Kufa Journal of Engineering
Vol. 16, No. 1, January 2025, P.P. 344-360
Article history: Received 2 August 2024, last revised 3 October 2024, accepted 3 October 2024

THEORETICAL AND EXPERIMENTAL INVESTIGATION OF CRITICAL BUCKLING LOAD OF COMBINED COLUMNS

Hayder Zuhair Zainy [1] , Aziz D. Almawash [1], Saddam K. Al-Raheem [1], Luay S. Alansari[1], and Mohammed Fakhir Al-Edhari [1,*]

[1] University of Kufa, College of Engineering, Department of Mechanical Engineering, Najaf, Iraq
*** Corresponding Author. Email Address: mohammedf.jaber@uokufa.edu.iq**

https://doi.org/10.30572/2018/KJE/160120

ABSTRACT

The work done in this research is dedicated to compute the critical buckling load of combined columns. This is performed using theoretical and experimental analyses. The combined column is made of two materials, namely copper and aluminum alloys. The theoretical approach involves numerical analysis using ANSYS and analytical analysis using the equivalent stiffness method. The study determines the critical buckling load by considering many parameters. These parameters include lengths of copper and aluminum in the combined columns, cross-sectional geometries (square, circle, and hexagon), and boundary conditions. The experimental work is carried out using a Strut Bucking Apparatus on a combined column with circular cross-sections and different end supports. The findings show that the critical buckling loads achieved from numerical and analytical analysis are in good agreement with those obtained from experimental analysis. It is found that the critical buckling load increases with increasing the length of copper in the combined column and with more restrictive end supports. There is a good match of critical buckling load between those achieved from the equivalent stiffness method and ANSYS model for any considered cross-sectional geometries (square, circle, and hexagon) and two types of support (fixed-fixed and fixed-pinned). The results reveal that the combined columns with square cross-sections have higher critical buckling loads than those with circular or hexagonal cross-sections. The highest percentage errors of critical buckling load between the experimental method and theoretical methods (equivalent stiffness method and ANSYS models) are (-13.120, -12.768, and -12.453) % and (-13.083, -18.039, and -19.052) % for pinned-pinned, fixed-pinned, and fixed-fixed columns respectively.

KEYWORDS

Critical Buckling Load, Combined Column, ANSYS Model, Equivalent Stiffness Method, Experimental Analysis.

1. INTRODUCTION

The structural integrity relies greatly on the strength and stiffness of the engineering structure. The lateral axial load applied on a structural engineering may cause a catastrophic failure because the structure becomes unstable leading to a sudden change in shape or collapse. This type of failure occurs due to buckling (Alansari et al., 2020). The studies have concentrated on maximizing the critical load and minimizing the weight of the structure. Choosing a proper cross-section area and geometry may lead to maximizing the critical buckling load and achieving weight reduction. There are two approaches to buckling analysis. The first approach is known as the eigenvalue buckling analysis (Saraçaoğlu and Uzun, 2020). The other approach is called nonlinear buckling analysis. This type is suitable for large-deflection buckling loads. It is more accurate compared to eigenvalue buckling analysis (ANSYS Inc, (2019)).

Euler made an assumption to estimate the critical buckling load of a column subjected to an axial concentrated load. His method is only valid for long and slender columns (Alansari et al., 2020, Khuder and Hussein, 2014). Timoshenko and Gere made considerable efforts to predict the elastic stability of any type of column (Saraçaoğlu and Uzun, 2020).

There are many studies available in the literature that studied the buckling behavior of structures. Abdel-Lateef et al. (Abdel-Lateef et al., 2001) performed an analytical analysis to investigate the stability of a variable cross-section column subjected to concentrated or distributed axial loads. The variation of the cross section was assumed to follow power law to estimate the load intensity and the changes in the moment of inertia. Šapalas et al. (Šapalas et al., 2005) used numerical and analytical analysis to study the buckling behavior of tapered columns under axial and bending moment loads. Pekbey et al. (Pekbey et al., 2007) performed analytical and experimental analysis to determine the critical buckling load. They obtained a good degree of discrepancy between the numerical and experimental results. Safa Bozkurt Cokun and Mehmet Tarik Atay (Coşkun and Atay, 2009) chose the variational iteration method to evaluate the critical buckling load for columns with different boundary conditions and cross-section areas. Coşkun (Coşkun, 2010) studied the elastic stability of tilt-buckled columns with varying flexural stiffness. They applied homotopy perturbation method to compute the critical buckling loads and mode shapes. They claimed that the proposed approach is an effective technique for the elastic stability of such problems. Darbandi et al. (Darbandi et al., 2010) introduced an analytical solution based on the Euler-Bernoulli beam theory and the singular perturbation method of Wentzel-Kramers-Brillouin to analyze the static stability of the variable cross-section column under distributed axial loads. Li et al. (Li et al., 2011) examined the stability of composite columns using the integral equation method. The effect of varying the material properties and changing the cross-section end forces and distributed axial forces on the stability of the column was studied. Coşkun and Öztürk (Coşkun and Öztürk, 2012) selected three analytical methods to assess the elastic stability of Euler columns. These methods are the domain decomposition method, variational iteration method, and homotopy perturbation method. Yilmaz, et al. (Yilmaz et al., 2013) applied the localized differential quadrature method to study the buckling performance of axially functionally graded non-uniform columns with elastic restraints. Avcar (Avcar, 2014) relied on a numerical analysis to model the elastic buckling of steel columns subjected to an axial load. The influence of the slenderness ratio, cross-section geometry, and boundary conditions on the critical buckling load was investigated. Ruocco et al. (Ruocco et al., 2016) explored the buckling behavior of non-uniform columns by using Hencky bar-chain model. They estimated the rotational spring stiffness through three methods for non-uniform columns. The finite difference column model was used to calibrate the end rotational spring stiffness of the Hencky bar-chain. In another study, Ruocco et al. (Ruocco et al., 2016) optimized the geometry of inhomogeneous columns with elastic restraints under concentrated and distributed loads. They discretized the differential equation of Euler columns by assuming the Hencky bar-chain model to evaluate the critical buckling load by looking for the minimum eigenvalue. Soltani and Sistani (Soltani and Sistani, 2017) adopted a finite difference method to look into the buckling load of columns. They considered varying the flexural rigidity, supported edges of columns, and the applied axial loads in their study. Saraçoğlu and Uzun (Saraçoğlu and Uzun, 2019) built an ANSYS model to examine the buckling performance of structural elements. They considered the impact on the buckling load. Botis et al. (Botis et al., 2023) analyzed the buckling behavior of beam-type components based on Cavalieri's principle. The stability of the changeable cross-section beam was simulated and studied using an indirect variational approach taking into account the stiffness of the support connections. Saeed et al. (Saeed et al., 2024) calculated the buckling load multiplier by considering long rectangular and circular structures with solid and hollow cross sections which are loaded by compressive loads. They computed the decrease in the buckling load and mass when the hollowness of the cross-section area increases. They discovered that the column with a rectangular cross-section displayed a 3% higher load multiplier than the column with a circular cross-section for the same volume of material.

In this research, numerical, analytical, and experimental analyses are used to evaluate the buckling load of two segments of combined columns. The numerical analysis is performed using ANSYS. An equivalent stiffness method is employed in the analytical analysis. Three different cross-section geometries (circle, square, and hexagon) and four boundary conditions (fixed-fixed, fixed-free, pinned-pinned, and fixed-pinned) are considered in the analytical and numerical analysis. The columns are assumed to be prismatic, isotropic, and homogenous. Experimentally, the circular combined columns made of aluminum and copper alloys are manufactured to measure the critical buckling load at different segment lengths with different boundary conditions.

2. EXPERIMENTAL WORK

2.1. Materials used:

In this work, a two-segment combined column with a circular cross-section area is made of aluminum and copper alloys. Table 1 shows the mechanical properties of these alloys. The diameter of the combined column is 8 mm. The combined columns are formed by pressing the outside threads of one column against the inner threads of the other. The total column length is kept at 552 mm. The length of each segment is varied and the effect of this variation on the buckling behavior was studied. The first and last columns are made entirely of pure aluminum and pure copper respectively, whereas the other columns consist of different percentages of aluminum and copper as displayed in Fig.1.

Table 1. Material Properties of Aluminum and Copper alloy (Diwan and Diwan, 2022).

Property	Value	
	Aluminum	Copper
Modulus of Elasticity (E)	7.1×10^{10} N/m^2	1.1×10^{11} N/m^2
Poisson Ratio (ν)	0.33	0.34
Density (ρ)	2770 kg/m^3	8300 kg/m^3

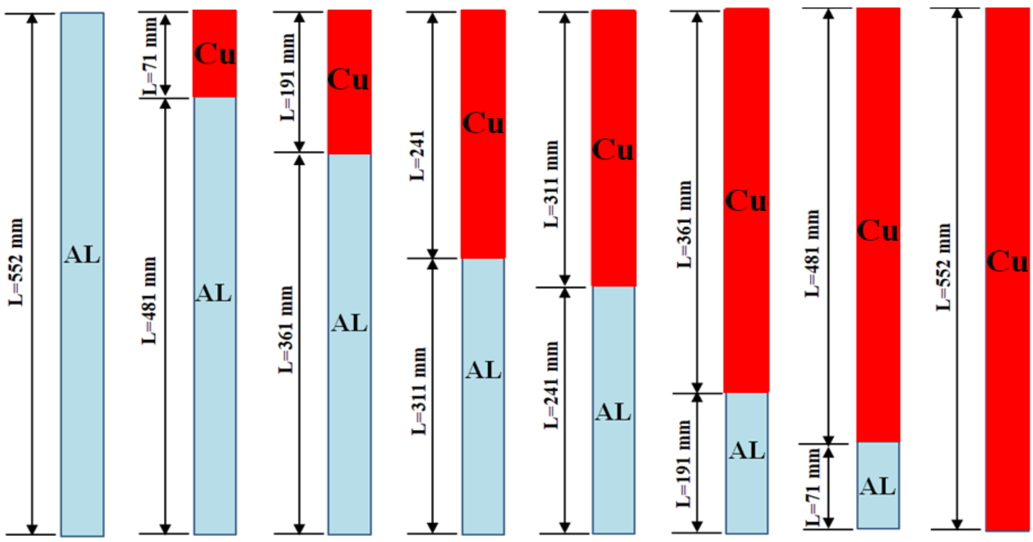


Fig.1 Combined column with different percent of Aluminum and Copper alloys.

2.2. Critical Buckling load measurement

The critical buckling load and deflection of single metal and combined columns were measured experimentally for three boundary conditions as demonstrated in Fig. 2. Fig. 3 shows that Strut Buckling Apparatus (SKU: 015) that was used in this study. This apparatus consists of a single vertical column with a sliding support that enables a screw to apply the load. The fixed or free-edge column support is a cylinder in a socket. The load cell measures the column load and the digital calipers (0.01mm) measure its deflection. Weight hangers and weights are used for side loading. The experiment was carried out on the column by following these steps:

1. Measuring the diameter and length of the columns.
2. Placing the column in the testing device between the two ends of the fixture.
3. Adjusting the force display and deflect measures to zero.
4. Applying a compressive load and then gradually raise the compressive force until a noticeable deformation is seen.
5. Recording the critical buckling load value.
6. Conducting the test for both single metal and combined columns, considering all end support conditions of columns.

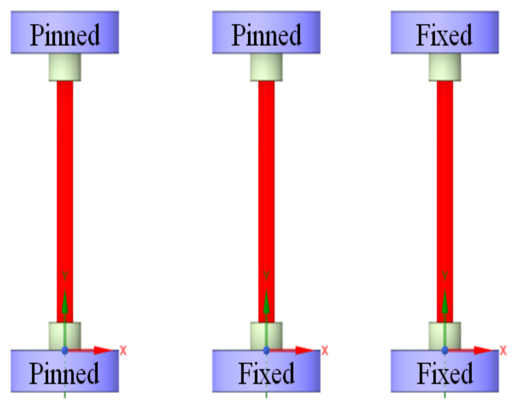


Fig. 2 Boundary conditions of column ends support used in experimental works.

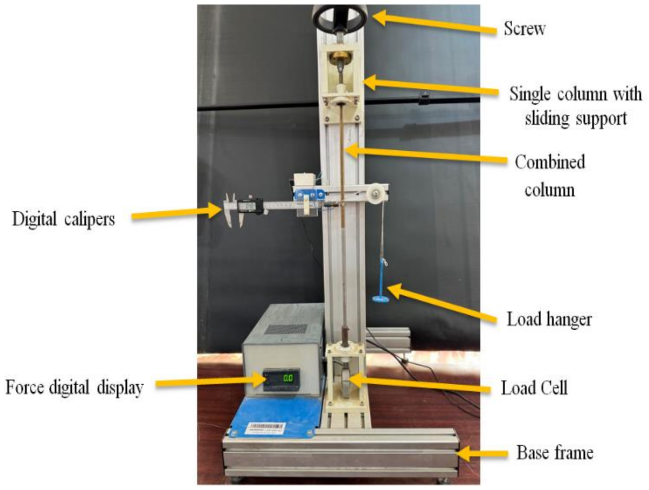


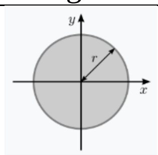
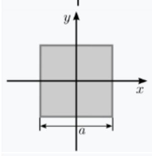
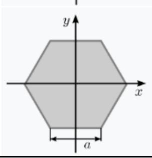
Fig.3 Strut Buckling Apparatus.

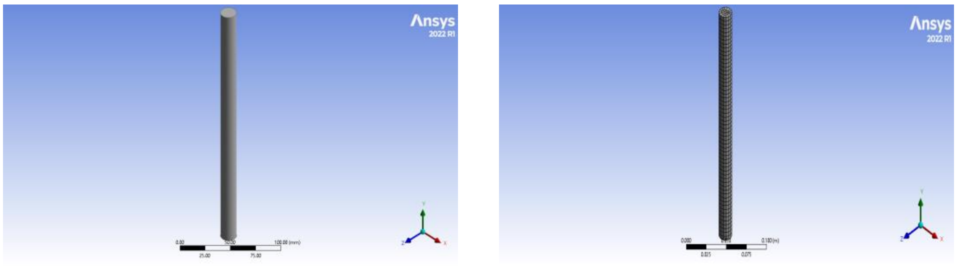
3. NUMERICAL MODELING

The ANSYS Workbench software version 2022R2 was employed in this research for calculating the critical buckling load of combined columns. The combined column has a length of 552 mm for various geometries of cross-sectional areas (circular, square, and hexagonal) with constant area. The dimensions of various cross-sectional areas of combined columns are given in Table 2. The combined column is composed of two materials (aluminum and copper alloys) in different percentages of length as illustrated in Fig. 1. Three different cross-section geometries of a combined column with constant area and length are utilized in the model of ANSYS. Fig. 4 presents the geometry and mesh of three combined columns with various cross-sectional geometries (circle, square, and hexagonal) when the area and length of the column are (50.265 mm^2 and 552 mm) respectively. The types of elements used in ANSYS model are SOLID186 with 20- Node for circular and square combined columns and SOLID187 with 10-Node for hexagonal combined columns. The appropriate element size is chosen using the convergent criterion (Hashim et al., 2022- Shukur et al., 2024). The least number of (nodes and elements) are (31284, 6468), (13144, 2106) and (7700, 3427) respectively when the cross-sections of the combined column are circular, square, and hexagonal respectively. This work employs the following four supporting edges types: (a) fixed-fixed support edges (F–F); (b) fixed-free support edges (F–Free); (c) fixed-pinned support edges (F–P); and (d) pinned-pinned support edges (P–P). The translational and rotational displacements for the three support types are as follows:

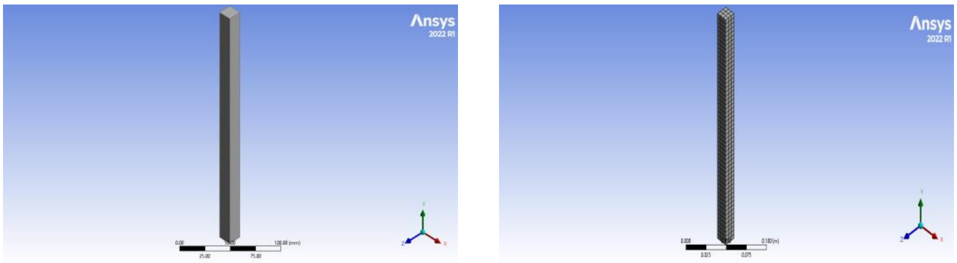
1. Fixed-fixed support: The displacements in the translation ($x, y,$ and z) and rotational on ($x, y,$ and z) are zero for two ends of the combined column.
2. Fixed-free support: The displacements within the translation (x, y and z) and rotational around (x, y and z) are zero for one end of the combined column and at the other end are free.
3. Fixed-pinned support: The displacements during the translation (x, y and z) and rotational concerning (x, y and z) are zero for one end and at the other end, the displacement through the translation (x and z) and rotational related to (x and y) are zero.
4. Pinned-pinned supported: The displacements under translation (x and z) and rotational around (x and y) are zero for two ends of the combined column.

Table 2. Dimensions of different cross sectional area of combined column.

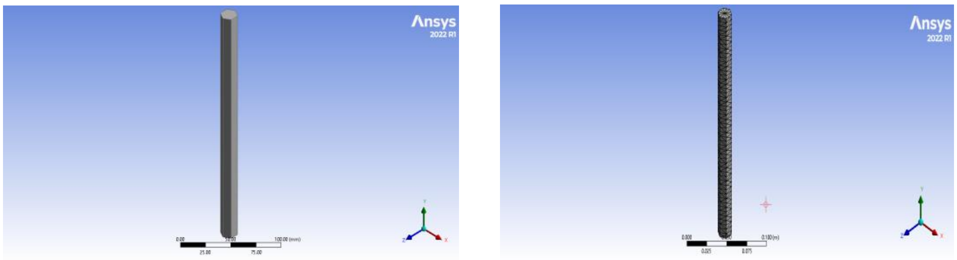
No.	Shape	Figure	Dimension
1	Circle		$r = 4\ mm$
2	Square		$a = 7.08\ mm$
3	Hexagon		$a = 4.3985\ mm$



(a)



(b)



(c)

Fig.4 Geometry and meshing of combined columns with different cross sectional geometries (a) circular, (b)square, and (c) hexagonal

4. ANALYTICAL MODELING:

The equivalent stiffness technique is applied to determine the equivalent stiffness of the combined column (Alansari et al., 2018 - Jebur and Alansari, 2023). In this method, the equivalent stiffness of the combined column depends on many parameters. These include the length of each part, cross-section area, and supporting type. As seen in Fig. 1, the cross-section area is assumed constant, while the properties and length of each part are varied and four supporting types are considered. Therefore, the equivalent stiffness of columns can be calculated depending on the supported type by using the following equations (Jebur and Alansari, 2023):

1. Fixed-free column: to calculate the equivalent stiffness (modulus of elasticity (E) and second moment of area (I)), the combined column consists of two parts and the equivalent stiffness of clamped –free column can be estimated using the following equation (for more details see (Jebur and Alansari, 2023)):

$$(EI)_{eq} = \frac{L^3}{\frac{(L_2)^3 - (L_1)^3}{(EI)_1} + \frac{(L_1)^3}{(EI)_1}} \quad (1)$$

a. Fixed-fixed, pinned-pinned, and fixed-pinned columns: for fixed-fixed, pinned-pinned, and fixed-pinned combined columns, the following four steps can be used to compute the equivalent second moment of areas.

b. The centroid of the column must be found in this case. The centroid equals half of the total length of the column (i.e. X=0.5*L=0.276 m)

c. Based on the centroid, the combined column is split into left and right parts. The left and right segments number depends on the length of each part but generally the number of parts in the left and right sides are either (1) or (2). Then, to determine the equivalent stiffness of the left and right sides, the following equation is applied:

i. If the left part consists of one material only and the right part consists of two materials (Jebur and Alansari, 2023):

$$(EI)_{left} = \frac{(L_{left})^3}{\frac{(L_1)^3}{(EI)_1}} = (EI)_1 \quad (2\text{-a})$$

$$(EI)_{right} = \frac{(L_{right})^3}{\frac{(L_2)^3 - (L_1)^3}{(EI)_1} + \frac{(L_1)^3}{(EI)_1}} \quad (2\text{-b})$$

ii. If the left part consists of two materials and the right part consists of one material (Jebur and Alansari, 2023):

$$(EI)_{left} = \frac{(L_{left})^3}{\frac{(L_2)^3 - (L_1)^3}{(EI)_1} + \frac{(L_1)^3}{(EI)_1}} \quad (3\text{-a})$$

$$(EI)_{right} = \frac{(L_{right})^3}{\frac{(L_1)^3}{(EI)_1}} = (EI)_1 \quad (3\text{-b})$$

iii. If the left and right parts consist of one material only (Jebur and Alansari, 2023):

$$(EI)_{right} = \frac{(L_{right})^3}{\frac{(L_1)^3}{(EI)_1}} = (EI)_1 \quad (4\text{-b})$$

$$(EI)_{left} = \frac{(L_{left})^3}{\frac{(L_1)^3}{(EI)_1}} = (EI)_1 \quad (4\text{-b})$$

d. The equivalent stiffness of the combined column is:

$$(EI)_{eq} = \frac{(L_{left} + L_{right}) * (L_{left})^2 * (L_{right})^2}{(EI)_{left} * (L_{left})^2 + (EI)_{right} * (L_{right})^2} \quad (5)$$

After calculating the equivalent stiffness of the combined column, the following formula can be used to determine the critical buckling load:

$$P_c = \frac{\pi^2 (EI)_{eq}}{(L_{eff})^2} \quad (6)$$

Where (L_{eff}) is the effective length of a column (in meters), and it equals to ($L_{eff} = K * L$) and K is:

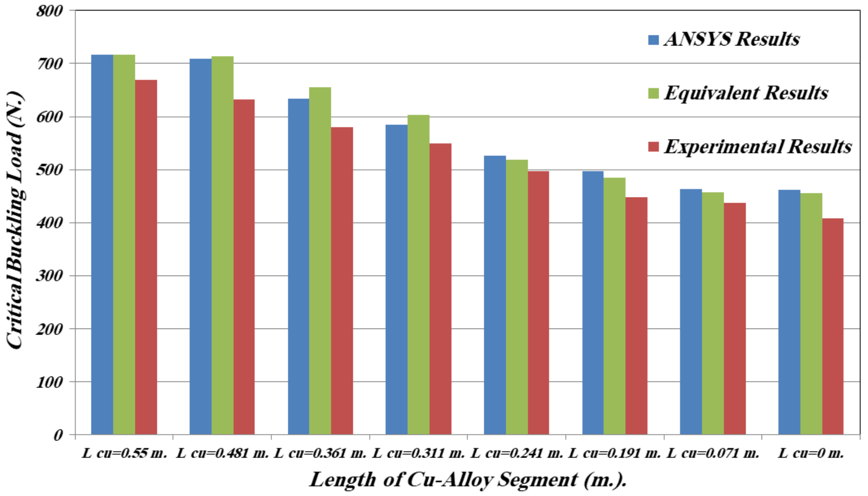
$$K = \begin{cases} 1 \quad \text{for Pinned} - \text{Pinned } Column \\ 2 \quad \text{for Clamped} - \text{ Free } Column \\ 0.5 \quad \text{for Fixed} - \text{Fixed } Column \\ 0.699 \quad \text{for Fixed} - \text{Pinned } Column \end{cases}$$

5. RESULTS AND DISCUSSION:

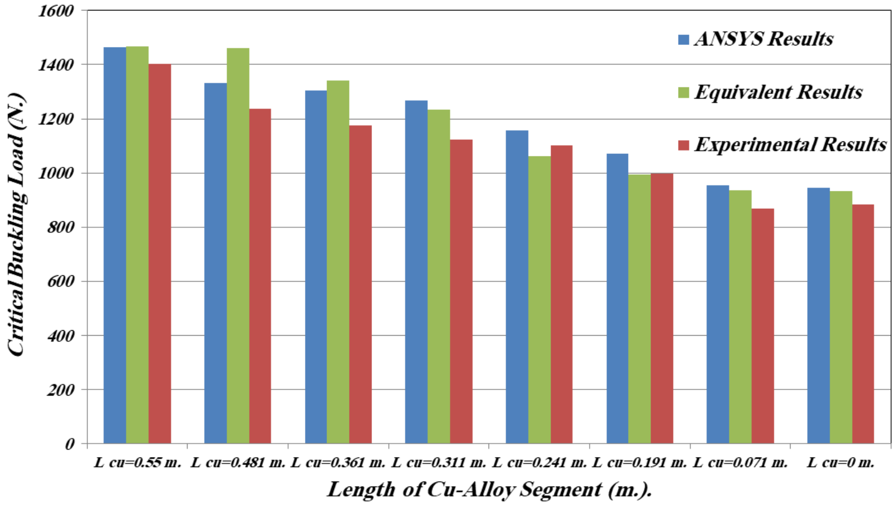
The findings are divided into two parts, the first one deals with the comparison among the experimental, analytical, and numerical results of circular columns for three types of support (pinned-pinned, fixed-pinned, and fixed-fixed column). The other parts cover the effect of the cross-section area of the column on the critical buckling load. A constant cross-section area is assumed with different geometry of it (circular, square, and hexagonal).

5.1. Comparison with Experimental Results:

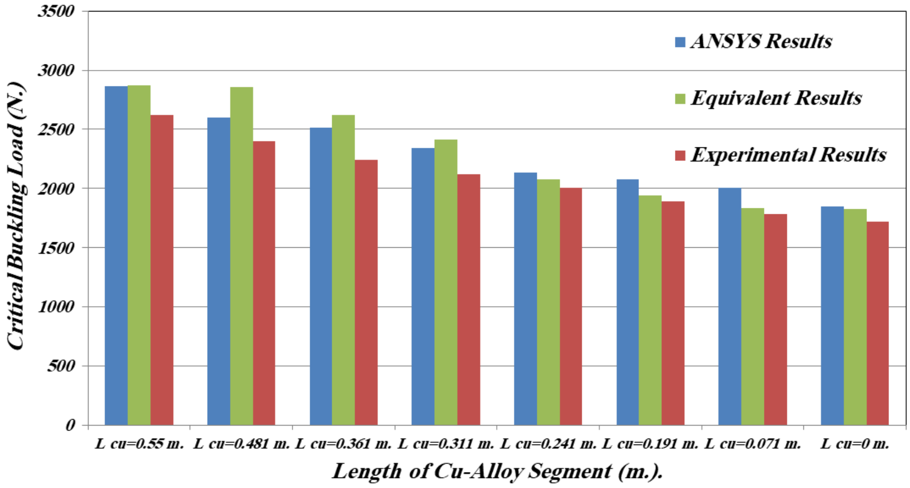
In this part, three types of support are used to validate the analytical and numerical models by comparing their results with the experimental results of the circular column. These types of support are pinned-pinned, fixed-fixed, and fixed-pinned supports. Fig. 5 displays the comparison among the critical buckling loads obtained by experimental, equivalent stiffness method, and ANSYS model when the length of the copper alloy decreases (i.e. aluminum alloy increases). Generally, the experimental results are less than those of analytical and numerical results. Very good agreements are noticed between experimental results and both of equivalent stiffness method and ANSYS results. The maximum error percentages between experimental and ANSYS critical buckling loads are (-13.083, -18.039and -19.052) % for pinned-pinned, fixed-pinned, and fixed-fixed columns respectively The maximum error percentages of critical buckling loads obtained by the equivalent stiffness method compared to experimental results are (-13.120, -12.768, and -12.453) % for pinned-pinned, fixed-pinned, and fixed-fixed columns respectively as listed in Table 3. From Table 3, it can be seen that the maximum error percentages are observed when the length of copper alloy is (0.25*L) or (0.75*L) approximately. Also, the maximum error percentages of ANSYS results compared to experimental results are greater than those obtained by the equivalent stiffness method.



(a) Pinned-Pinned Column.



(b) Fixed-Pinned Column.



(c) Fixed-Fixed.

Fig. 5 The Comparison among the critical buckling loads of combined circular column obtained by experimental work, equivalent stiffness method, and ANSYS model assuming different boundary conditions.

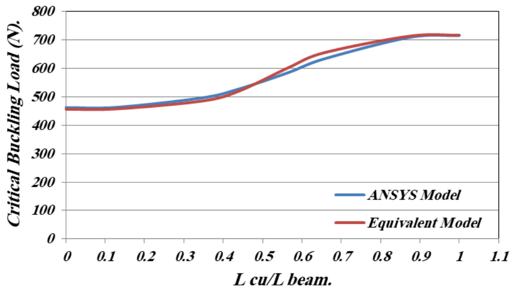
Table 3. The comparison among the critical buckling loads of combined circular columns obtained by experimental work, equivalent stiffness method, and ANSYS model assuming different boundary conditions.

No.	L_{cu}	Critical Buckling Load (N).			Error % w.r.t. Exp. Results	
		Experimental	Equivalent	ANSYS	Equivalent	ANSYS
1- Pinned-Pinned Support.						
1	0.55	669.800	716.110	717.246	-6.914	-7.084
2	0.481	632.600	709.560	713.775	-12.166	-12.832
3	0.361	579.400	633.550	655.205	-9.346	-13.083
4	0.311	549.700	585.540	602.617	-6.520	-9.626
5	0.241	496.500	527.000	519.289	-6.143	-4.590
6	0.191	448.300	496.760	485.696	-10.810	-8.342
7	0.071	437.200	463.390	457.847	-5.990	-4.722
8	0	408.600	462.210	456.430	-13.120	-11.706
2- Fixed-Pinned Support.						
1	0.55	1,401.700	1,464.400	1,467.960	-4.473	-4.727
2	0.481	1,237.600	1,331.800	1,460.854	-7.612	-18.039
3	0.361	1,176.900	1,305.700	1,340.982	-10.944	-13.942
4	0.311	1,124.700	1,268.300	1,233.352	-12.768	-9.660
5	0.241	1,103.500	1,156.900	1,062.808	-4.839	3.688
6	0.191	998.600	1,070.800	994.055	-7.230	0.455
7	0.071	867.700	953.310	937.056	-9.866	-7.993
8	0	883.800	945.200	934.156	-6.947	-5.698
3- Fixed-Fixed Support.						
1	0.55	2,624.800	2,861.600	2,868.986	-9.022	-9.303
2	0.481	2,398.200	2,602.000	2,855.099	-8.498	-19.052
3	0.361	2,239.700	2,511.800	2,620.820	-12.149	-17.017
4	0.311	2,121.500	2,342.800	2,410.467	-10.431	-13.621
5	0.241	2,006.200	2,132.300	2,077.156	-6.286	-3.537
6	0.191	1,892.500	2,077.700	1,942.785	-9.786	-2.657
7	0.071	1,784.300	2,006.500	1,831.387	-12.453	-2.639
8	0	1,718.600	1,847.100	1,825.718	-7.477	-6.233

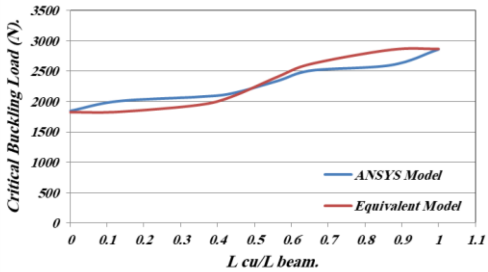
5.2. Effect of Cross-Section Area of Column:

In Figs. 6-8, the comparison between critical buckling loads obtained from ANSYS and equivalent stiffness method of different supporting types when the cross-section area of the column is a circle, square, and hexagon shape respectively. It can be observed that an excellent agreement between the findings of the two theoretical models for pinned-pinned and fixed-free columns for any cross-section geometry. However, the equivalent stiffness method must be improved to get an agreement with the ANSYS model for fixed-fixed and fixed-pinned columns for any cross-section geometry. Because of the effect of support is not considered when the equivalent stiffness of the beam is calculated. In other word, the position of maximum deflections of buckling in both fixed-fixed and fixed-pinned columns are not similar to that of pinned-pinned column.

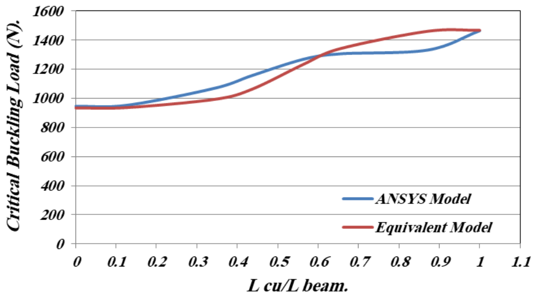
The effect of cross-section shape on the critical buckling loads obtained from the ANSYS model for different supporting types are compared as shown in Fig. 9. The square column has a higher critical buckling load compared to circular and hexagonal columns. The critical buckling load of the square column is about (1.04) times the critical buckling load of the circular column at different supporting types. On the other hand, the hexagonal column has a higher critical buckling load compared to circular and hexagonal columns. The critical buckling load of the square column is about (1.008) times the critical buckling load of the circular column at different supporting types. The effect of cross section area appears in the second moment or area as shown in equation (6) and the second moment or area causes increases the critical buckling load of square column. On the contrary, the effect of equivalent of stiffness of combined column appears in Fig. 9, the critical buckling load increases with increasing the length of copper part considering constant cross sectional area. This increasing in critical buckling load occurs due to the high elastic modules of copper (i.e. increasing the modulus of combined material).



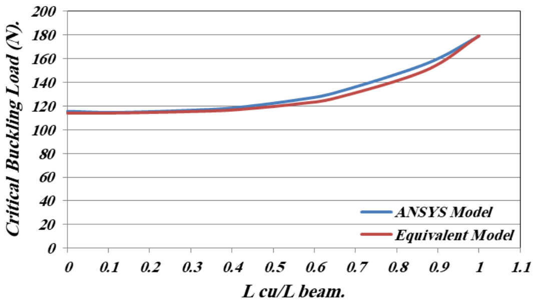
(a) Pinned-Pinned Support



(b) Fixed-Fixed Support

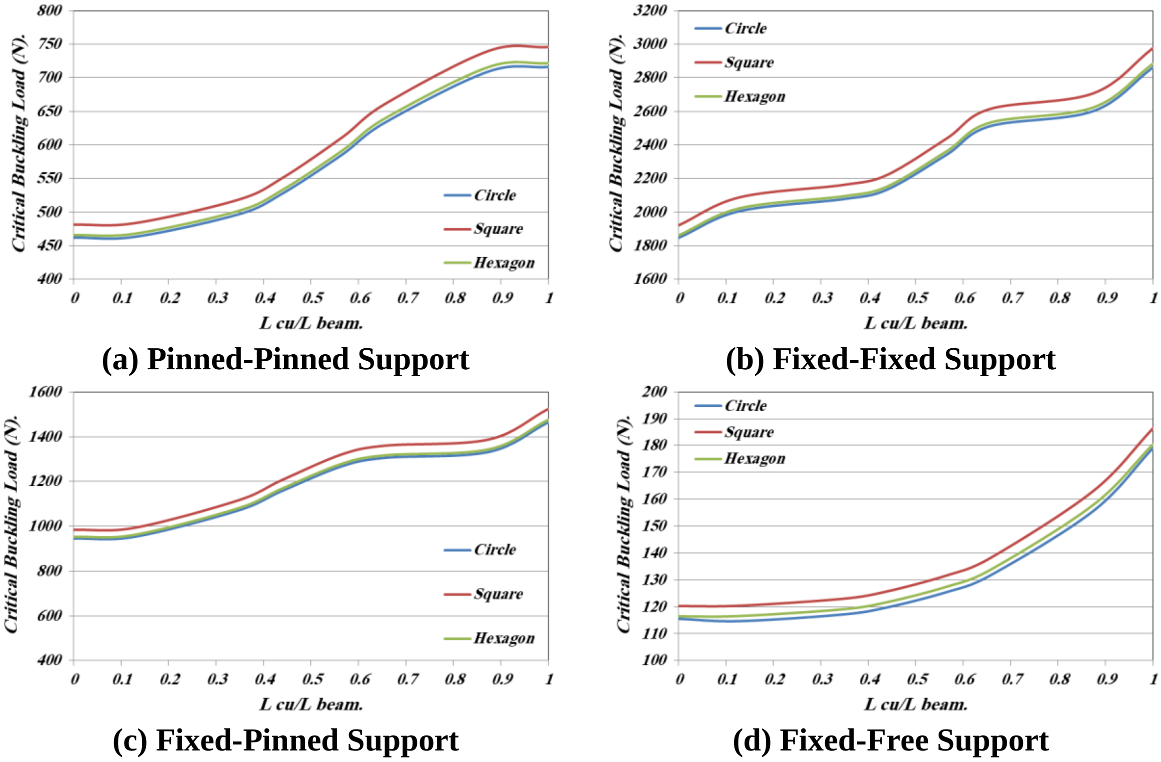


(c) Fixed-Pinned Support



(d) Fixed-Free Support

Fig. 6 The comparison between the critical buckling loads of ANSYS and equivalent stiffness models when the column have circular cross-section.



(a) Pinned-Pinned Support **(b) Fixed-Fixed Support**

(c) Fixed-Pinned Support **(d) Fixed-Free Support**

Fig.9 The comparison between the critical buckling loads of ANSYS model for different cross section geometries and different supporting types.

6. CONCLUSIONS:

The critical buckling load of combined columns made of copper and aluminum alloys with different cross-sectional geometries and various supported edges of the combined column were studied. The critical buckling load was calculated using theoretical methods (the equivalent stiffness method and the ANSYS model) and compared to the experimental method. From the obtained results, the following conclusions can be drawn:

1. The critical buckling load increases as the column ends support becomes more restricted because the freedom of movement of the column reduces the buckling load.
2. The critical buckling load in the case of a square cross-section is greater than in the cases of circular and hexagonal cross-sections because the moment of inertia in the case of a square section is greater.
3. The critical buckling load increases as the copper length increases due to the stiffness of copper is higher than that of aluminum.
4. There is good agreement of values of critical buckling loads between the equivalent stiffness method and ANSYS model for any cross-sectional areas (square, circle, and hexagon) and two types of support (pinned-pinned and fixed-free).

5. There is a good match of critical buckling load between those achieved from the equivalent stiffness method and ANSYS model for any cross-sectional geometries (square, circle, and hexagon) and two types of support (fixed-fixed and fixed-pinned)
6. A good degree of discrepancy was found between the experimental and theoretical results (analytical and numerical analysis).
7. The highest percentage errors of critical buckling load between the experimental method and theoretical methods (equivalent stiffness method and ANSYS models) are (-13.120, -12.768, and -12.453) % and (-13.083, -18.039, and -19.052) % respectively for pinned-pinned, fixed-pinned, and fixed-fixed columns.
8. The highest percentage errors of critical buckling load between the experimental method, and theoretical methods (equivalent stiffness method and ANSYS models) were found at the quarter and three quarters of the length of copper in the combined columns.

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