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# A COMPARISON STUDY BETWEEN TWO MATHEMATICAL MODELS OF GROUNDWATER FLOW SIMULATION IN TEEB AREA, MISSAN PROVINCE

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## ABSTRACT

Two-dimensional mathematical models are developed to simulate the flow regime of the upper part of Quaternary Deposits by two approaches finite difference and finite element. The suggested conceptual model, which is advocated to simulate the flow regime of aquifer is fixed for one layer, i.e. the activity of the deeper aquifer is negligible. The models are calibrated using trial and error procedure in two stages, steady state followed by unsteady state. Calibrated value of hydraulic conductivity and specific yield using MODFLOW (finite difference) and MICROFEM (finite element) simulations for both steady and unsteady states ranged (1-10) m/day while the specific yield ranges between (0.1- 0.4). For steady state condition, the mean absolute errors are 0.249 and 0.133 for MODFLOW and MICROFEM respectively. For unsteady state condition, the mean absolute errors are 0.025 and 0.02 for MODFLOW and MICROFEM respectively. The results of MODFLOW and MICROFEM shows that the flow in the study area from northeast to southwest.

## KEYWORDS

Two-dimensional mathematical models, Groundwater, Teeb area, aquifer

# دراسة مقارنة بين نموذجين رياضيين لمحاكاة جريان المياه الجوفية في منطقة الطيب، محافظة ميسان

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**الخلاصه**

تم تطوير نماذج رياضية ثنائية الأبعاد لتمثيل حركة المياه الجوفية للحشرج الأعلى لتكوين الرسوبيات الرباعية, باستخدام طريقة الفروقات المحددة وطريقة العناصر المحددة. اقترح نموذج مفاهيمي محدد لطبقة واحدة، أي أهمل تأثير طبقات المياه الجوفية السفلى. أجريت عملية المعايرة للنماذج باستخدام طريقة المحاولة والخطأ للحالتين، الجريان الثابت وغير الثابت. وفقا لعملية المعايرة، أعيد توزيع الخصائص الهيدروليكية لمنطقة الدراسة، حيث تراوحت قيم معامل الايصالية الهيدروليكية لنموذج MODFLOW (الفروقات المحددة) ونموذج MICRFEM (العناصر المحددة) بين (1،10) متر/يوم, بينما كانت قيم معامل العطاء النوعي متغيرة بين (0.1،0.4). للحالة الجريان الثابت بلغ معدل الخطا المطلق (0.249, 0.133) في برنامج MODFLOW وبرنامج MICROFEM على التولي. اما في الحالة غير المستقرة فقد كان معدل الخطا المطلق (0.02, 0.025) في برنامج MODFLOW وبرنامج MICROFEM على التولي. اثبتت النتائج ان حركة جريان المياه الجوفية من الشمال الشرقي الى الجنوب الغربي.

## 1. INTRODUCTION

Recently, the study of water resources becomes basic necessity that must be done, because it is indispensable an economical fortune. Increasing demands on water make revive development in groundwater usage. Any development adds new problems, and efforts begin to resolve these problems. Sources of groundwater are numerous and its quality and quantity is affected by geological formations, in addition to different hydrological conditions and human activities. These processes involve the study of geological formations and the different hydrological conditions in order to make an important advantage from this water.

Groundwater flow model attempt to represent the essential features and operation of the actual groundwater system by means of mathematical counterpart. Mathematical models include analytical and numerical models. Usually assumptions necessary to solve mathematical model analytically are fairly restrictive. Many analytical solutions require that the medium should be homogeneous and isotropic, for this reason using numerical methods is very realistic solution (Wang and Anderson, 1982). Numerical models have been extensively used for groundwater analysis since the mid-1960 as high speed digital computers become widely available (Mercer and Faust, 1981).

Several important studies were conducted in the study area, Al-Dabage and Murad, (1998) made hydrological and hydrochemical studies about Bai-Hassan and Mukdadia (Bakhtiari) formations in Kut and Ali Al-Grabi. They mentioned that this area may be subdivided into two hydrogeological regions. First one represents the mountains and hills area which lies in the boundary region between Iraq and Iran. This region is considered as recharge area into the main aquifers, while the other one is Al-Kut region. Lazim (2002) presented a two-dimensional mathematical model for representing the groundwater flow in both steady and unsteady states. She showed that the groundwater may be used for injecting to maintain the pressure in the oil reservoir in Buzurgan oil field. Al-Jaburi (2005), showed through a hydrogeological and hydrochemical study of Ali Al-Garbi area, this area can be divided into two major aquifers depending on chemical composition of groundwater. Al-Kaabi (2009) indicated in his study, that the general flow direction of groundwater is in a concordance with topography, which means from northeast to southwest. Developments of both conceptual and computational models for groundwater hydrology have started since the beginning of the twentieth century to the present day. Initially, computational models relied on analytical methods but there is now a greater use of numerical models. Real groundwater problems are frequently so complex that they can only be analyzed when simplified assumptions are introduced. Imaginations and experiences are required to identify the key process which must be included in conceptual models. The overall goal of this research is to develop mathematical models by using finite difference (MODFLOW program) and finite element (MICROFEM) for simulating groundwater flow to meet the future demands of groundwater in the study region and determine the basic difference between finite difference and finite element approach.

## 2. STUDY AREA

The study area is located in north and north east of Missan province as shown in Fig. 1. It occurs along the foot of mountains of the Iraqi-Iranian frontier in south of Iraq, between longitudinal-line (47°06'-47°36') and latitude-line (32°06'-32°30'). The considered area is about 1860 $km^2$. It extended from Teeb area close to the Iraqi-Iranian border to Shikh Fars area. There are two rivers in this region, Teeb and Duiraige river. Teeb river crosses the area from north to south and ends in a local marsh. Duiraige river lies in south-east of this area. Marshes receive their water from the distributaries of Teeb and Duiraige river, and other small streams that generally flow toward the west and southward from the foothills of Himreen along the Iraqi-Iranian frontier in south of Iraq.

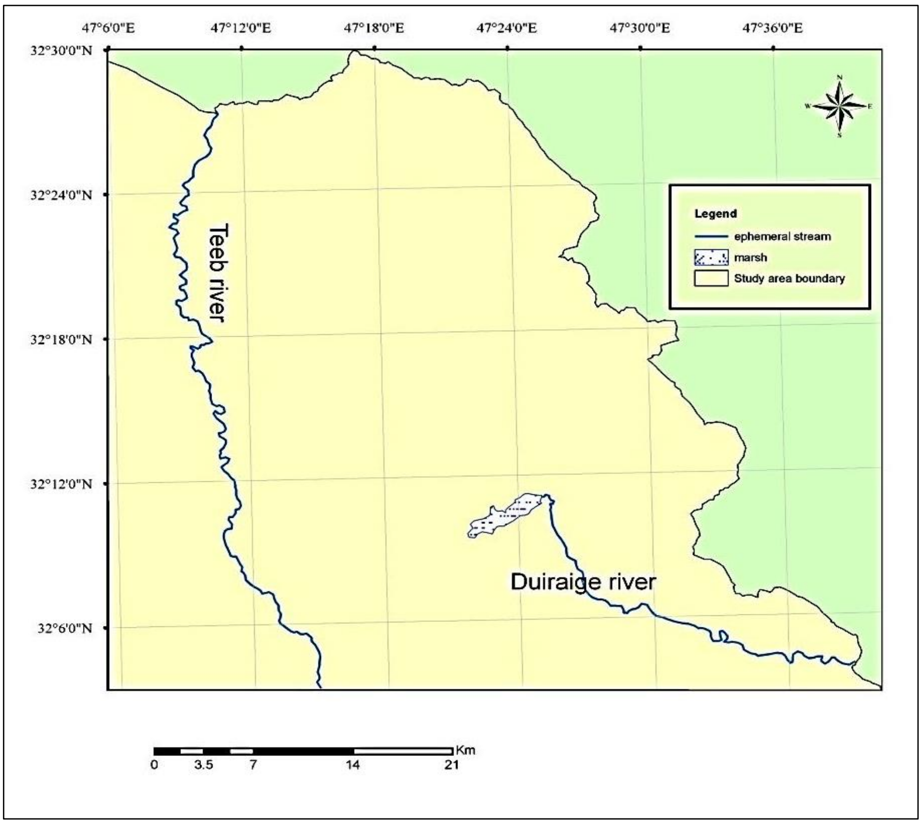


**Fig. 1. Location of the study area with reference to the coordinate Cartesian**

## 3. MODEL STRUCTURE

The governing partial differential equation used in MODFLOW is (McDonald and Harbaugh, 1988).

$$\frac{\partial}{\partial x}\left(Kxx\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(Kyy\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(Kzz\frac{\partial h}{\partial z}\right)-W=Ss\frac{\partial h}{\partial t} \qquad (1)$$

Where:

Kxx , Kyy, Kzz : are values of hydraulic conductivity along the x, y, and z coordinates axes, which are assumed to be parallel to the major axes of hydraulic conductivity ($LT^{-1}$).

h: is potentiometric head (L).

W : is a volumetric flux per unit volume and represents sources and/or sinks of water ($T^{-1}$).

Ss is the specific storage defined as the ratio of the volume of water which can be injected per unit volume of aquifer material per unit change in head ($L^{-1}$), and

t is time (T).

In general Ss, Kxx , Kyy, and Kzz may be functions of space (Ss, $= Ss(x, y, z)$, $Kxx = Kxx(x, y, z)$ , etc.) and W may be a function of space and time ($W = W(x, y, z, t)$; equation (1) describes groundwater flow under nonequilibrium condition in a heterogeneous and anisotropic medium, provided the principal axes of hydraulic conductivity are aligned with the coordinate directions.

Analytical solutions of equation (1) are possible for very simple systems, so various numerical methods must be employed to obtain approximate solutions for complex systems. One such approach is the finite difference method, where in the continuous system described by equation (1) is replaced by a finite set of discrete points in space and time, and the partial derivatives are replaced by terms calculated from the differences in head values at these points. The process leads to systems of simultaneous linear algebraic difference equations; their solution yields of head at specific points and times.

***Input Data***

**1. Initial Water Table Elevations**

Ten monitoring wells distributed over the study area are selected to measure the initial and historical groundwater level for one year. Fig. 2 shows the local distribution of observed wells. Al-Aboodi (2011) measured the groundwater level per month for the period (April, 2010; March, 2011).

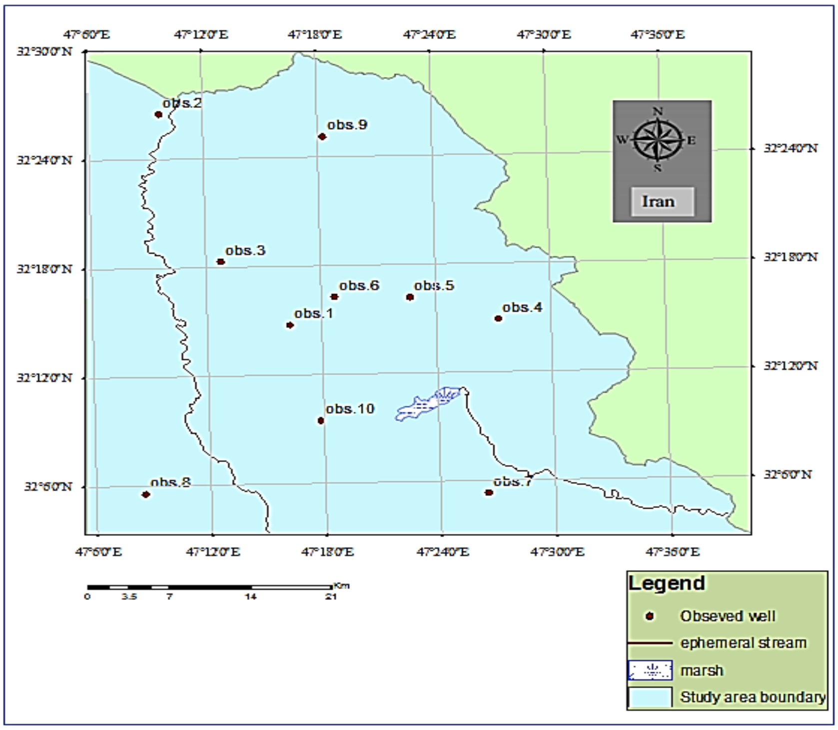


**Fig. 2. Local distribution of (10) observed wells in the study area**

**2. Initial Assessment of Hydraulic Characteristics**

Twenty soil samples are selected to obtain the soil texture classes by (Al-Aboodi, 2011). The average values of minimum infiltration rates are then interpolated using Kriging techniques in Geostatistical analysis extension of Arc GIS 9.3 to produce the soil hydrologic group layer in the study area as shown in Fig. 3. Based on the covered area by soil hydrologic groups and Table 1, the initial assessment of hydraulic conductivity and specific yield are supplied to the numerical program.

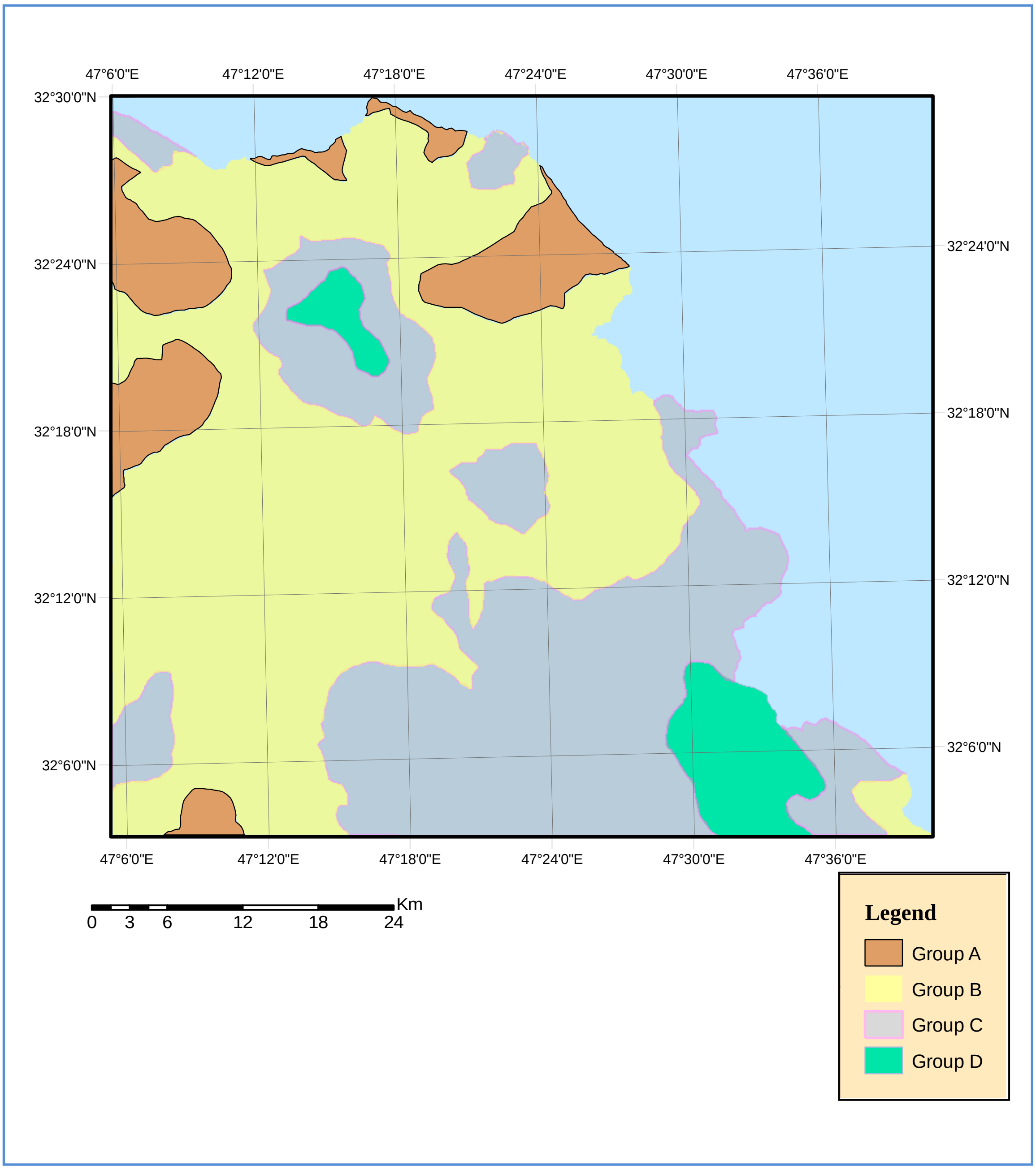


**Fig. 3. Hydrological soil groups in the study area**

**Table 1. Saturated hydraulic conductivity, soil porosity, and effective porosity by the hydrologic soil group (National Resources Conservation Service (NRCS), 2001**

| Soil Type | Saturated Hydraulic Conductivity (cm/hr), Ks | Porosity | Effective Porosity |
|---|---|---|---|
| Hydrologic soil group: a | 23.56 | 0.437 | 0.417 |
| Hydrologic soil group: b | 1.32 | 0.463 | 0.434 |
| Hydrologic soil group: c | 0.20 | 0.398 | 0.330 |
| Hydrologic soil group: d | 0.06 | 0.475 | 0.385 |

### 1- Distribution of Pumped Wells

The average discharge of the existing wells in the western parts of the study area ranged from 4 l/sec to 8 l/sec. Most existing wells in the eastern part of the study are along the foothill of Hemrin abstracted groundwater from the Bai Hassan and Mukdadiya Formations by artesian wells. All hand-dug and tube wells in the study area withdraw groundwater for domestic and stock use only. Tube wells are commonly used for abstracting groundwater compared with large diameter hand dug wells. Hand dug wells are conducted randomly with non-uniform shapes. Normal pumping periods have been almost in a year, they attain about tweleve hours per day (i.e, only in daylight hours).

### 2- Boundary Conditions and Model Grid

Fig. 4 shows a spatial distribution of the aquifer in the study area by MODFLOW and MicroFEM program. All boundaries in the present model was modeled as head- dependent boundary to allow inflow to the modeled region at a rate proportional to the head difference between the aquifer outside the simulated area and the model boundary. The top of the model was represented as unconfined aquifer. The water table elevation changes as part of the model solution. The bottom of the model was represented as a no flow condition. The vertical location of this boundary was selected to correspond with the base of the aquifer (aquitrad).

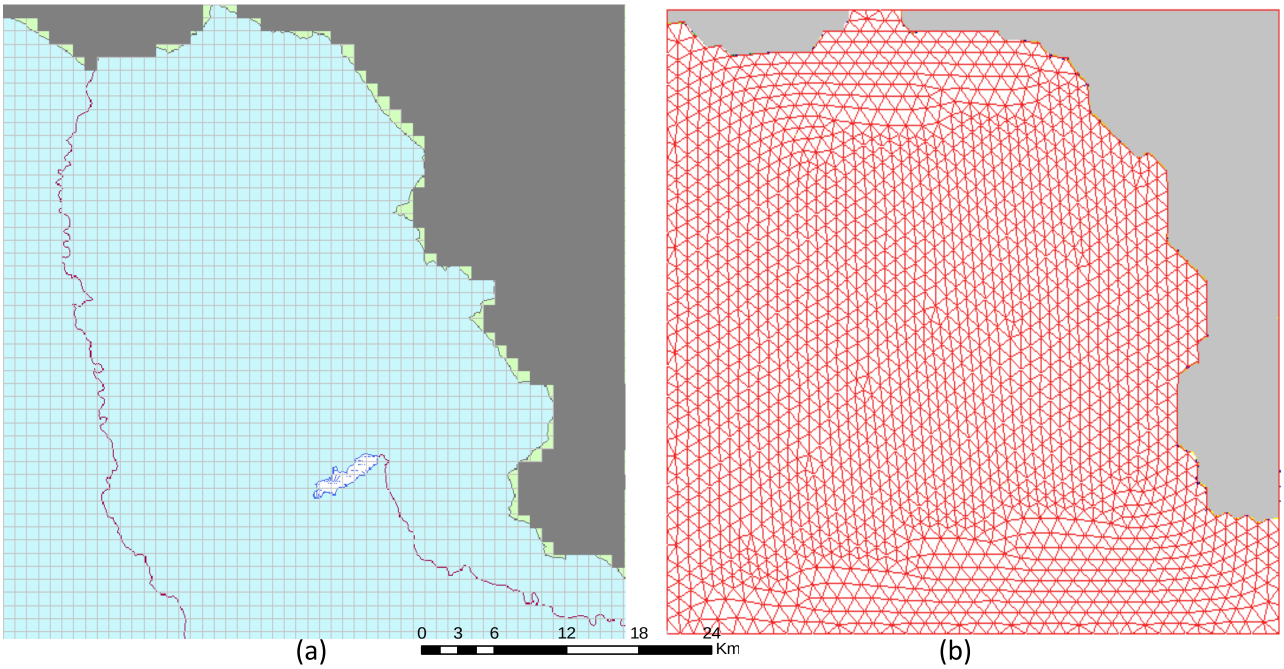


**Fig. 4. Configuration of nodal network by MODFLOW program (a) and MICROFEM program (b) of the study area**

## 4. CALIBRATION OF THE MODELS

Calibration is a process of selection model parameter to accomplish a good matching between simulated and observed hydraulic head. Once, calibration of model has been completed through systematic redistribution of hydraulic properties. In the trial-and-error calibration process, the independent variables (parameters and fluxes) of a model are adjusted manually, in successive model runs, to produce the reasonable match between the simulated and measured data. Calibration of present model was carried out in two sequential stages, a steady state calibration followed by unsteady state calibration. Steady state calibration permits the adjustment of hydraulic conductivity, where aquifer storage changes are not significant. Thus, dynamic stress and storage effects are excluded. Comparison between the observed and calculated heads for this process is shown in Table 2 and Table 3. A trial and error calibration based on mean absolute error and root mean squared error as the following formulas

$$Mean\ absolute\ error = \frac{1}{n}\sum_{i=1}^{n}|(h_m - h_s)_i| \quad (2)$$

$$Root\ mean\ squared\ error = \left[\frac{1}{n}\sum_{i=1}^{n}(h_m - h_s)_i^2\right]^{0.5} \quad (3)$$

Where:

n: number of observation, $h_m$ : measured head and $h_s$ : simulated head.

**Table 2. Comparison between observed and calculated heads in steady state in MODFLOW.**

| Well No. | Simulated head (m) | Observed head (m) | Absolute difference between simulated and observed head (m) |
|---|---|---|---|
| 1 | 25.12 | 25.5 | 0.37 |
| 2 | 37.10 | 36.7 | 0.40 |
| 3 | 33.68 | 33.4 | 0.28 |
| 4 | 62.09 | 62.3 | 0.21 |
| 5 | 56.11 | 56.4 | 0.29 |
| 6 | 43.21 | 43.3 | 0.09 |
| 7 | 47.48 | 47.6 | 0.12 |
| 8 | 18.57 | 18.4 | 0.17 |
| 9 | 72.03 | 72.2 | 0.17 |
| 10 | 28.49 | 28.1 | 0.39 |

Mean absolute error : 0.249

Root mean squared error : 0.312

**Table 3. Comparison between observed and calculated heads in steady state in MICROFEM**

| Well No. | Simulated head (m) | Observed head (m) | Absolute difference between simulated and observed head (m) |
|---|---|---|---|
| 1 | 25.74 | 25.5 | 0.24 |
| 2 | 36.82 | 36.7 | 0.12 |
| 3 | 33.55 | 33.4 | 0.15 |
| 4 | 62.42 | 62.3 | 0.12 |
| 5 | 56.41 | 56.4 | 0.01 |
| 6 | 43.35 | 43.3 | 0.05 |
| 7 | 47.68 | 47.6 | 0.08 |
| 8 | 18.47 | 18.4 | 0.07 |
| 9 | 72.32 | 72.2 | 0.12 |
| 10 | 28.27 | 28.1 | 0.17 |

Mean absolute error : 0.113

Root mean squared error : 0.129

After the steady state calibration was achieved, unsteady state calibration was undertaken to calibrate the aquifer storage and direct recharge. The unsteady state calibration results were evaluated by comparing the temporal variation in simulated heads with those of observed ones at ten observed wells. The calibration process shows that the best value of direct recharge as a percentage from rainfall is equal to 10%.

## 5. RESULTS AND DISCUTION

Figs. 5 to 14 show comparisons between head values simulation by MODFLOW and MICROFEM models and those observed in the ten wells in study area. Fig. 5 shows a comparison between observed and simulated head for well No.1. From this Fig. it can be show that the observed head values are low during the pumping duration between (April – September), while they rise during the pumping duration between (October –march), because the low rainfall in the first pumping duration the values of simulation head are not increase and in the second period the simulation head increase because the rainfall increase and the location of well No.1 in the study area are near from well No.3 and No.6 and the elevation of well No.1 less than from this wells the increase in wells No.3 and well No.6 lead to increase in well No.1.

Fig. 6 shows that the difference in hydraulic head values between observed and simulated values increased during the pumping duration between (October – March). The value of mean absolute error of well No.2 equal to 0.163. This value indicates a relatively good agreement between observed and simulated head values. The reason behind the difference between observed and simulated head can be assumed distributed of wells and other hydrogeologic characteristics of the aquifer.

Fig. 7 shows a good degree of agreement between observed and simulated hydraulic head values for well No.3 (the mean absolute error 0.057). Both observed and simulated head values have been affected by the rainfall.

Fig. 8 shows a comparison between observed and simulated heads for well no.4. This well is located near well No.5 and the elevation of well No.4 is greater than well No.5 and in Fig. 15 shows the direction of flow vector from well No.4 to well No.5 because the elevation. The mean absolute error equal to 0.050 and good degree of agreement between observed and simulated head, the percentage of rainfall that percolate to groundwater can be effect on the hydraulic head.

Fig. 9 shows that the simulated head increase slightly during the period (April- October) because the rainfall in this period are very little and because the elevation of well No.5 is low than well No.4 the direction of flow vector from well No.4 to well No.5 as shown in Fig. 15. in the second period the simulated head increase and the mean absolute error equal to 0.055 this value refer to good agreement between observed and simulated hydraulic head.

Fig. 10 shows a comparison between observed and simulated head for well No.6. From this figure it can be shown that the simulation head during the period (April – October) increase slightly because the location of well no.6 near well no.5 and elevation of well No.6 is less than No.5 as shown in Fig. (15) the flow vector direct from well No.5 to well No.6 and in this period the rainfall are low and low number of well abstraction are not effect on the hydraulic head . In the second period (November – March) the observation and simulation head increase because the rainfall increase in this period.

Fig. 11 shows a comparison between observed and simulated hydraulic head values were obtained for well No.7 (the mean absolute error equal to 0.045). The simulation head values during the period (April – October) increase slightly and increase during the period (November –March) because the rainfall increase.

Fig. 12 shows a comparison between observed and simulated heads for well no.8. the elevation of well No.8 is less than other wells in the steady area , during the period (April – October ) the simulated head increase slightly because the low rainfall and low elevation of this well (the mean absolute error equal to 0.087 ).

Fig. 13 shows a good degree of agreement between observed and simulated hydraulic head values were obtain for well No.9. (the mean absolute error equal to 0.031 ). Both observed and simulated head values have been affected by the rainfall, the elevation of this well is greater than other wells in the study area lead to decrease the simulated head during the period (April – October). The simulated head increase when the rainfall increase during the period (November – March).

Fig. 14 shows a good degree of agreement between observed and simulated hydraulic head values were obtain for well No.10. (the mean absolute error equal to 0.02). This value refer to good similarity between simulated and observed hydraulic head. The simulated head have been affect by the rainfall and location the well in the study area.

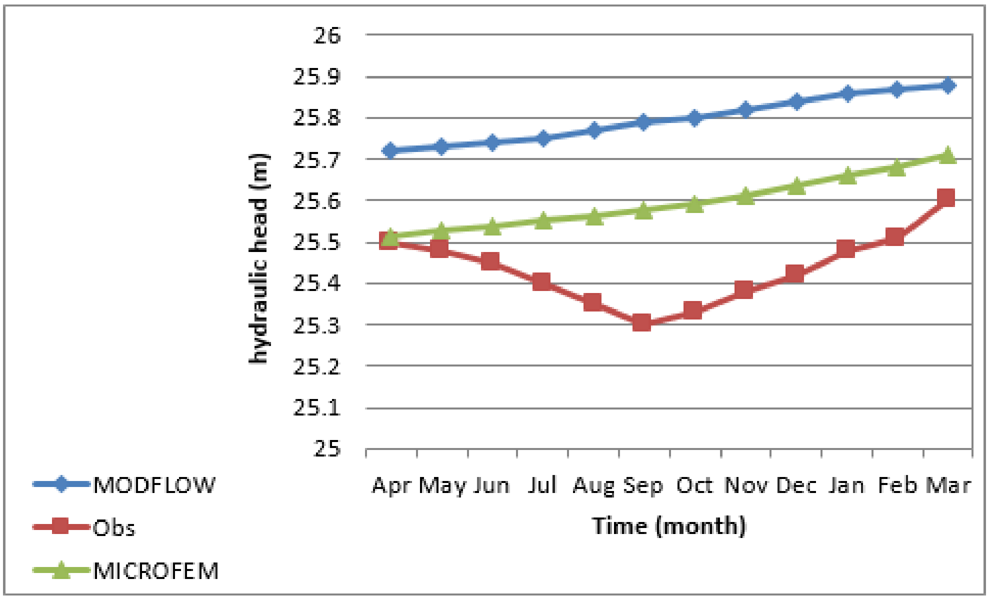


**Fig. 5. Comparison between observed and simulated heads (in both models) for well No.1**

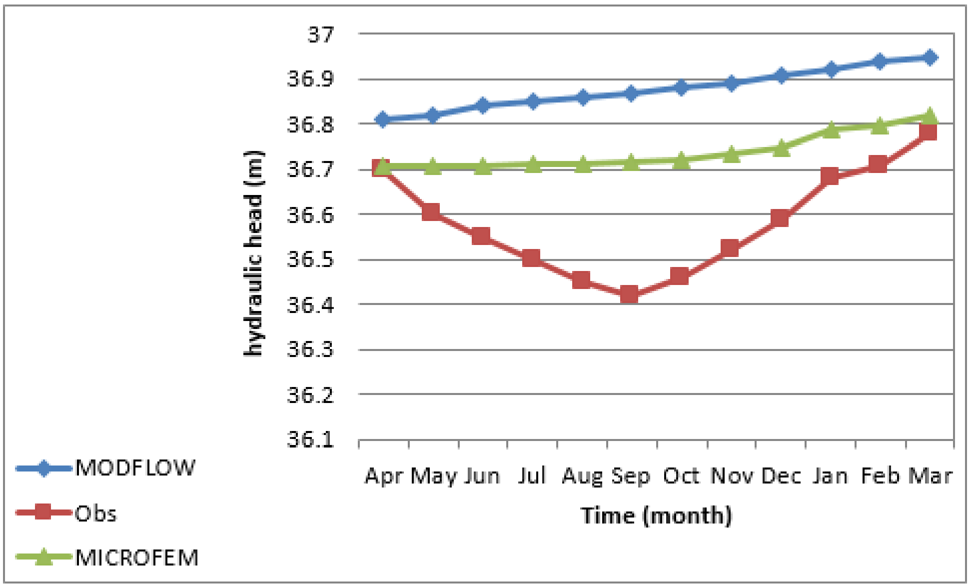


**Fig. 6. Comparison between observed and simulated heads (in both models) for well No.2**

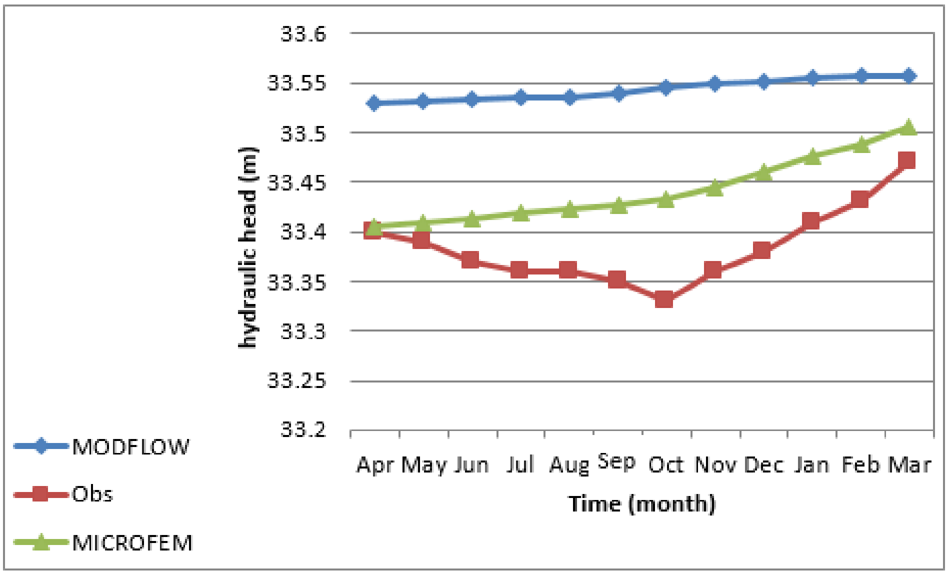


**Fig. 7. Comparison between observed and simulated heads (in both models) for well No.3**

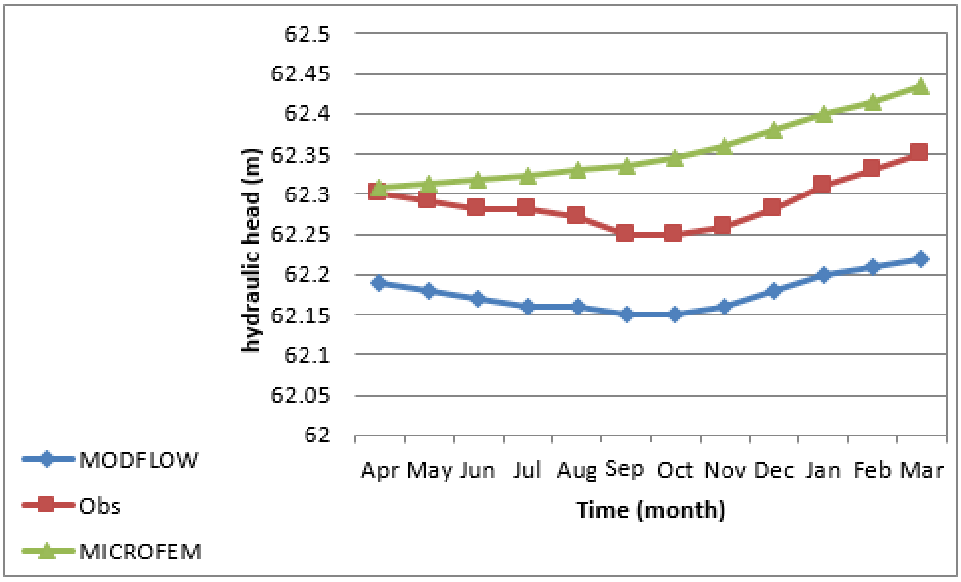


**Fig. 8. Comparison between observed and simulated heads (in both models) for well No. 4**

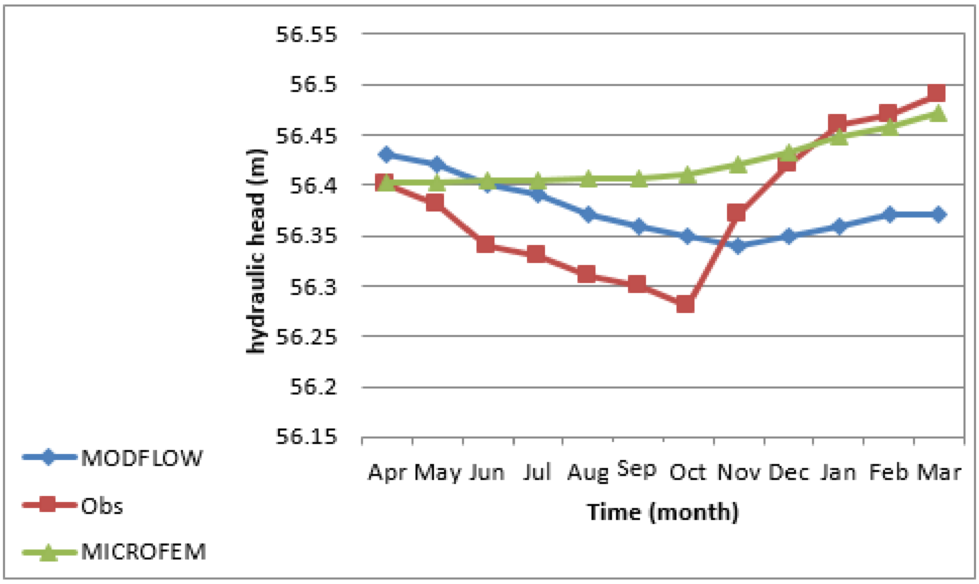


**Fig. 9. Comparison between observed and simulated heads (in both models) for well No. 5**

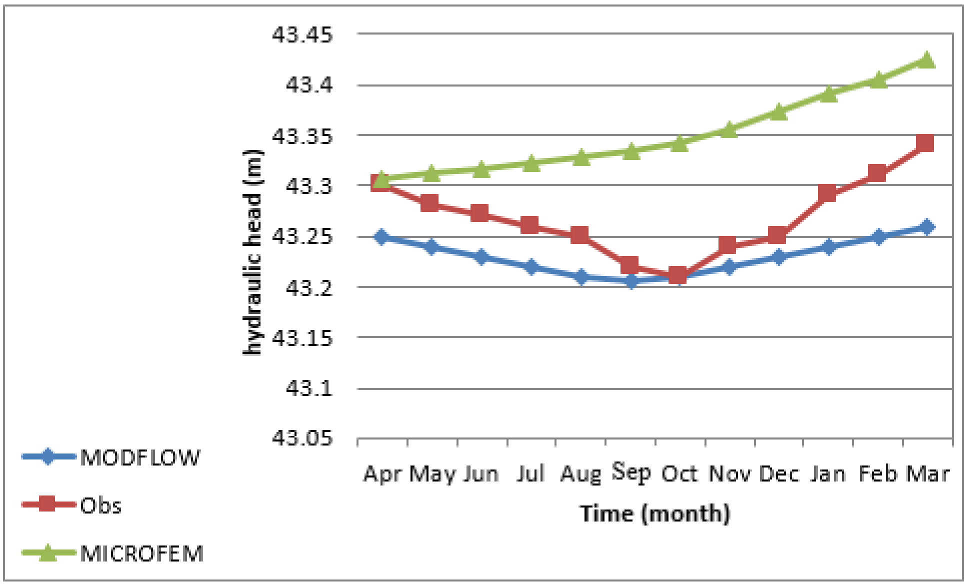


**Fig. 10. Comparison between observed and simulated heads (in both models) for well No. 6**

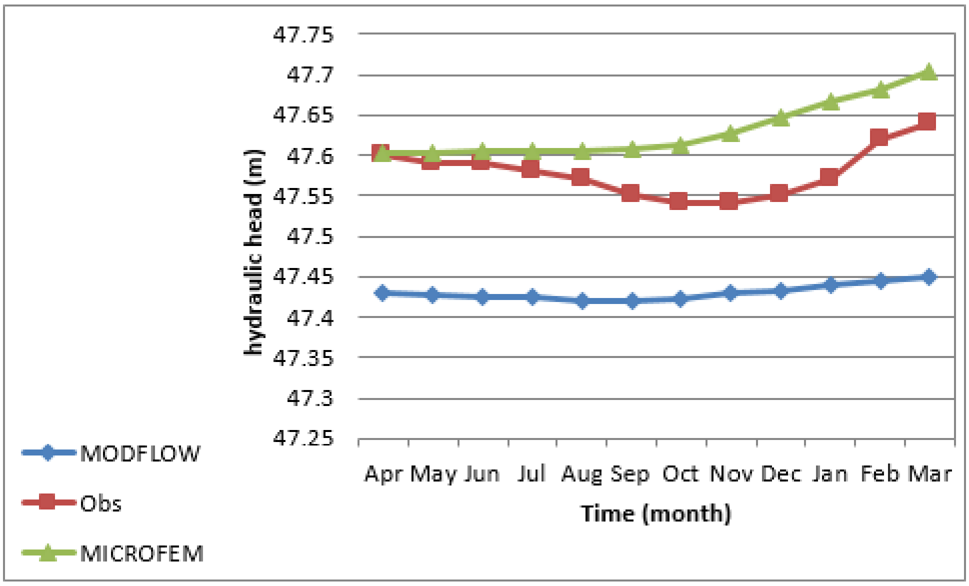


**Fig. 11. Comparison between observed and simulated heads (in both models) for well No. 7**

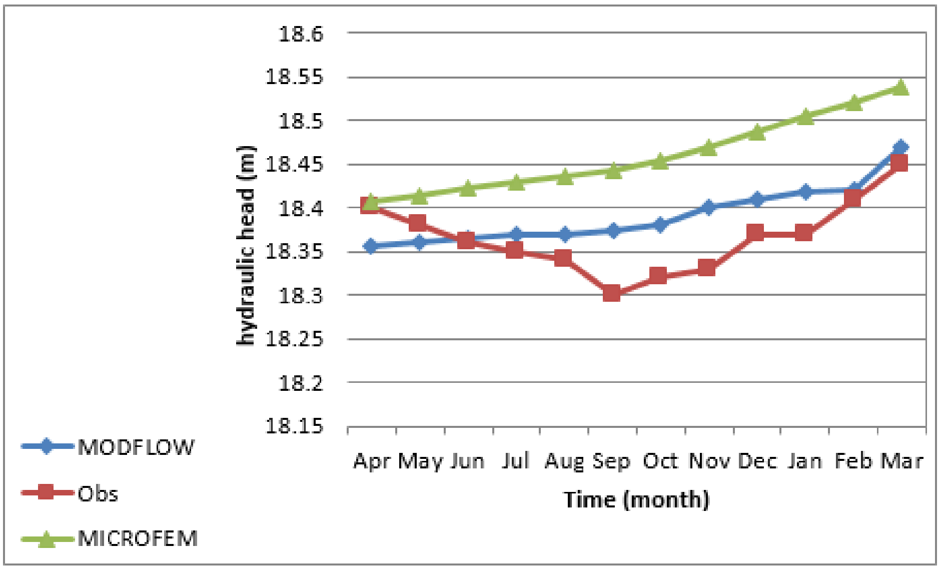


**Fig. 12. Comparison between observed and simulated heads (in both models) for well No. 8**

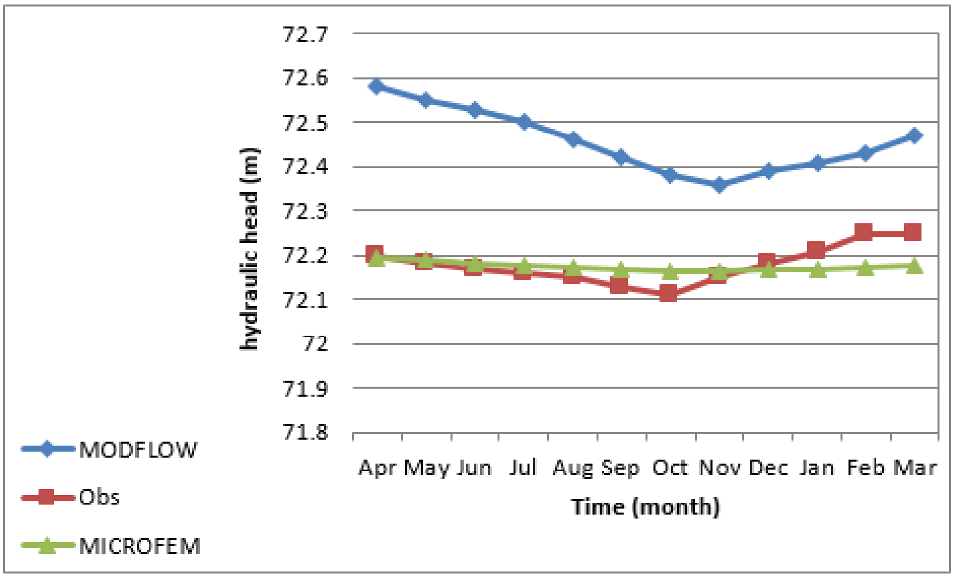


**Fig. 13. Comparison between observed and simulated heads (in both models) for well No.9**

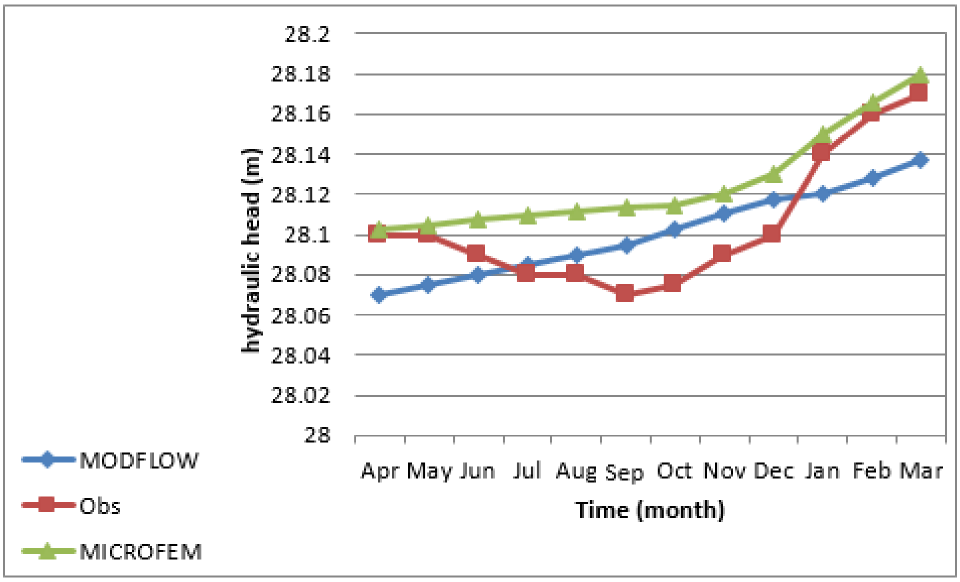


**Fig. 14. Comparison between observed and simulated heads (in both models) for well No.10**

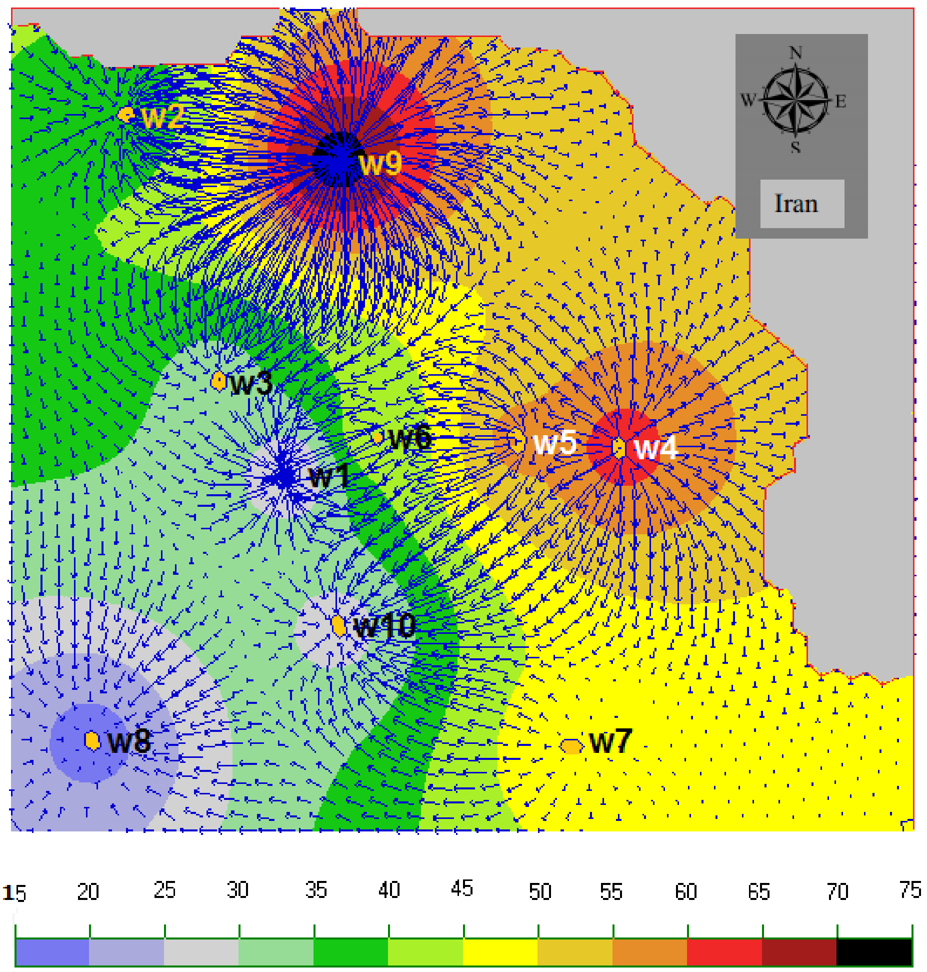


**Fig. 15. Distribution of the calibrated hydraulic head and flow vector**

## 6. CONCLUTION

MODFLOW and MICROFEM are capable for simulating groundwater flow of Teeb area in Missan province with reasonable accuracy, and for both steady and unsteady flow conditions. Based on the calibration process and the comparison between actual values of groundwater hydraulic heads and calculated heads, the hydraulic characteristics (hydraulic conductivity and specific yield) are redistributed on the study area. Calibrated value of hydraulic conductivity and specific yield using MODFLOW and MICROFEM models for both steady and unsteady states are in the order of (1-10) m/day and (0.1-0.4) respectively. For steady state condition, the mean absolute errors between actual hydraulic heads values and calculated values are 0.249 and 0.133 for MODFLOW and MICROFEM, respectively. For unsteady state condition, the mean absolute errors are 0.025 and 0.02 for MODFLOW and MICROFEM, respectively. The results of MODFLOW and MICROFEM show that the flow in the study area from northeast to southwest.

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