



HEALTH MONITORING ANALYSES OF CRACKED PLATES BASED ON FORCED VIBRATION

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ABSTRACT

This research presents a SHM (Structural Health Monitoring) techniques that are based on harmonic and transient analyses of cracked plate structure. Finite element models for the different cracked steel plate have been created by considering the length and the orientations of the crack, and a cantilever supported plates (parallel to the crack) are used.

The cracked structures are examined under harmonic concentrated lateral load. The frequency responses show that the lateral deflections of the crack plate decrease when increasing the crack length, whereas, the lateral deflections increase when increasing orientation angle until 90°. The models are also examined under the effect of transient concentrated lateral load. The time history responses show that the lateral deflections of the crack increase when increasing the crack length.

Finally, SHM analysis is applied in order to predict the early behaviour of the plates in accordance with the results above. The extrapolation for certain symbols of the results is performed to derive numerical formulas of the lateral deflections. The lateral deflection's formula is used as a function for the half-crack length, however, there is a formula for each orientation angle.

KEYWORDS: Structural Health Monitoring, Transient Analysis, Harmonic Analysis, Cracked Plate, Deflections, Finite Element Models.

مراقبة سلامة الصفائح المشققة و تحليلها بالاعتماد على الاهتزاز القصري

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الخلاصة

قدم هذا البحث تقنيات SHM (مراقبة سلامة الهياكل) بالاعتماد على الاهتزاز التوافقي، والانتقالي لهيكل صفيحة محتواة على شق. باستخدام طريقة العناصر المحددة تم إنشاء نماذج مختلفة لصفائح من الحديد الصلب و المحتواة على شق مع الاخذ بنظر الاعتبار طول و زاوية ميلان الشق و قد ثبتت الصفائح من جهة واحدة و تركت حرة من الجهات الاخرى.

إن الهياكل التي تم اختبارها تحت تأثير الاحمال الترددية المركزة أظهرت ان التشوهات العرضية تقل بزيادة طول الشق في حين تزداد بزيادة زاوية ميلان الشق لتصل الى 90 درجة. أيضاً، تم فحص النماذج تحت تأثير الاحمال الانتقالية المركزة والمسلسلة عرضياً حيث أظهرت الاستجابات الزمنية ان التشوهات العرضية تزداد بزيادة طول الشق. وأخيراً، تم تطبيق تحليل SHM من أجل التنبؤ بسلوك الصفيحة في وقت مبكر وفقاً للنتائج المذكورة أعلاه حيث اظهرت عملية الاستقرار لبعض النتائج لاشتقاق الصيغ العددية للتشوهات العرضية معادلات خاصة بها. حيث ان لكل زاوية ميلان شق صيغ خاصة و هي دالة لنصف طول الشق.

1. INTRODUCTION

Structural Systems in civil, mechanical and aerospace engineering, or any other, are susceptible to sudden damage, deterioration and aging. Therefore, a health monitoring system that is able to detect and identify any damage in real time in its earliest stage is essential to maintain the structural stability, integrity and to maximize the life span of the structure as much as possible. Therefore, the process of implementing a damage detection strategy for aerospace, civil and mechanical engineering infrastructure is referred to as Structural Health Monitoring (SHM), [Hoon Sohn et al. \(2004\)](#).

The health monitoring is studied by several researchers in recent years, where [Chen and Swamidas \(1994\)](#) and [Swamidas and Chen \(1995\)](#) presented FEM (Finite Element Model) results for a cantilever plate containing a crack. Their best data for locating damage were determined to be strain mode shapes. A methodology to detect and locate damage in a plate structure presented by [Choi S., et al. \(2005\)](#). The methodology utilizes damage indices based on the changes in the distribution of the modal compliance of the plate structure due to damage. Also, [Rainah Ismail \(2012\)](#) studied a vibration analysis for a thin isotropic plate containing an arbitrarily orientated surface crack. It is found that the vibration characteristics and nonlinear characteristics of the cracked plate structure can be greatly affected by the orientation of the crack in the plate.

The above researchers focused on modal parameters to detect the defect in structures. But, the present paper represents a harmonic and transient analyses for elastic plate models to extract the behaviours of these models under effect of these analyses. Therefore, the basic items which is assumed to be as an objective for this work is the prediction of the plates' behaviours' by using Structural Health Monitoring's techniques to predict the damage history. This has been verified by finding lateral deflections of the cracked thin plates.

2. THEORETICAL CONSIDERATION

Plates and shells are initially flat and curved structural elements, respectively, for which the thicknesses are much smaller than the other dimensions. It is commonly to divide the plate thickness (h) into equal halves by a plane parallel to the faces. This plane is called the midplane of the plate. The plate thickness is measured in a direction normal to the midplane. The flexural properties of a plate depend greatly upon its thickness in comparison with its other dimensions, [A. C. Ugural \(1981\)](#).

The geometric model of this paper is a square thin plate with a central crack with different length and angles as shown in [Fig. 1](#). Isotropic steel plate with mechanical properties shown in [Table 1](#) of dimensions (30 cm *30 cm) and (1mm) thickness is used in the present work. The central crack has half crack-length ranges from (0.015 m - 0.12 m) increased each (0.015 m) and also has the angle ranges from (0°- 90°) increased by (15°).

The theoretical part includes two types of analyses which are the harmonic and transient analyses and as follows:

2.1. Harmonic Analysis

Any sustained cyclic load will produce a sustained cyclic response (a harmonic response) in a structural system. Harmonic response analysis gives the ability to predict the sustained dynamic behaviour of the structure under consideration, thus enabling the designer to verify whether or not the designs will successfully overcome resonance fatigue, and other harmful effects of forced vibrations, [Wisam Auday Hussain \(2005\)](#).

The basic equation of motion for damped system solved by a harmonic analysis is of the form

$$[\mathbf{M}]\{\ddot{\mathbf{x}}\} + [\mathbf{C}]\{\dot{\mathbf{x}}\} + [\mathbf{K}]\{\mathbf{x}\} = \{\mathbf{F}_0\} \sin \bar{\omega} t \quad 1$$

The complete solution of this equation consist of the complementary solution $x_c(t)$ and the particular solution $x_p(t)$. The total response is obtained by summing the complementary solution (transient response) and the particular solution (steady-state response), that is,

$$\mathbf{x}_i(t) = e^{-\xi \omega_i t} (A \cos \omega_{iD} t + B \sin \omega_{iD} t) + \frac{x_{st} \sin(\bar{\omega}_i t - \theta_i)}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}, (i= 1, 2, 3, 4, \dots, n) \quad 2$$

where, n is the number of degrees of freedom. It must be warned that the constants of integration A and B should be evaluated from initial conditions using the total response given by eq. (2) and not from just the transient component of the response given in eq. (1), [Mario P. \(1985\)](#).

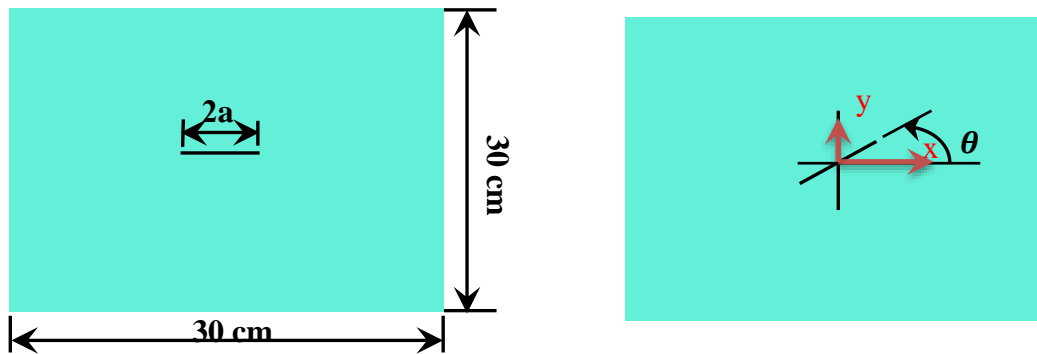


Fig. 1. Location and orientations of crack

Table 1. Mechanical properties of steel plate, [N. V. Srinivasulu, et al. \(2012\)](#)

Parameters	Value
Modulus of Elasticity (E)	207 Gpa
Density (ρ)	7850 kg/m ³
Poisson's Ratio (ν)	0.29

2.2. Transient Analysis

If the system is subjected to a suddenly applied nonperiodic force, the response will be transient, since steady-state vibrations are not usually produced. The transient response of a system can be found using what is known as the convolution integral.

Consider a system subjected to a unit impulse at $t=0$ as shown in [Fig. 2](#) for an under-damped system, the solution of the equation of motion, [Singiresu S. Rao \(2000\)](#):

$$[\mathbf{m}]\{\ddot{\mathbf{x}}\} + [\mathbf{c}]\{\dot{\mathbf{x}}\} + [\mathbf{k}]\{\mathbf{x}\} = 0 \quad 3$$

is given as follows:

$$\mathbf{x}(t) = e^{-\xi \omega t} \left\{ \mathbf{x}(0) \cos \omega_d t + \frac{(\dot{\mathbf{x}}(0) + \xi \omega \mathbf{x}(0))}{\omega_d} \sin \omega_d t \right\} \quad 4$$

where, $\omega_d = \omega \sqrt{1 - \xi^2}$, for under-damping vibration.

If the mass is at rest before the unit impulse is applied ($x = \dot{x} = 0$ for $t < 0$ or at $t = 0$), we obtain, from the impulse-momentum relation,

$$\text{Impulse} = f = 1 = [\mathbf{m}]\{\dot{\mathbf{x}}\}(t = 0) - [\mathbf{m}]\{\dot{\mathbf{x}}\}(t = 0 -) = [\mathbf{m}]\{\dot{\mathbf{x}}\}(0) \quad 5$$

Thus the initial conditions are given by:

$$\mathbf{x}(t = 0) = \mathbf{x}(0) = 0$$

$$\dot{\mathbf{x}}(t = 0) = \dot{\mathbf{x}}(0) = \frac{1}{[\mathbf{m}]} \quad 6$$

Assuming that at τ , the force $F(\tau)$ acts on the system for a short period of time $\Delta\tau$, the impulse acting at $t = \tau$ is given by $F(\tau).\Delta\tau$. At any time t , the elapsed time since the impulse $t - \tau$, so response of the system at t due to this impulse alone is given by

$$\mathbf{x}(t) = \frac{F}{m\omega_d} g(t - \tau) = \frac{F}{m\omega_d} e^{-\xi\omega(t-\tau)} \sin(\omega_d(t - \tau)) \quad 7$$

by solving this equation, for zero initial conditions we obtain

$$\mathbf{x}(t) = \frac{1}{m\omega_d} \int_0^t F(\tau) e^{-\xi\omega(t-\tau)} \sin \omega_d(t - \tau) d\tau \quad 8$$

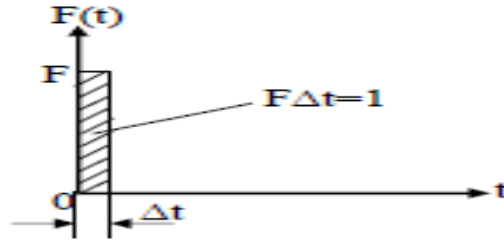


Fig. 2. Impulse force excitation

3. RESULTS AND DISSCUSSION

The results include numerical results. These results include the harmonic deflections, transient deflections, predicting harmonic deflections and predicting transient deflections.

The predicting process is performed with the aid of extrapolation method. An additional models have been built in order to obtain the lateral deflections due to harmonic and transient loads. The purpose of building these models is to find the best fit and to produce the equations as a function of half crack-length in order to predict the maximum lateral deflections for the remaining models when the crack propagates. The models that introduced are the intact and cracked plate models with half-crack length of (0.0075, 0.0225 and 0.0375 m). The equations produced are based on the results of models with half-crack length of (0, 0.0075, 0.015, 0.0225, 0.03, 0.0375 and 0.045 m).

3.1. Harmonic Analysis Results

The lateral deflections for cracked and intact plates is shown in Fig. 3. It is found that the maximum value occurred at the first mode of the natural frequencies while it decreases at other modes. Also, the lateral deflection decreases when the crack length increases to the lowest value on the biggest half crack length ($a = 0.12$ m) as shown in Fig. 4. This decrease in the lateral deflection is attributed to the crack area which causes a discontinuity area due to harmonic load (cyclic load), however, the deflection increases when the orientation of crack increases and the maximum value occurs in (90°) of crack inclination as shown in Fig. 5.

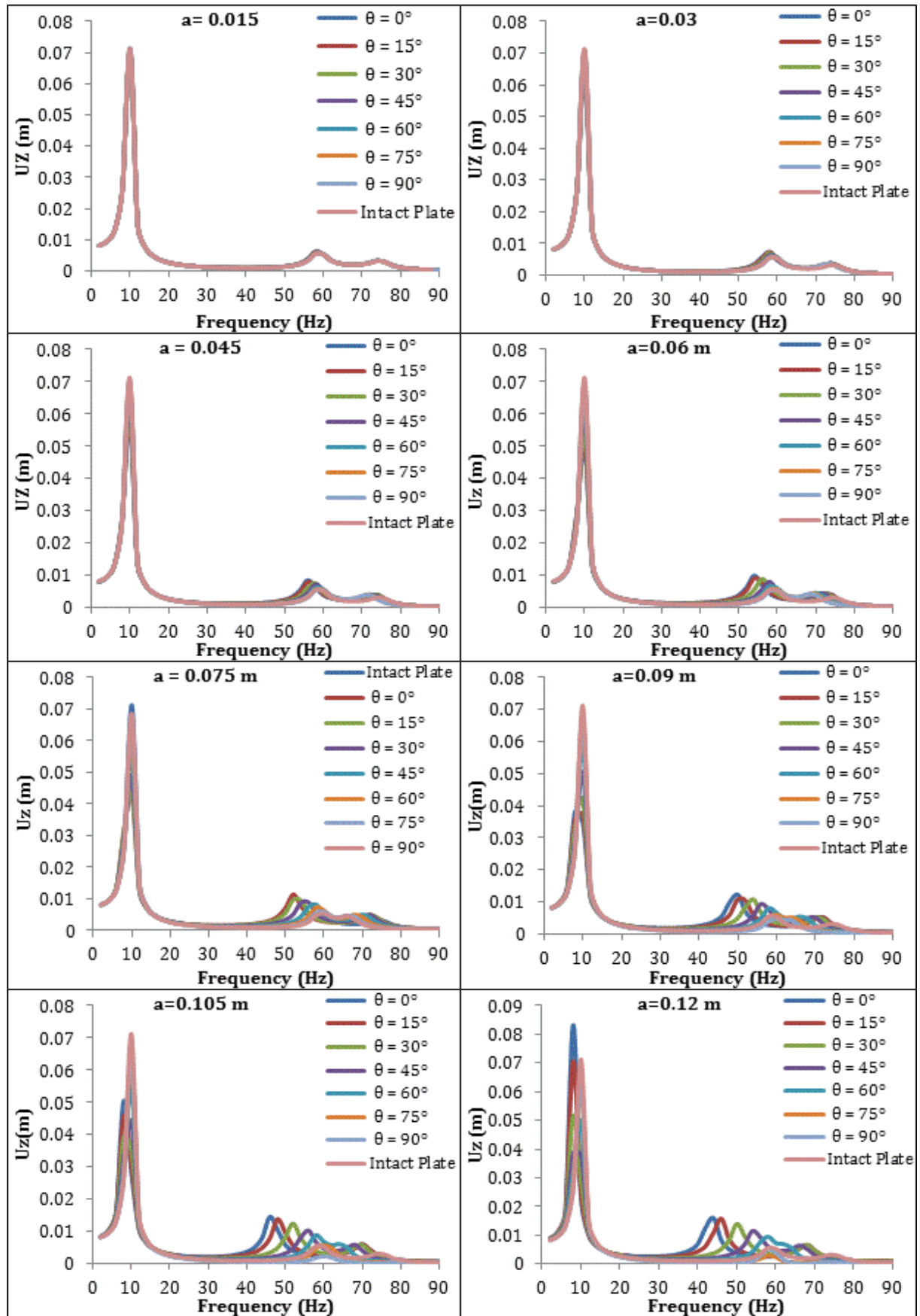


Fig. 3. Lateral displacements due to harmonic load for intact and cracked plates Transient Analysis Results

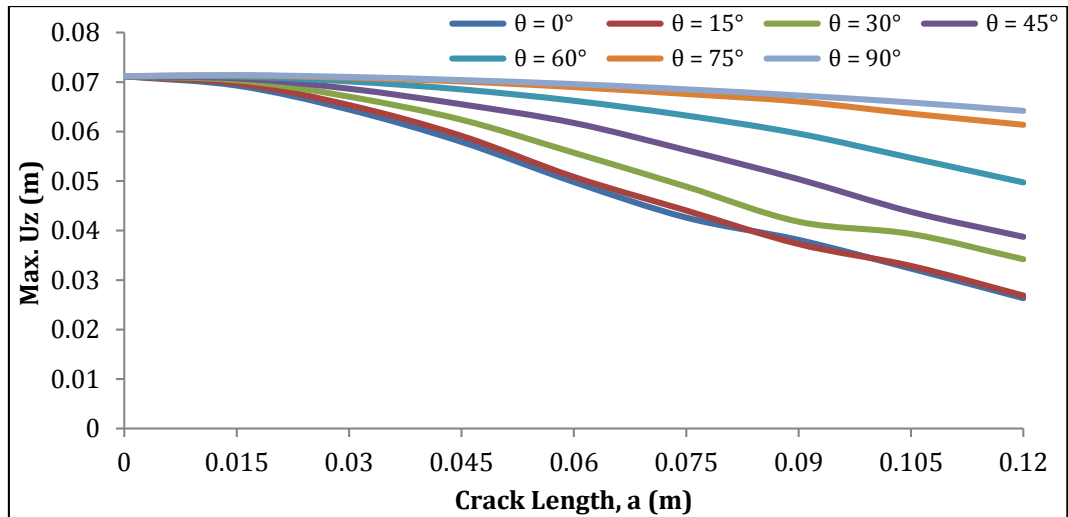


Fig. 4. Maximum lateral displacement due to harmonic load as a function of half crack length for different crack orientations

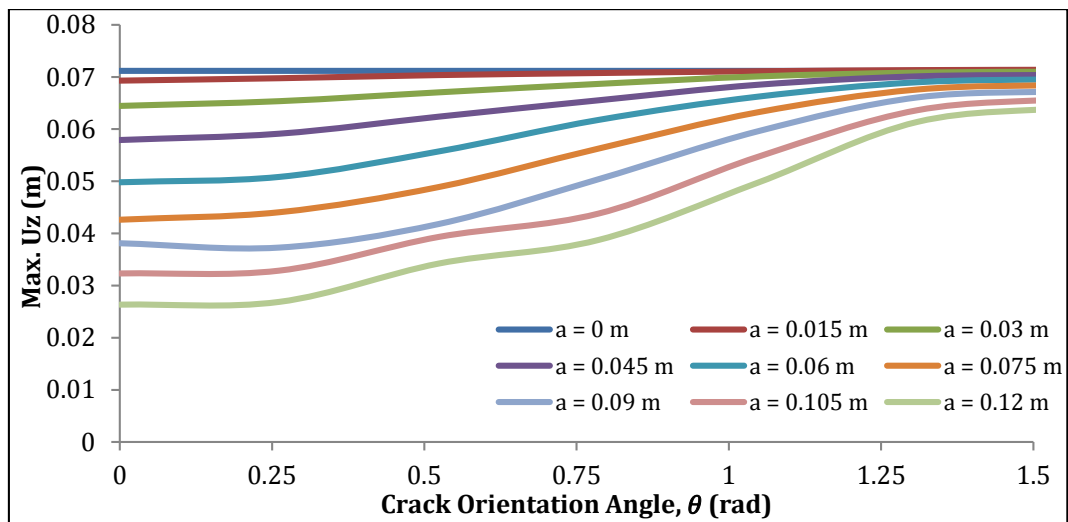


Fig. 5. Maximum lateral displacement due to harmonic load as a function of crack orientations for different crack length

Various FEMs were examined under action of stepped transient load as shown in Fig. 6. The impulsive transient load applied to plate models as a lateral concentrated load during a period of (0.1 sec) for total time of (3.5 sec).

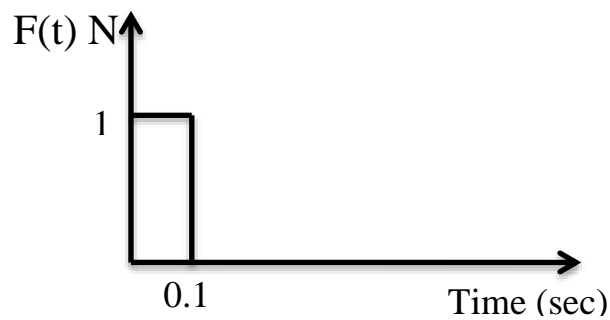


Fig. 6. Impulsive transient load

The lateral deflections of different FEMs (cracked and intact plates) is shown in Fig. 7. The maximum value of the deflection occurs at the moment the impulse is applied to the structures and it is decreasing until reaching a proper stability condition. Also, when the load is removed, the deflections on models increase as the crack length increases to have the largest value at (0.12 m) of half crack length. In addition, the maximum lateral deflection increases when the crack length of plate models increases (as an absolute value) but it decreases as the orientation of crack increases as shown in Fig. 8. So, the behaviour of the lateral deflection is attributed to the shortage period of the impulse force.

3.2. Predicting the Harmonic Deflections

Most structures are subjected to different dynamic (may be a harmonic) load and this causes a dynamic deflections. So, the maximum lateral deflections caused by the lateral harmonic load applied to various models are predicted.

The equations of maximum lateral deflections that are produced depend on some numerical results as shown in Table 2. These equations are formulated as a function of half crack length for different crack orientations.

The comparison between numerical and predicted results is shown in Fig. 9 to give the maximum difference between which that estimated by (6 %) for different models. Also, the maximum harmonic deflections that are predicted is shown in Fig. 10.

Table 2. Formulas of prediction the maximum harmonic deflections for different plate models

Angle Orientations	The Equations
$\theta = 0^\circ$	$U_z(a) = -5.2117a^2 - 0.0658a + 0.0713$
$\theta = 15^\circ$	$U_z(a) = -5.7284a^2 - 0.0129a + 0.0712$
$\theta = 30^\circ$	$U_z(a) = -4.4444a^2 + 0.0052a + 0.0712$
$\theta = 45^\circ$	$U_z(a) = -3.468a^2 + 0.0338a + 0.0712$
$\theta = 60^\circ$	$U_z(a) = -1.8063a^2 + 0.0232a + 0.0712$
$\theta = 75^\circ$	$U_z(a) = -1.1053a^2 + 0.0258a + 0.0712$
$\theta = 90^\circ$	$U_z(a) = -0.9577a^2 + 0.025a + 0.0712$

3.3. Predicting the Transient Deflections

The structures may be excited by transient load causing deformation. Therefore, the mathematical equations to predict deflections of different models are used in the present research. These equations are shown in Table 3. These equations are the function of half crack length for different crack orientations of cracked plate structures.

The results of maximum deflection caused by transient load are compared with the numerical results as shown in Fig. 11. The divergence in behaviours between the numerical and predicted curves is found. Also, Fig. 12 shows the maximum predicted deflection, as an absolute value, decreases when crack length increases as it is compared with Fig. 8. So the prediction of the maximum lateral deflections under transient force can't be reliable to predict the early behaviour.

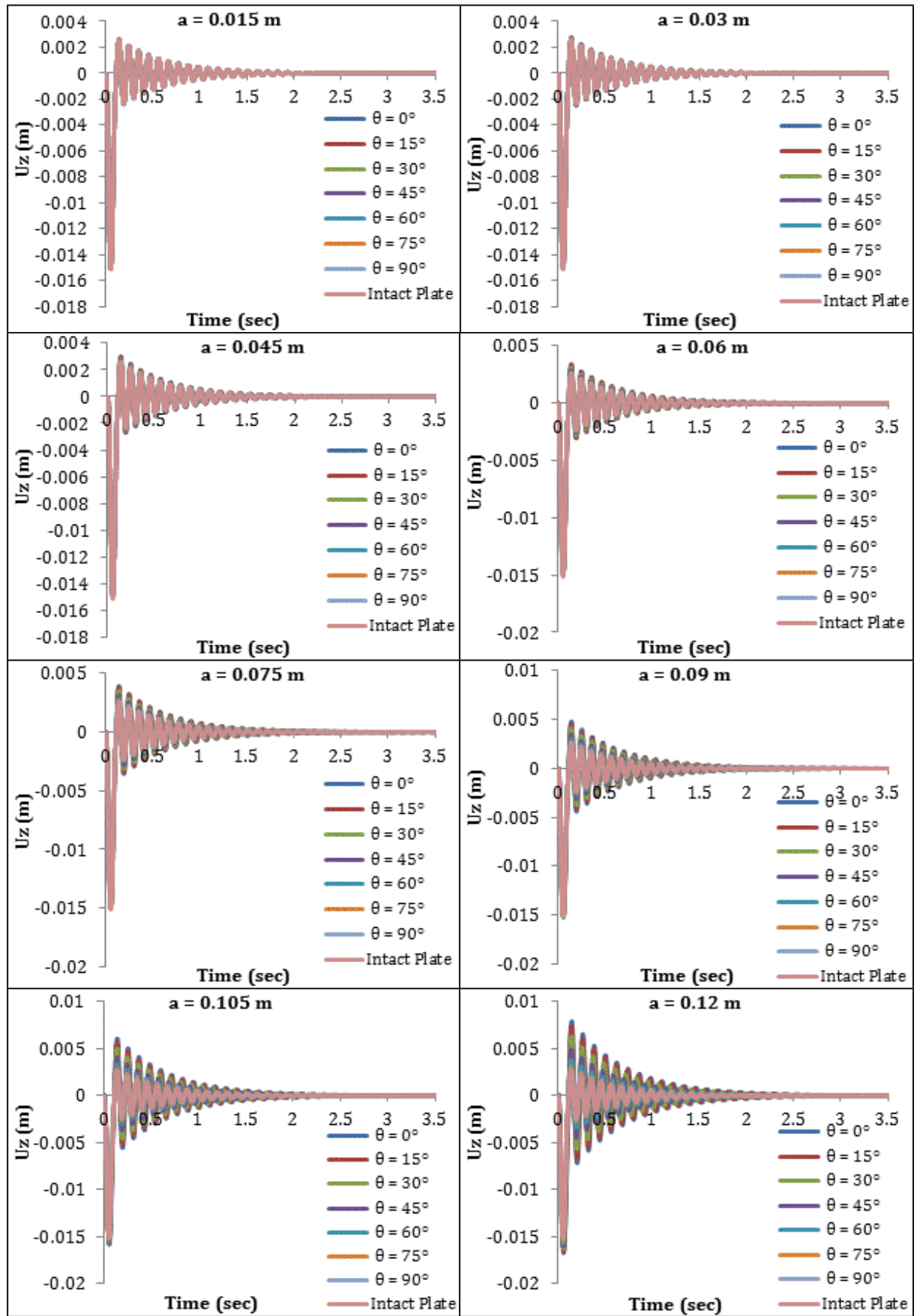


Fig. 7. Lateral displacements due to transient load for intact and cracked plate models

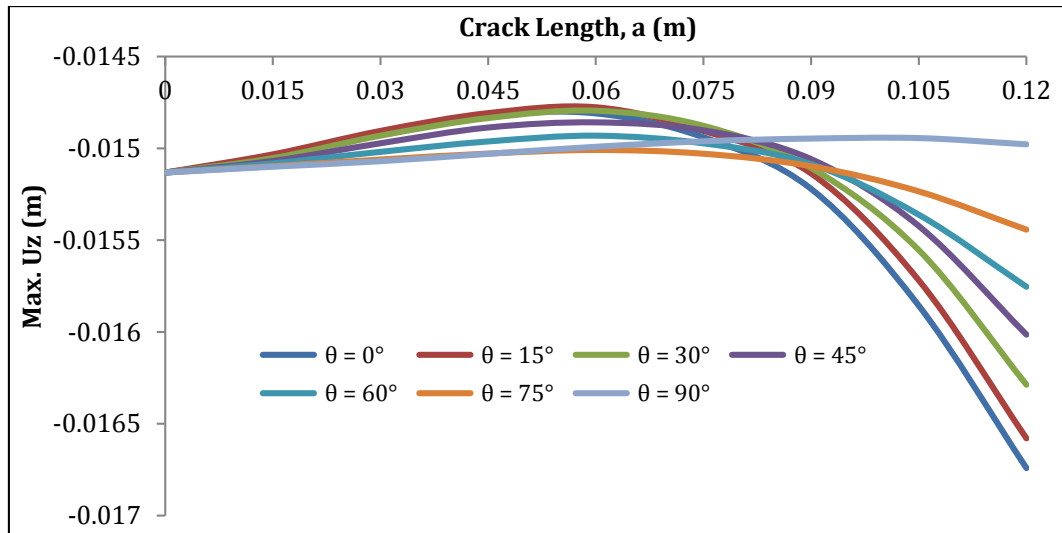


Fig. 8. Maximum lateral displacement due to transient load as a function of half crack length

Table 3. Formulas of prediction the maximum transient deflections for different plate models

Angle Orientations	The Equations
$\theta = 0^\circ$	$U_z(a) = 0.0053a^2 + 0.0071a - 0.0151$
$\theta = 15^\circ$	$U_z(a) = 0.0134a^2 + 0.0069a - 0.0151$
$\theta = 30^\circ$	$U_z(a) = 0.0302a^2 + 0.0055a - 0.0151$
$\theta = 45^\circ$	$U_z(a) = 0.0389a^2 + 0.0039a - 0.0151$
$\theta = 60^\circ$	$U_z(a) = 0.0139a^2 + 0.0031a - 0.0151$
$\theta = 75^\circ$	$U_z(a) = -0.0048a^2 + 0.0025a - 0.0151$
$\theta = 90^\circ$	$U_z(a) = 0.013a^2 + 0.0015a - 0.0151$

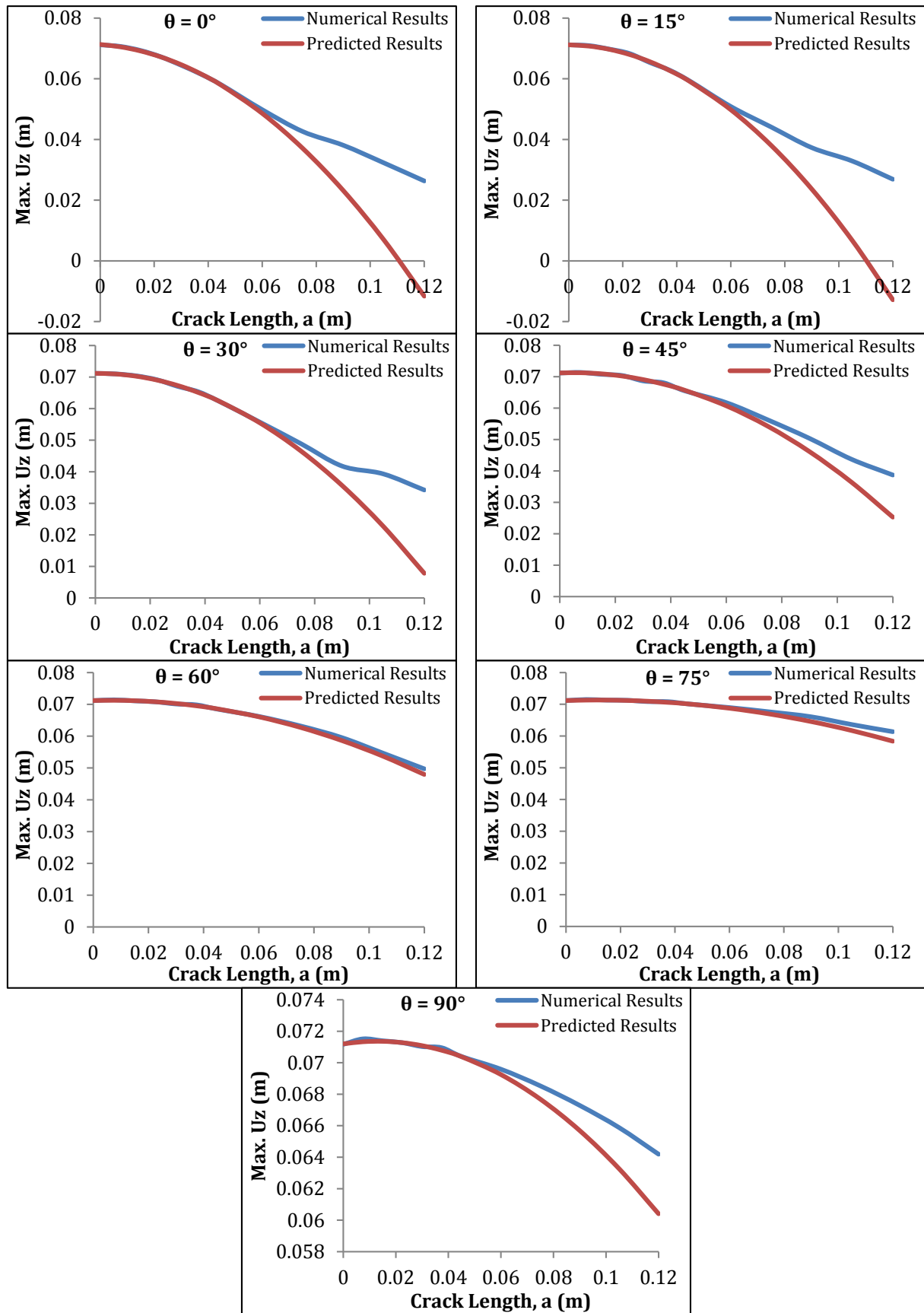


Fig. 9. A comparison between numerical and predicated results of maximum harmonic deflections for various models

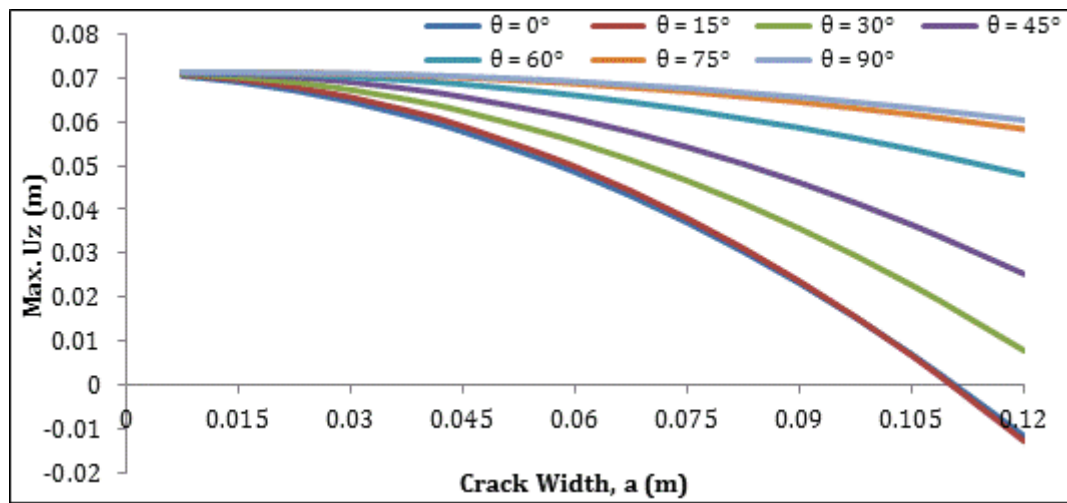


Fig. 10. Predicted of maximum harmonic deflection for different cracked plate structures

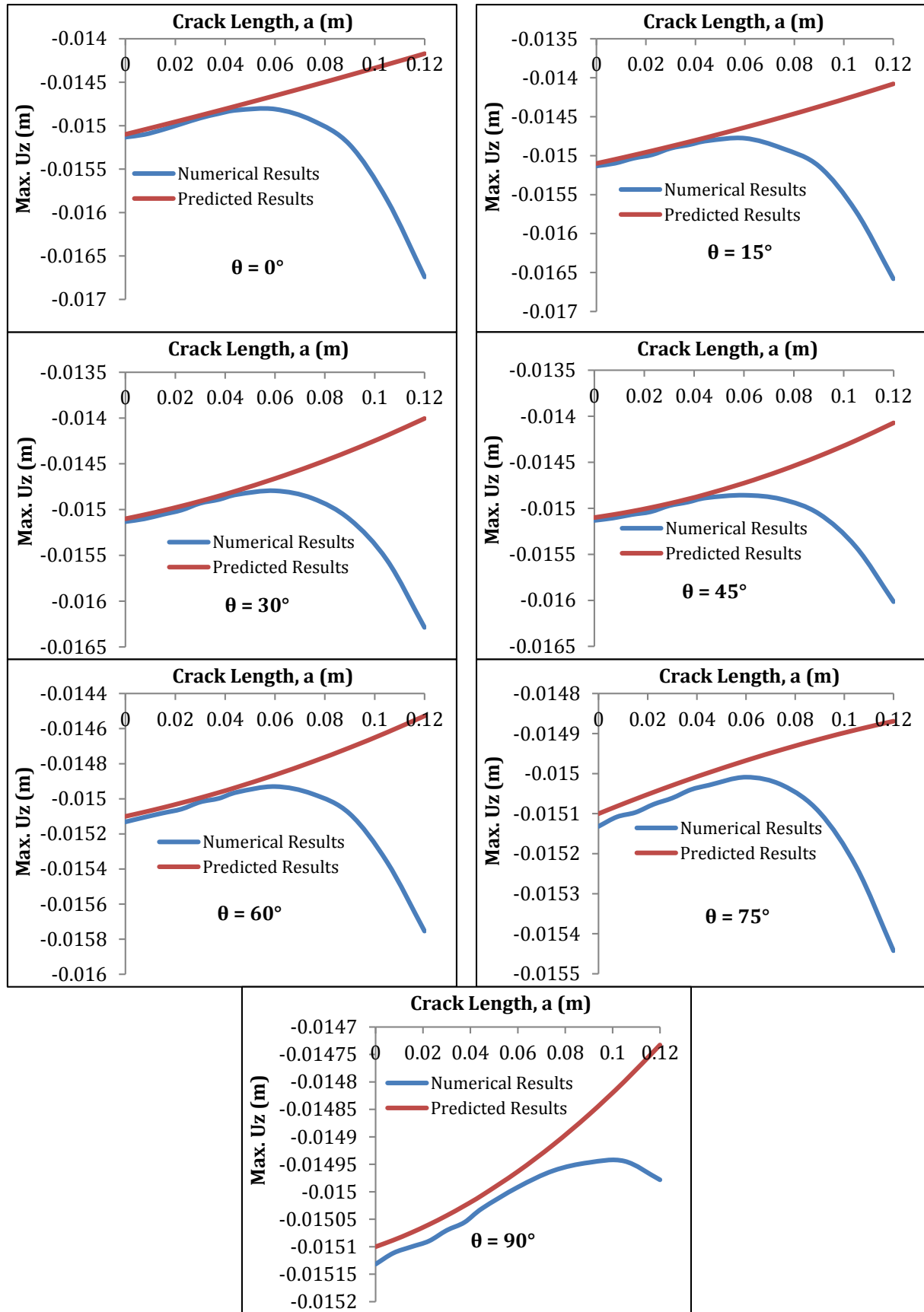


Fig. 11. A comparison between numerical and predicated results of maximum harmonic deflections for various models

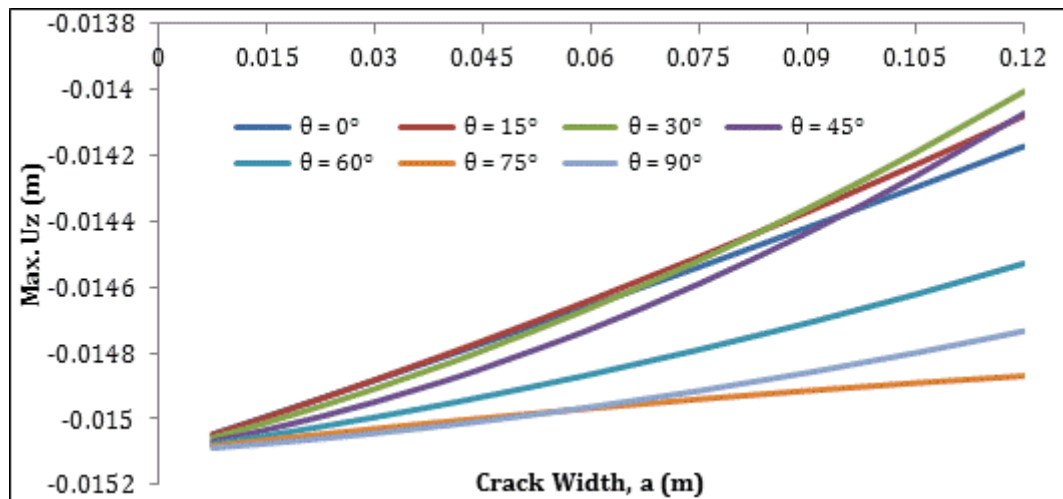


Fig. 12. Predicted of maximum transient deflection for different cracked plate structures

4. CONCLUSIONS

Most of the important conclusions obtained under the influence of each of the harmonic and transient loads effects for the cracked plate structure are listed below:

1. The harmonic lateral deflection decreases when the crack length increases to be minimum at (0.12 m) of half crack length.
2. The lateral harmonic deflections predicted are compared with numerical ones and it was found that the discrepancy between them is estimated by (6 %).
3. A comparison between the predicted and numerical results of lateral transient deflection is made, and a divergence in behaviours between them is detected.
4. Hence, the best analyses to predict the early behaviour of the structure is harmonic analysis. But, the transient analysis cannot be used as a reliable method to predict the early behaviour of the structure.

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