numerical Applications

In this section, we use reduced differential transform method (RDTM)for solving heat equations.

Example 3.1 Consider the initial value problem which describes the heat equation

$$u_t(x,t) = u_{xx}(x,t) - u_x(x,t) \qquad \dots (4),$$

with the initial condition

$$u(x,0) = e^{\frac{1}{2}x} \qquad \dots (5)$$

where $u = u(x,t)$ is a function of the variables x and t .
Taking the reduced differential transform of (4), we obtain the

$$(k+1)U_{k+1}(x) = \frac{\partial^2}{\partial x^2}U_k(x) - \frac{\partial}{\partial x}U_k(x) \qquad \dots (6)$$

where the t -dimensional spectrum function is the transform function. From the initial condition (5), we write

$$U_0(x) = e^{\frac{1}{2}x} \qquad \dots (7)$$

Substituting (7) into (6), we obtain the following $U_k(x)$ values successively

$$U_1(x) = \frac{-1}{4}e^{\frac{1}{2}x}$$
$$U_2(x) = \frac{1}{2!}\frac{1}{4^2}e^{\frac{1}{2}x}$$
$$U_3(x) = \frac{1}{3!}\frac{1}{4^3}e^{\frac{1}{2}x}$$
$$U_4(x) = \frac{1}{4!}\frac{1}{4^4}e^{\frac{1}{2}x}$$
$$\vdots$$

Finally the differential inverse transform of $U_k(x)$ gives

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)\,t^k = \left(1 - \frac{1}{4}t + \frac{1}{2!}\frac{1}{4^2}t^2 + \frac{1}{3!}\frac{1}{4^3}t^3 + \cdots\right)e^{\frac{1}{2}x}$$
$$= e^{\frac{-1}{4}t}e^{\frac{1}{2}x}$$

which is the exact solution .

Example 3.2

To solve the equation:

$$u_t(x,t) = u_{xx}(x,t) + \frac{x}{2} \qquad \dots (8),$$

with the initial condition

$$u(x,0) = x \qquad \text{…(9)}$$

Applying the reduced differential transform to (8) we obtain the recurrence equation

$$(k+1)U_{k+1}(x) = \frac{\partial^2}{\partial x^2} U_k(x) - \frac{x}{2}\delta(k) \qquad \text{…(10)}$$

where the t -dimensional spectrum function is the transform function. From the initial condition (5), we write

$$U_0(x) = x \qquad \text{… (11)}$$

Substituting the initial condition (11) into (10), and using the recurrence relation (10), we can
obtain the following $U_k(x)$ values ;

$U_1(x) = \frac{x}{2}$

$U_2(x) = 0$

$U_3(x) = 0$

$U_4(x) = 0$

$\vdots$

i.e $U_k(x) = 0\ ,\ \forall k \geq 1$

Finally the differential inverse transform of $U_k(x)$ gives

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)\, t^k = \left(1 + \frac{1}{2}t\right)x$$

which is the exact solution .

Example 3.3 Consider the initial value problem which describes the heat equation

$$u_t(x,t) = u_{xx}(x,t) + \sin(x) \qquad \text{… (12) ,}$$

with the initial condition

$$u(x,0) = 0 \qquad \text{… (13)}$$

Similarly, by using the reduced differential transform to (12), we obtain the recurrence equation

$$(k+1)U_{k+1}(x) = \frac{\partial^2}{\partial x^2} U_k(x) + (\sin(x))\delta(k) \qquad \text{… (14)}$$

From the initial condition by given of (13)

$$U_0(x) = 0 \qquad \text{… (15)}$$

Substituting (15) into (14), we have

$U_1(x) = \sin(x)$
$U_2(x) = \frac{-1}{2}\sin(x)$
$U_3(x) = \frac{1}{6}\sin(x)$
$U_4(x) = \frac{-1}{24}\sin(x)$
$U_5(x) = \frac{1}{120}\sin(x)$
$\vdots$

The series solution is given by

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)\,t^k = \left(t - \frac{1}{2!}t^2 + \frac{1}{3!}t^3 - \cdots\right)\sin(x)$$
$$= (1 + e^{-t})sinx$$

which is the exact solution .

Example 3.4 Consider the initial value problem which describes the heat equations

$$u_t(x,t) = u_{xx}(x,t) - u(x,t) + x \qquad \dots (16) ,$$

with the initial condition

$$u(x,0) = 0 \qquad \dots (17)$$

Apply the reduced differential transform method to (16)

$$(k+1)U_{k+1}(x) = \frac{\partial^2}{\partial x^2}U_k(x) - U_k(x) + x\delta(k) \qquad \dots (18)$$

From the initial condition (17) , we can write
$U_0(x) = 0$
$U_1(x) = x$
$U_2(x) = \frac{-1}{2}x$
$U_3(x) = \frac{1}{6}x$
$U_4(x) = \frac{-1}{24}x$
$\vdots$

By using definition(2.2), we get the exact solution

$$u(x,t) = x\left(t - \frac{1}{2}t^2 + \frac{1}{6}t^3 - \frac{1}{24}t^4 + \cdots\right) = -xe^{-t} + x .$$

4. Conclusions

In this research, reduced differential transform method has been applied to solving the heat equation. The method is applied in a direct way without using linearization, transformation , discretization or restrictive assumptions. The result show RDTM needs small size of computation contrary to other numerical methods(classical differential transform method (DTM), Adomain method and homotopy perturbation method) and powerful and efficient techniques in finding the exact solution.

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استخدام طريقة التحويل التفاضلي المختزل لحل المعادلات التفاضلية الجزئية

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الخلاصة :-
طبقنا في هذا البحث طريقة التحويل التفاضلي المختزل لحل المعادلات التفاضلية الجزئية (معادلات الحرارة المؤثر عليها قوى خارجية)وبينت النتائج فعالية وسهولة الطريقة المستخدمة.

الكلمات المفتاحية : معادلة الحرارة ، طريقة التحويل التفاضلي المختزل .