Mathematical Estimation for the Bearing Capacity of Sand Column Inserted in Soft Clay Soil

Dr. Awf Abdul Rahman Al-Kaisi
Building and Construction Engineering Department/ University of Technology/Baghdad
Hiba Hussein Ali
Building and Construction Engineering Department / University of Technology/Baghdad
Email: hiba@uotechnology.edu.iq

Received on: 4/2/2012 & Accepted on: 4/10/2012

ABSRTUCT

Sand column is one of the soft ground improvement methods. It is made up of well compacted sand pile, it is used for enhance the load capacity of soft clay soils, reduce the settlement and accelerate the consolidation process of the native soft soil surrounding it, also the sand column is used to minimize the likelihood of liquefaction when installed in loose sand soils.

This research deals with the evaluation of a mathematical equation depending on the results obtain from series of triaxial test in order to calculate the bearing capacity of sand column which inserted in soft clay soil and for two different cases (floating case condition and other is when the sand column is resting on a firm base).

The results indicate that the calculation of the shaft resistance for sand column mainly depends on the clay medium properties, while the calculation of the end bearing depends on the sand property (depending on the mode of failure for the sand column, which was observed). Another group of mathematical relations was suggested to calculate the load capacity of sand column inserted in soft clay for two area replacement ratios (11% and 6%) and depending on the confining pressures considered (100-400) kPa, and for two cases, the first is when the sand column in the floating case condition and other is when the sand column is resting on a firm base.

Keyword: Sand column, triaxial test, End bearing, Shaft resistance

تقدير قابلية تحمل الاعمدة الرملية المغروزة في التربة الطينية الضعيفة

الخلاصة

الأعمده الرمليه هي احد الطرق لمعالجه الترب الرخوه.وتتكون من الرمل المرصوص جيدا, وتستخدم لزياده تحمل الترب الطينيه الرخوه, تقليل الهبوط وتعجيل عمليه الانضمام للتربه الرخوه المحيطه بها. وكذلك تعمل على تقليل ظاهره ال(liquefaction) عندما تغرس بالترب الرمليه الضعيفة.

https://doi.org/10.30684/etj.31.5A2
2412-0758/University of Technology-Iraq, Baghdad, Iraq

يتناول هذا البحث ايجاد علاقات رياضية من اجل حساب قابلية تحمل العمود الرملي المغروس في التربة الطينبة الرخوة ولحالتين (في حالة كون العمود الرملي غير مستند على طبقة قوية والاخرى مستند على طبقة قوية)وبالاعتماد على نتائج استحصلت من سلسلة فحوصات الانضغاط الثلاثي.

أشارت النتائج الى إن حساب المقاومه السطحيه(Shaft Resistance) للعمود الرملي تعتمد أساساً على خصائص الوسط الطيني الرخو، بينما حساب تحمل القاعده (End Bearing) تعتمد على خصائص الرمل المستخدم في الأعمده الرمليه (اعتماداً على نمط الفشل الملاحظ للأعمده الرمليه بعد التحميل).

ولقد تم اقتراح مجموعه أخرى من العلاقات الرياضيه لحساب تحمل الأعمده الرمليه المغروزه بالترب الطينيه الرخوه اعتمادا على نسبه الاشغال للعمود الرملي المستخدمه (١١% او٦%)واعتمادا على الضغوط الجانبيه المستخدمه(١٠٠-٤٠٠)كيلوباسكال وفي حالتين, الأولى عندما تكون الأعمده الرمليه غير مستنده على طبقه قويه(Floating sand column), والأخرى عندما تكون تلك الأعمده مستنده على طبقه قويه

INTRODUCTION

The use of granular columns in soft clay deposits has been found to enhance soil foundation load capacity respects which are to provide vertical support for overlying structures or embankments (i.e. increase the bearing capacity), also accelerate the consolidation process of the native soft soil surrounding the granular columns. On the other hand it improves the load-settlement characteristics of the foundation.[13]. In addition the sand column is used to minimize the likelihood of liquefaction when installed in loose sand soils [20].

Existing researches in sand pile show that the soil improvements through using sand columns lead to high shear strength and low compressibility[14] The sand column and the tributary area of soil surrounding each sand column are simplified as a cylinder having the same total area although different available patterns and having an equivalent diameter or an effective diameter (De). This diameter was chosen so that the cross-sectional area of the unit cell is equal to that of the tributary area per column, i.e. (De=1.05 S) for the triangular pattern, (De=1.13 S) for a square pattern and (De=1.29 S) for the hexagonal pattern, where (S) is the spacing as defined on Figure (1) [10, 15].

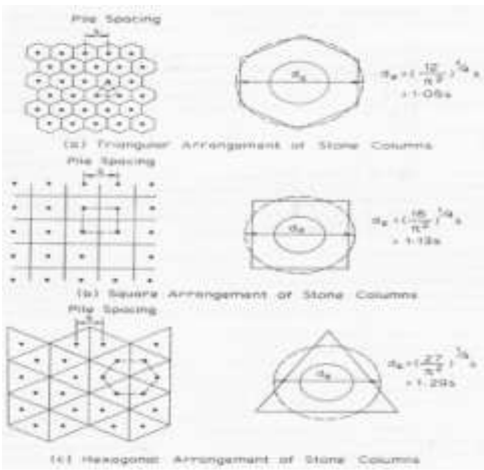


Figure (1) Various Pile Arrangements Showing the Domain of Influence of Each Column(after Balaam and Booker, 1981).

There are many methods for analyzed the behavior of sand column considered by many others such like (Rasheed, 1992)[19] analyzed the behavior of stone columns in the soft clay soil by using the finite element method. The linear and nonlinear hyperbolic models are used to present both soil and stone column materials. The effect of the ratio of modulus of elasticity of stone to the modulus of soil, spacing to the diameter of column, length of column to the thickness of soil layer and Poisson's ratio of soil on settlement behavior of the treated soft soil were studied and presented. The results show that the most effective parameter is the ratio of spacing to the diameter of column, see Figure (2).

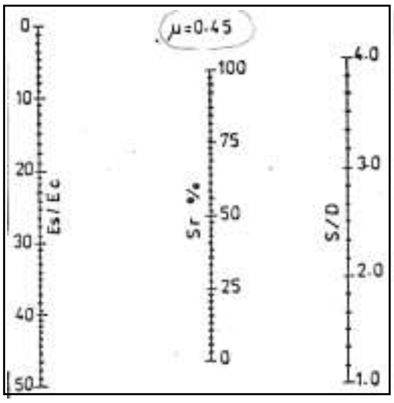


Figure (2) Nomogram of Relationship Between Es/Ec and Ratio of Settlement for Soil with ν=0.45.

(Vesic) [21] developed a general cylindrical cavity expansion solution extending earlier work to include soil with both friction and cohesion. The ultimate lateral stress (σ3) developed by the surrounding soil can be expressed as :

$$\sigma_3 = cF'_c + \bar{q}F'_q$$

where: c: cohesion of the surrounding soil., $\bar{q}$: mean isotropic stress at the failure depth =(σ1+ σ2+ σ3)/3., F'_c, F'_q: cavity expansion factors. The cavity expansion factors are shown in Figure (3)

(Ahmed) [6] presented a new approach of analyzing stone columns reinforced soft soil. The author predicts the ultimate bearing capacity of the stone column by evaluating the equivalent shear parameter of composite soil by the following equations:-

$$\phi^{eq} = 2\left[\tan^{-1}\sqrt{k_p^s a_s + k_p^c(1-a_s)} - \frac{\pi}{4}\right]$$

and

$$\frac{c^{eq}}{c_u} = \sqrt{\frac{k_p^c}{k_p^{eq}}} * (1-a_s)$$

where: ϕ^{eq}, c^{eq} : the equivalent friction and cohesion of the c_u material.,:

undraind cohesion of the native soil., and as: reinforcement ratio=(AS/At).

$$k_p^{eq} = k_p^s a_s + k_p^c (1 - a_s)$$

where: k_p^{eq} : the equivalent coefficient of passive earth pressure., k_p^s : the coefficient of passive earth pressure for stone column material, k_p^c :the coefficient of passive earth pressure for native soft soil.

(Al-Hity)[9] used the finite element to analyze the behavior of stone columns, the axisymmetric quadrilateral element is adopted to simulate the soft soil and the stone column while the one-dimensional element is used to simulate the stone column interface. The effect of some of the parameters concerning the geometry of the stone column and the material of column adjacent soil is investigated, and studied improvement efficiency of the stone column by adopting certain measures especially at the stone column bulging zone. The results show that the increase in stone column length and relative stiffness of stone column material to soil play an important role in increasing ultimate capacity of the stone column and in reducing settlement, and the addition of concrete steel discs within stone column material provides additional increase in stone column capacity (200%) and casing the upper part of the stone column with concrete can provide another (86%) improvement in the column capacity.

(Baquir)[12], carried out a number of triaxial tests by making a modification in the top cap of the triaxial cell in order to permit the drainage through the sand column during the compression stage. The results of his research show that the resistance of the reinforced soil is proportional to the sand column diameter, so increase in the sand column diameter causes increasing in the resistance of reinforced soil to the applied vertical load. Also the undrained column carried a less load than the drained one.

(Al-Mersomy)[8]carried out a series of triaxial tests for three different samples; which were soft clay samples, sand samples and soft clay samples reinforced with sand column in order to evaluate the shear strength parameters for these soils; which were considered for the assessment numerically the effect of using sand columns on the stability of earth embankments erected on soft clay soil foundation, From the tests results, she could obtain the following conclusions that The useing of sand column increase the strength of soft clay by about (200-300)% relative to unreinforced soft clay and a simple equation{c=b+(a-b)/4} is formulated to relate the shear strength of the soft clay reinforced with a sand column with respect to the shear strength of a sand and unreinforced soft clay and based on sand reinforced ratio (a_s) equal to (11%) where: a: the deviator stress for sand sample, at failure, b: the deviator stress for unreinforced soft clay sample, at failure, c: the estimated deviator stress for a reinforced soft clay sample reinforced with sand column, at failure.

The aim of this study is to establish some relations for the effect of the confining pressure on the load carrying capacity of the sand column.

TRIAXIAL TEST RESULTS

The triaxial test results which considered in this study were obtained from experimental work carried by (Heba)[18] , Where carried out several series of triaxial tests with the aim to prepare soft clay samples and to evaluate shear strength parameters of soft clay in addition to evaluation of shear strength parameters of the sand used on reinforcement sand column. Other series of the tests were used to evaluate strength parameters of clay samples reinforced with sand columns, with full or partial penetration. The results can be summarized in Table (1).

Table (1) Summary of Triaxial test Results (Heba H.Ali, 2007).

Type of samples series	a_s (%)	$\frac{L_s}{L_c}$	Test type	$\sigma_c = 100kPa$			$\sigma_c = 200kPa$			$\sigma_c = 300kPa$			$\sigma_c = 400kPa$			c (kPa)	ϕ (Degree)
				$(\sigma_1-\sigma_3)_f$ (kPa)	ε_f (%)	$\frac{(\sigma_1-\sigma_3)_r}{(\sigma_1-\sigma_3)_{r_o}}$	$(\sigma_1-\sigma_3)_f$ (kPa)	ε_f (%)	$\frac{(\sigma_1-\sigma_3)_r}{(\sigma_1-\sigma_3)_{r_o}}$	$(\sigma_1-\sigma_3)_f$ (kPa)	ε_f (%)	$\frac{(\sigma_1-\sigma_3)_r}{(\sigma_1-\sigma_3)_{r_o}}$	$(\sigma_1-\sigma_3)_f$ (kPa)	ε_f (%)	$\frac{(\sigma_1-\sigma_3)_r}{(\sigma_1-\sigma_3)_{r_o}}$		
Soft clay samples	—	—	C.U	49	13	—	66	13	—	86	15	—	110	16	—	12	6
Sand samples	—	—	C.D	370	10	—	771	11	—	1015	15	—	1400	15	—	0	39
Soft clay samples reinforced with sand column (tested using standard cap).	11	1	C.U	125	15	2.55	175	16	2.65	225	15	2.61	275	16	2.5	21	10
			C.D	225	16	4.59	300	16	4.54	400	17	4.65	475	19	4.31	34	14
	6	1	C.U	60	9	1.22	80	11	1.2	100	12	1.62	130	13	1.18	13	6
			C.D	66	11	1.4	93	14	1.4	110	13	1.27	140	15	1.3	25	6
Soft clay samples reinforced with sand column (tested using modified cap).	11	1	C.U	950	6	—	1350	6.2	—	1700	6	—	2000	6	—	—	—
			C.D	1100	10	—	1527	8	—	2000	8	—	2400	8.2	—	—	—
	6	1	C.U	800	4	—	1159	5	—	1500	4	—	1850	5	—	—	—
			C.D	950	7	—	1291	10	—	1650	10	—	2000	8	—	—	—
	11	0.5	C.U	225	10	—	310	12	—	—	—	—	600	18	—	—	—
	6	0.5	C.U	190	10	—	275	9	—	—	—	—	500	10	—	—	—

L_s: Length of sand column. $(\sigma_1-\sigma_3)_r$: Deviator stress at failure of clay sample reinforced with sand column. σ_c: Confining pressure.
L_c: sample length. $(\sigma_1-\sigma_3)_{r_o}$: Deviator stress at failure of unreinforced clay sample.

MATHEMATICAL RELATION

In an aim to establish some relations for the effect of confining pressure on the load capacity of the sand column, the end bearing for full penetration sand column is separated from the shaft resistance throughout the following mathematical calculations: -

Because there is just shaft resistance in case of half penetration sand column as mentioned earlier, so at failure the interface surface between the sand column and the soft clay soil surrounding it will be considered as a failure surface and it can be supposed that the normal stress equal to the confining pressure applied on the sample; and the (Mohr- Coulomb) equation will be as follows:

$$\tau = c + \sigma_c \tan(\phi) \qquad \ldots(1)$$

where: τ: the shear strength of the soil (soft clay or sand) (kPa).
c: the cohesion of the soil (kN/m2).
σ_c: the anticipated confining pressure (kPa).

ϕ : the angle of internal friction (degree).

$$f_s = M * \tau \qquad \ldots(2)$$

where: f_s : the shaft resistance of the sand column (kPa).

M : the shaft resistance factor.

The shaft resistance for floating (partial penetration) sand column is equal to the load capacity of the sand column (which is obtained from triaxial tests results), where there is no end bearing resistance as concluded earlier, while for the full penetration sand column, the shaft resistance is equal to the twice of the load capacity of the partial penetration sand column. For calculation example, considering soft clay samples reinforced with sand column of area replacement ratio (11%), the calculation will be as follows: -

For half penetration sand column

The shaft resistance factor is calculated based on considering the soft clay material or the sand column material

Considering clay medium

$$\tau = c + \sigma_c \tan(\phi)$$

$$\tau = 12 + 100 * \tan(6) = 22.5kPa.$$

$$f_s = \frac{load}{A_s} \quad , \quad A_s = \pi * 1.3 * 3.8 = 15.5cm^2 = 0.00155m^2$$

$$f_s = M * \tau \longrightarrow M = \frac{19}{22.5} = 0.84$$

Considering sand medium

$$\tau = c + \sigma_c \tan(\phi)$$

$$\tau = 100 * \tan(39) = 81kPa.$$

$$f_s = M * \tau \longrightarrow M = \frac{19}{81} = 0.23$$

The calculation summary of shaft resistance factor is illustrated in Table (2).

For full penetration sand column

$$f_b = \frac{F_b}{A_b} \qquad \ldots(3)$$

$$F_b = F_{TX} - 2 * F_s \qquad \ldots(4)$$

where: f_b : end-bearing stress for full penetration sand column (kPa).

F_b : end load capacity (kN) .

F_{TX} : load capacity of the full penetration sand column, which is obtained from triaxial tests results (kN).

F_s: shaft load resistance for half penetration sand column obtained by triaxial tests results (kN).

Table (2) Summary of End Bearing Capacity (f_b) of Sand Column Calculations.

For a_s = 11%								
σ_c (kPa)	$(F_s)_{half}$ (kN)	$(F_s)_{full}$ (kN)	$(F_{TX})_{full}$(C.U) (kN)	$(F_{TX})_{full}$(C.D) (kN)	(F_b)(C.U) (kN)	(F_b)(C.D) (kN)	(f_b)(C.U) (kPa)	(f_b)(C.D) (kPa)
100	0.029	0.058	0.125	0.1452	0.067	0.087	507.5	659
200	0.041	0.082	0.17	0.2	0.08	0.118	606	894
400	0.79	0.158	0.264	0.32	0.1006	0.106	803	1204.5
For a_s = 6%								
σ_c (kPa)	$(F_s)_{half}$ (kN)	$(F_s)_{full}$ (kN)	$(F_{TX})_{full}$(C.U) (kN)	$(F_{TX})_{full}$(C.D) (kN)	(F_b)(C.U) (kN)	(F_b)(C.D) (kN)	(f_b)(C.U) (kPa)	(f_b)(C.D) (kPa)
100	0.15	0.03	0.624	0.74	0.0324	0.044	415	564
200	0.125	0.43	0.09	0.1	0.047	0.057	602	731
400	0.039	0.078	0.144	0.156	0.066	0.078	800	1000

As an example for calculation, consider a case of sand column with area replacement ratio of (11%): -

$$F_b = 0.125 - 2*(0.029) = 0.1452$$

$$f_b = \frac{0.1452}{0.000132} = 507.5kPa.$$

The calculation summary of end bearing capacity is illustrated in Table (3).

Table (3) Summary of Bearing Capacity Factor (N_q) Calculations.

For $a_s = 11\%$				
σ_c (kPa)	(f_b) (C.U) (kPa)	(f_b) (C.D) (kPa)	N_q (C.U)	N_q (C.D)
100	507.5	659	626.7	814
200	606	894	748	1104
400	803	1204.2	992	1487
For $a_s = 6\%$				
σ_c (kPa)	(f_b) (C.U) (kPa)	(f_b) (C.D) (kPa)	N_q (C.U)	N_q (C.D)
100	415	564	512.5	696
200	602	731	743	903
400	800	1000	1045	1235

Referring to Figures (3 to 6) which represent the relationship between shaft resistance factor (M) and the confining pressure based on considering the soft clay media material or the sand column material, it is clear that the decreasing relation obtained for the shaft resistance factor (M) values is to be neglected as the tests results reveal that the shaft resistance of the sand column increases with increasing the confining pressure. These results lead to the conclusion that when the shaft resistance is calculated depending on soft clay medium properties, the shaft resistance is to be calculated using the following equation:

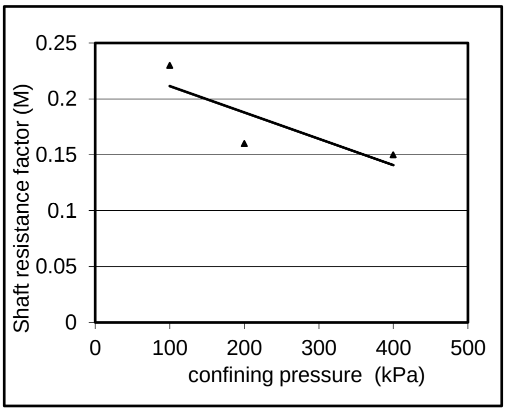
$$f_s = M * \tau$$

where: f_s : shaft resistance of the sand column (kPa)

M : shaft resistance factor obtained from Figs.(3-6) according to the area replacement ratio use (11% or 6%).

τ: the shear strength of the clay medium (kPa)

$$\tau = c + \sigma_c \tan(\phi)$$



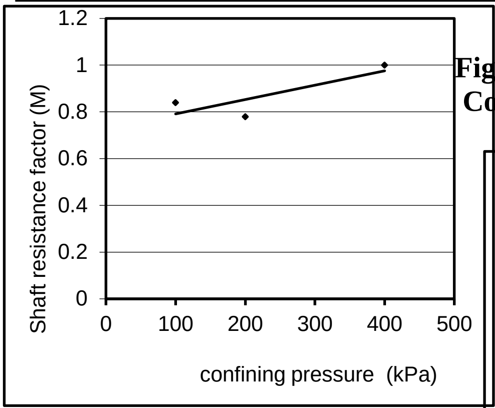


Figure (3) Shaft Resistance Factor (M) Versus Confining Pressure for Clay Media- a_s=11%.

Figure(4) Shaft R Versus Confining Pr a_s=

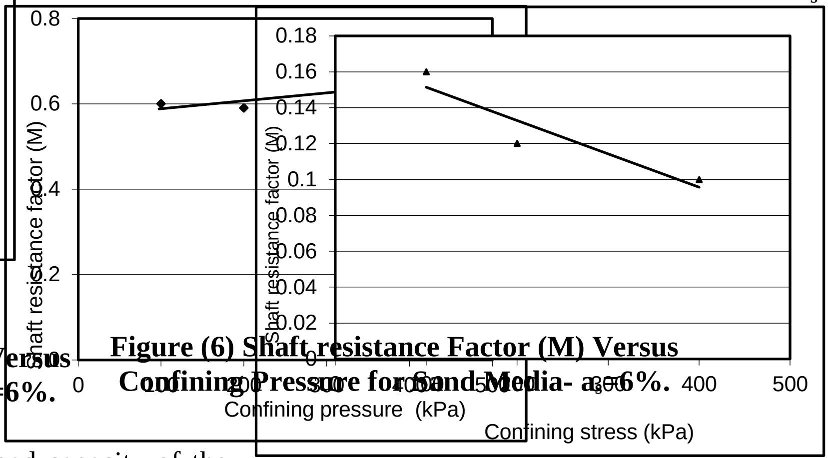


Figure (5) Shaft Resistance Factor (M) Versus Confining Pressure for Clay Media- a_s=6%.

Figure (6) Shaft resistance Factor (M) Versus Confining Pressure for Sand Media- a_s=6%.

Considering the end bearing load capacity of the sand column, Figs. (7 and 8) show that there is a clear increase in the end bearing capacity of the sand column due to the increase in the confining pressure. If these results are examined together with the mode of failure according to (Baraksdale and Bachus, 1983) of the sand column illustrated in the theses of (Heba,2004); it can be deduced that for the case where the sand column is resting on stiff media the failure is expected to appear within the

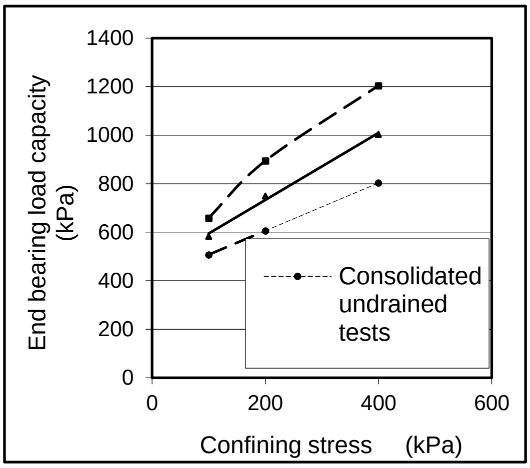


Figure (7) End Bearing Load Capacity Versus Confining pressure for Full Penetrating Sand column- as=11%.(Heba H.Ali, 2007)

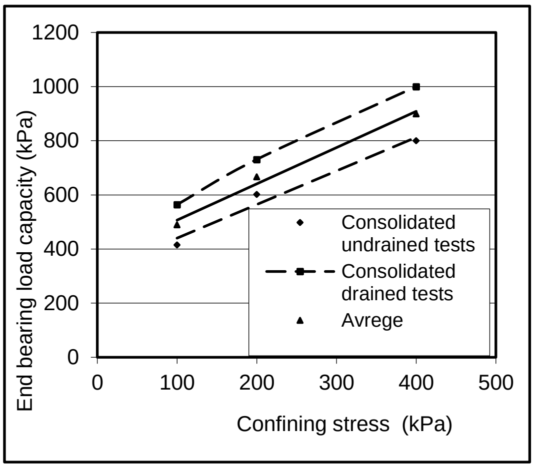


Figure (8) End Bearing Load Capacity Versus Confining pressure for Full Penetrating Sand column- as=6%. (Heba H.Ali, 2007)

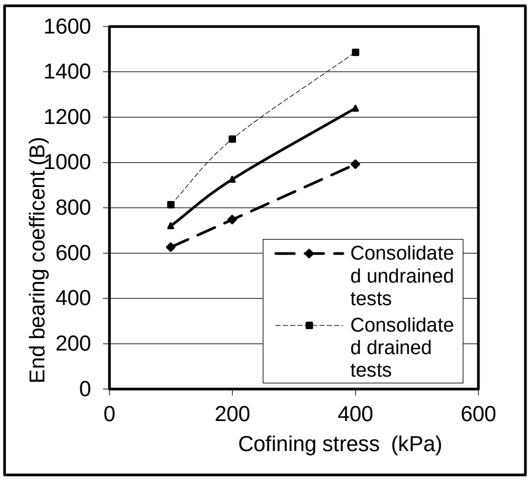


Figure (9) End Bearing Coefficient Versus Confining Pressure for Full penetration Sand column- as=11%. (Heba H.Ali, 2007)

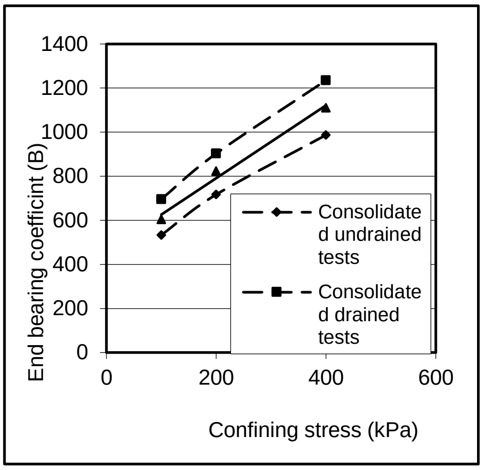


Figure (10) End Bearing Coefficient Versus Confining Pressure for Full penetration Sand column- as=6%. (Heba H.Ali, 2007)

column material (sand) and this implies that the end bearing load capacity is a function of the sand property (Ø). The following equation can be used to estimate end bearing capacity: -

$$f_b = N_q * \tan(\phi) \qquad ...(5)$$

where: f_b : the end bearing capacity (kPa).

N_q : bearing coefficient.

ϕ : angle of internal friction of sand (Degree).

The bearing coefficient (N_q), which its calculation summarized in Table (3) is related with the confining pressures as illustrated in Figs. (9 and 10) and can be calculated by suggesting the following equations for two area replacement ratios (11% or 6%): -

For area replacement ratio=11%

$$N_q = 533 + 1.7\sigma_c \qquad ...(6)$$

For area replacement ratio=6%

$$N_q = 430 + 1.7\sigma_c \qquad ...(7)$$

The following equations are suggested for calculating the end bearing load capacity of sand column when founded on stiff layer depending on the Triaxial results: -

For area replacement ratio=11%

$$f_b = 475 + 1.3\sigma_c \qquad ...(8)$$

For area replacement ratio=6%

$$f_b = 360 + 1.4\sigma_c \qquad ...(9)$$

Impact Between Striker and a Slab of Distributed Mass.

To get more accurate solution of the slab impact problem, a more accurate description of the slab vibration than that given by the effective or rigid mass models should be considered. This means that the free and forced vibration of slabs should be considered.

CONCLUSIONS

1. An equation is formulated to calculate the sand column load capacity for the case of floating column in soft clay medium as follows: -

   $$f_s = M * \tau$$

   In which (f_S) is shaft resistance, (τ) shear strength and (M) is a shaft resistance coefficient.
2. Equations are formulated to calculate the end bearing load capacity (f_b) of a sand column resting on firm base, taking into consideration: -
   - The confining pressure (σ_c) applied on sand column and for each area replacement ratio: -

     - For area replacement ratio=11%: - $f_b = 475 + 1.3\sigma_c$

     - For area replacement ratio=6%: - $f_b = 360 + 1.4\sigma_c$

   - The sand column media properties: -

     $$f_b = N_q * \tan(\phi)$$

   where N_q : end bearing coefficient and can be calculated by suggesting the following equations, taking into consideration The confining pressure on sand column and for each area replacement ratio: -

     - For area replacement ratio=11%: - $N_q = 533 + 1.7\sigma_c$
     - For area replacement ratio=6%: - $N_q = 430 + 1.7\sigma_c$
3. The use of sand column with area replacement ratio of (11%) is more effective than (6%) in enhancing soft clay soil as a foundation material.
4. Maintaining good and effective drainage path is also effective in enhancing the load carrying capacity of soft clay-sand column matrix.

REFERENCES

[1].Aboshi, H., Ichimoto, E., Enoki, M. and Harada, K., (1979), “The Composer- A Method to Improve Characteristics of Soft Clays by Inclusion of Large Diameter Sand Columns” Proceedings International Conference on Soil-Reinforcement; Reinf. Earth and Other Technique, Paris, pp. 211-216.

[2].Ahmmed, A.A, (1998), “Performance of Stone Columns in Soft Soils”, Ph.D. Thesis, College of Engineering, University of Baghdad.

[3].Al-Kafaji, Z.A., (1996), "Reinforced of Soft Clay Using Granular Columns", Ph.D Thesis, University of Manchester, U.K.
[4].Al-Mersomy, H.M., (2004), "Assessment of Stability of Embankment Erected on Soft Clay Soils Reinforced with Sand Columns", M.Sc. Thesis, University of Technology.
[5].Al-Hity, A.A.J., (1999), "Analysis of Stone Columns In Soft Soil By The Finite Element Method", M.Sc. Thesis, Building and Construction Engineering Department University of Technology.
[6]. Balaam, N.P. and Booker, J.R., (1981), "Analysis of Rigid Rafts Supported by Granular Piles", International Journal for Numerical and Analytical Methods in Geomechanics, Vol.5, pp. 379-403.
[7].Baraksdale, R.D., Bachus, R.C., (1983), "Design and Construction of Stone Columns", Federal Highway Administration, Vol.1, School of Civil Engineering, Georgia Institutes of Technology, Atlanta (www.fhwa.dot.gov).
[8].Baquir, H.H., (1990), "Stabilization of Fao Clay", M.Sc. Thesis, University Of Baghdad.
[9].Craig, W.H. and Al-Khafaji, Z.A., (1997), "Reduction of Soft Clay Settlement By Compacted Sand Columns", International Conference on Ground Improvement Geosystems, Thomas Telford, London, Third Conference, pp. (218-224).
[10].Cho, S.M., Kim, B., Lee, S., (2005), "Effect of Soil Compaction Piles on Settlement Reduction in Soft Ground", International Journal of Offshore and Polar Engineering, Vol.15, No.3, September, pp. 235-240. (www.isope.org).
[11].Goughnour, R.R., (1983), "Settlement of Vertically Loaded Stone Columns in Soft Ground", Proc.8th. European Conference on SMFE, Helsinki, Finland, Vol.1, pp. 235-240.
[12].Head, K.H, (1982), "Manual of Soil Laboratory Testing", Vol. 2, Pentech press, London.
[13].Head, K.H, (1986), "Manual of Soil Laboratory Testing", Vol. 3, Pentech press, London.
[14].Heba Hussain Ali,(2007), "Effect of Lateral Confinement on Load Carrying Capacity of Sand Column", M.Sc. Thesis, University of Technology.
[15].Rasheed, A.H., (1992), "Efficiency of Stone Column in Improving the Behavior of Footing Resting on Soft Soils", M.Sc. Thesis, University of Technology.
[16].Vanimpe, W.F., Madhav, M.R. and Vandercruyssen, J.P., (1997), "Considerations in Stone Column Design", Int. Conf. On Ground Improvement Geosystems, Thomas Telford, London, pp. 190-196.
[17].Vesic, A.S., (1972), "Expansion of Cavities in Infinite Soil Mass", Journal of Soil Mech. and Foundation Engineering Div. ASCE, Vol.98, No. SM3, pp. 265-290