



FLOW PATTERN, PRESSURE DROP, AND VOID FRACTION IN VERTICAL TWO COMPONENT TWO-PHASE PIPE FLOW

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Received: 3 / 11 / 2013

Accepted: 23 / 6 / 2014

Abstract

Two-phase flow pattern, pressure drop, and void fraction in vertical transparent pipe of 0.0254 m internal diameter and 3.65 m length is investigated experimentally. The rig is designed to achieve the measurements of pressure drop and void fraction for different combinations of phase superficial velocities such that the regimes encountered are bubbly, slug and annular, which required a wide range of water and air superficial velocities. The flow patterns are investigated by recording video movies for each test achieved. The pressure is measured by using five pressure sensors distributed through the pipe, while the void fraction values are measured by using two quick closing valves at pipe terminals. The effect of heating liquid to 60C° on the pressure drop values is also discussed. The results are compared with many pressure drop and void fraction correlations. The pressure drop results in bubbly flow regime are nearest to the Lockhart and Martinelli correlation with an average difference of 3.93 %., while in slug and annular flow regimes the results are well predicted by Steinhagen and Heck correlation with an average difference of 18.4 % and 26%. Hughmark correlation is the nearest to the void fraction results in bubbly flow regime with an average difference of 2.4% ,while Chisholm and Smith correlations are the best in slug and annular flow regimes with an average difference of 6.91% and 1.71% respectively.

Keywords: Two-Phase Flow, Vertical Pipe, Void Fraction, Pressure Drop, Flow Pattern.

هيئة الجريان, انخفاض الضغط و نسبة الهواء لجريان ثنائي الطور دون الوصول إلى درجة الغليان في أنبوب عمودي

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الخلاصة

تم دراسة شكل نظام الجريان, انحدار الضغط ونسبة الهواء لجريان ثنائي الطور (هواء-ماء) داخل أنبوب شفاف بقطر داخلي يساوي 0.0254 م وطول يساوي 3.65 م عمليا". أجريت هذه الاختبارات من خلال تصميم منظومة متكاملة. تم الحصول على عدة أشكال لأنظمة الجريان ثنائي الطور وهي: الجريان الفقاعي , الدفعي , والحلقي الأمر الذي تطلب استخدام مدى واسع من السرعات لطوري الجريان. الهدف من البحث هو اختبار شكل الجريان من خلال تصويره بواسطة كاميرا رقمية وتم تحويل تلك الأفلام إلى صور متتابعة لتجسيد أشكال الجريان. كما يهدف إلى قياس الضغط باستخدام خمس مجسات للضغط وزعت على طول الأنبوب. بينما نسبة الهواء فقد تم إيجادها باستخدام صمامين سريعي الغلق وضعا في طرفي الأنبوب. وقد تمت دراسة تأثير تسخين الطور السائل على قيم الضغط التي تم قياسها إذا ما قورنت بقيم الضغط عند نفس سرع الغاز والسائل ولكن من غير تسخين. وجد من خلال التجربة إن هبوط الضغط يتناسبان طرديا" مع سرعة الماء وعكسيا" مع سرعة الهواء في نظامي الجريان الفقاعي والدفعي. أما في نظام الجريان الحلقي فان سرعتي الماء



والهواء يتناسبان طردياً" مع هبوط الضغط . كما إن تسخين الماء يؤدي إلى زيادة قيمة الضغط إذا ما قورنت مع قيمته في حالة استخدام نفس سرعة الماء والهواء ولكن من غير تسخين الطور السائل. كما وجد إن نسبة الهواء في كل أنظمة الجريان المدروسة تتناسب طردياً مع سرعة الهواء. تم مقارنة النتائج العملية مع عدة علاقات تجريبية لانحدار الضغط ونسبة الهواء ووجد إن علاقة Lockhart and Martinelli هي الأفضل لإيجاد انحدار الضغط في نظام الجريان الفقاعي بنسبة اختلاف تصل إلى 3,93% بينما علاقة Steinhagen and Heck هي الأمثل في نظامي الجريان الدفعي والحلقي بنسب اختلاف تصل إلى 18% و 26%. كما وجد إن علاقة Hughmark هي الأفضل لحساب نسبة حجم الهواء لنظام الجريان الفقاعي بنسبة اختلاف تصل إلى 2,4% بينما علاقة Chisholm هي العلاقة الأمثل في حالة الجريان الدفعي و علاقة Smith هي الأقرب للنتائج العملية في حالة الجريان الحلقي بنسب اختلاف تصل إلى 6,91% و 1,71% بصورة متتابة.

1. Introduction

Two-phase has a continuing interest in engineering situations. It occurs extensively throughout industries such as tubular boilers, reboilers, oil and geothermal wells, gas and oil transport pipelines, refrigerators, heat exchangers and condensers. It is widely encountered in petroleum, chemical, civil and nuclear industries [Ghajer 2004]. The ability to quantify void fraction and pressure drop are of considerable importance in systems involving two-phase flow. In addition, void fraction plays an important role in the modeling of two-phase pressure drop, flow pattern transition, and heat transfer and it is the key physical parameter for determining other two-phase parameters, namely two-phase density, and gas and liquid velocities [Zhao 2005]. Many studies on two-phase, gas-liquid flow were published through horizontal, vertical, and inclined pipes. Some of these studies were concerned with finding an experimental data. Other, they found analytical relations or used these experimental data to drive empirical correlations, and the others compared many of the pressure drop and void fraction correlations to select the best one which able to find these parameter at a specific flow regime [Noora 2013]. None of these researchers studied the effect of heating at one of the two phases on pressure values. Also most of the researchers used a two pressure transducers or manometers to calculate the pressure drop.

In this work water was heated separately and mixed with the air in the mixing chamber to find the effect of water heating on the pressure values. The pressure was measured by using five sensors distributed along the test pipe to give a more realistic readings, and to reduce errors. The void fraction was also measured by using two quick closing valves. The two-phase flow pattern visualized by using digital camera. The recorded video was districited by video converter program. Experimental results are compared with many correlations.

2. Experimental Apparatus and Procedure

Experiments were carried out in a smooth transparent circular pipe of 0.0254 m internal diameter and 3.65m length. The schematic description of the experimental facility is presented in Figure 1 which had been built at the fluid mechanics lab. in engineering college Babylon university. The water and air supply systems work independently. The water was supplied from the water reservoir (1) to the test section (21) by a pump (3) which had been manufactured by Hitachi Ltd. with maximum head of 5m , maximum discharge of 0.2 m³/min and power of 0.4kW. The water flow rate was controlled by a flow regulator (10) and measured by water flow meter (5) which has a range of 0.002 to 0.030 m³/min, and then the measured values were calibrated. The air was supplied by a compressor (13) (which was designed by Ingersoll-Rand Company and has maximum rate of 150 L/s, maximum pressure of 16.5 bar and operating temperature of -10 to 120 °C) through the air reservoir (12) and

pressure regulator (11). The air flow rate was regulated by a valve (10) and measured by air flow meter (9) (two air flow meters were used in order to get three types of flow regimes. The first has a range of 0.006 to 0.050 m³/min for bubble and slug flow, the other has a range of (0.0834 to 0.834) m³/min for annular flow), the measured values were calibrated. The water and the air were mixed together in chamber (7). The mixture passed through the test section (21). For each run video movie was recorded the flow pattern with time about 30 seconds. The pressure in test section had been measured by the pressure sensors (16) which are converted the pressure into an electrical signal of 4 to 20 mA and they have a range of 0 to 0.6 bar. The results were collected by the data logger (17) which is used to make log of analog signals by converting them to digital signals. This device is worked with the help [DALI 08](#) software. The pressure values were read every second, then the total time of the test was about 30 seconds. The average of these 30 readings was calculated for each air and water flow rates. The results were stored in the personal computer hard. The first sensor is located at a distance of 60 cm from test section edge. The second, third and forth are located at a distance of 60 cm from each other, while the distance between the fourth and fifth is 90 cm. Then the measuring of pressure drop was repeated for another condition, when heating the water until 60 C° in order to show the effect of heating on these readings. The water was recirculated by recirculation pipe and collected in the accumulation tank. The recirculation pump (20) delivered the water from the accumulation tank(19) to the water reservoir (1). These flow patterns were obtained depending upon the values of air and water flow rate according to the flow map which was detailed by [\[Ghajer 2004\]](#) as shown in [Figure 2](#).

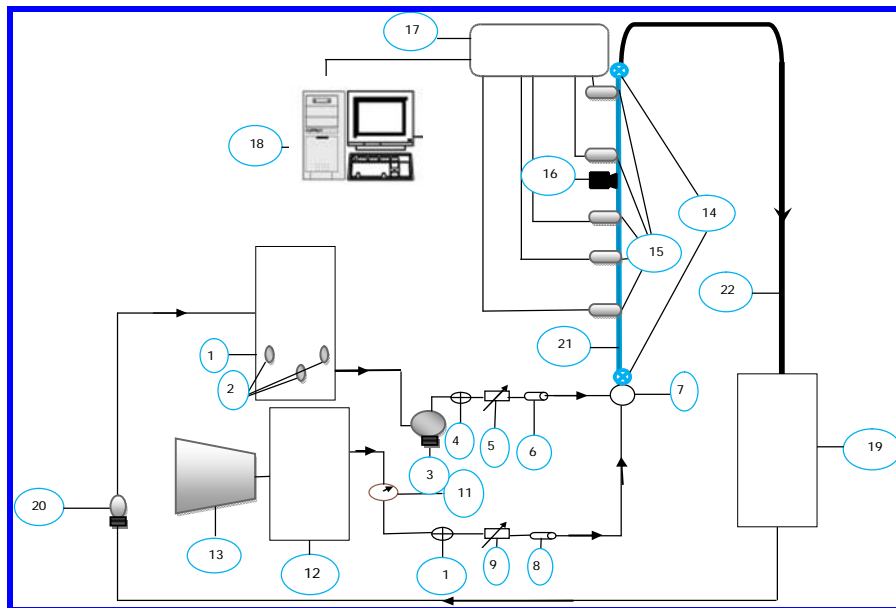


Fig. 1: Schematic Diagram of Rig.

(1)Water reservoir (2)Heaters (3)Pump (4)Water flow regulator (5)Water flow meter (6)Water check valve (7)Mixing chamber (8)Air check valve (10)Air flow regulator (11)Pressure regulator (12)Air reservoir (13)Compressor (14)ball valve (15)pressure sensor (16)Digital camera (17)Data logger (18) Personal computer (19)Accumulation tank (20) Recirculation pump (21)Test section (22)Recirculation pipe.

2.1 Experimental Analysis

The void fraction was measured by using two ball valves situated at the test section terminals. The two valves were closed at the same time to trap the water and the air in the test section. The trapped air released and the water was collected to measure its volume. The volume of the test section between the two valves was given by equation:

$$V = \frac{\pi}{4} D^2 L_{1 \rightarrow 5} \dots \dots \dots \text{eq.(1)}$$

So, the void fraction can be calculated by:

$$\alpha = \frac{V_G}{V_G + V_L} \dots \dots \dots \text{eq.(2)}$$

The pressure drop was calculated by fixing the first sensor reading as reference. Then, the reading of the other sensors was subtracted from this reference reading. The second sensor from the first, the third from the first, the fourth from first, and the fifth from the first through each test run. The difference percentages between the experimental results and empirical results are given by the following equation:

$$DP = \frac{\text{experimental value} - \text{empirical value}}{\text{experimental value}} 100\% \dots \dots \dots \text{eq.(3)}$$

While the average absolute difference percentage values are given by following equation as:

$$AADP = \frac{1}{n} \sum_{i=1}^n |DP_i| \dots \dots \dots \text{eq.(4)}$$

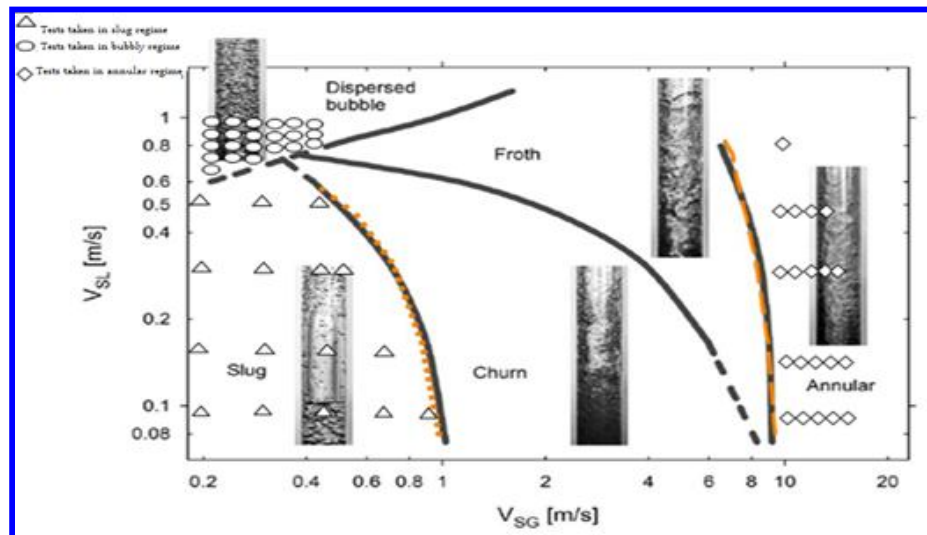


Fig.2:Flow Regime Map for Vertical Pipe. [Ghajar 2004].

3. Results and Discussions

Figure 3 visualizes bubbly flow regime at different operating conditions. **Figure 3a** shows the flow pattern at very small quantity of air. The flow looks as continuous water for a while, then small quantity of air bubbles appear. In fig. 3b the same thing is seen but the bubbles appear faster than in fig. 3a. This is due to the slightly increase in gas superficial velocity. **Figure 3c** shows the bubbles appear from the beginning of the frame but this is not necessary mean that these bubbles appear from the beginning of the test pipe because the camera was situated at 1.5 m from the pipe inlet. In **Figure 3d** it can be observed the bubble's quantity increases and the bubbles size is smaller than in the previous Figures. In **Figures 3e and 3f** the bubbles quantity is increased and the bubbles size is being very fine.

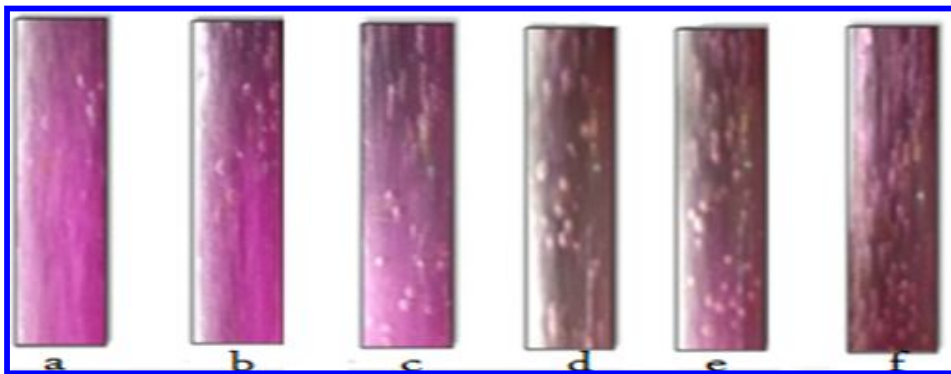


Fig.3 : Flow Patterns for Bubbly Flow. ($V_{SL}=0.888\text{m/s}$ at a-f, a- $V_{SG}=0.197\text{m/s}$, b- $V_{SG}=0.263\text{m/s}$, c- $V_{SG}=0.328\text{m/s}$, d- $V_{SG}=0.394\text{m/s}$, e- $V_{SG}=0.526\text{ m/s}$, f- $V_{SG}=0.594\text{ m/s}$).

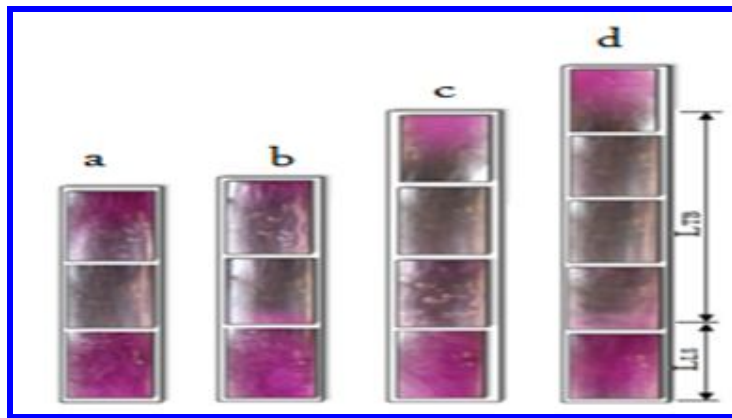


Fig.4 : Flow Patterns for Slug Flow($V_{SL}=0.164\text{m/s}$ at a-d, a- $V_{SG}=0.197\text{m/s}$, b- $V_{SG}=0.263\text{m/s}$,c- $V_{SG}=0.493\text{m/s}$, d- $V_{SG}=0.675\text{m/s}$).

Figure 4 demonstrates the slug flow regime. The first consequent frames which make a complete slug unit are displayed in this flow regime. In **Figure 4a** the first frame represents the liquid slug region. The second and part of the third frame represent the Taylor bubble region. A small value of increasing in Taylor bubble length can be noticed in **Figure 4b** due to the slightly increase in superficial gas velocity. In **Figure 4c** the increasing in Taylor

bubble region is very obvious due to the relatively large increase in superficial gas velocity. The first frame and small part of the second frame represent the liquid slug region in **Figure 4d**, while the third, fourth and parts of the second and fifth frames represent the Taylor bubble region. It can be observed from these figs., the liquid slug approximately is constant, and it is not affected by the change of gas superficial velocity.

Figure 5 illustrates the annular flow regime. The gas core can be seen obviously in fig.5a because the liquid film is very thin due to the small value of superficial liquid velocity. In fig.5b the liquid film thickness is greater than in the previous figure but the gas core also can be recognized. Due to the increase in liquid film thickness the gas core cannot be recognized. In **Figures 5d and 5e** there is a gradual change in the liquid color due to the change in liquid thickness. The gas core cannot be recognized absolutely, and the flow looks like a continuous colored liquid.

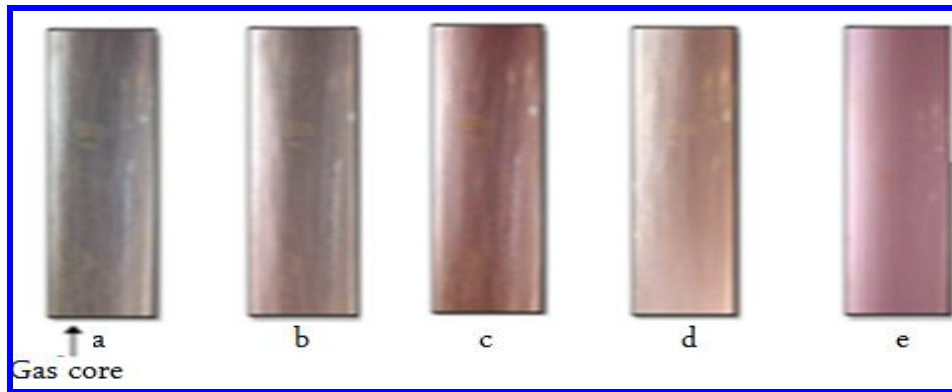


Fig.5 : Flow Patterns for Annular Flow. ($V_{SG}=9.98\text{m/s}$ at a-e, a- $V_{SL}=0.164\text{m/s}$, b- $V_{SL}=0.328\text{m/s}$, c- $V_{SL}=0.493\text{m/s}$, d- $V_{SL}=0.675\text{m/s}$, e $V_{SL}=821\text{m/s}$).

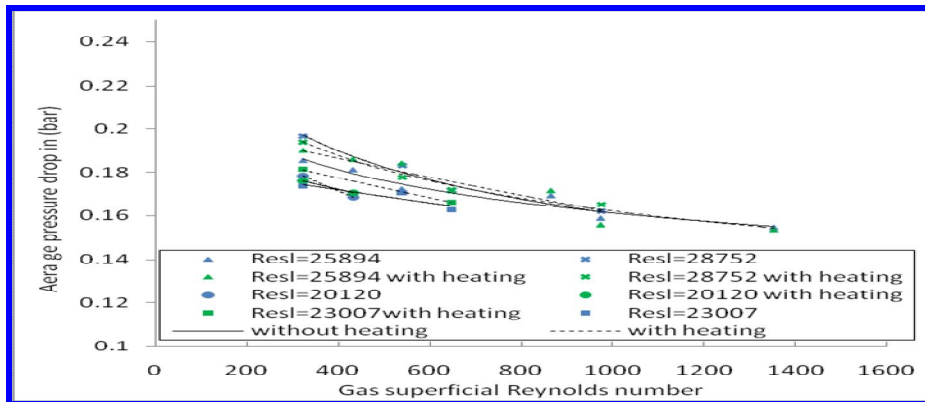


Fig.6 : The Relation Between the Average Pressure Drop and Gas Superficial Reynolds in Bubbly Flow Regime.

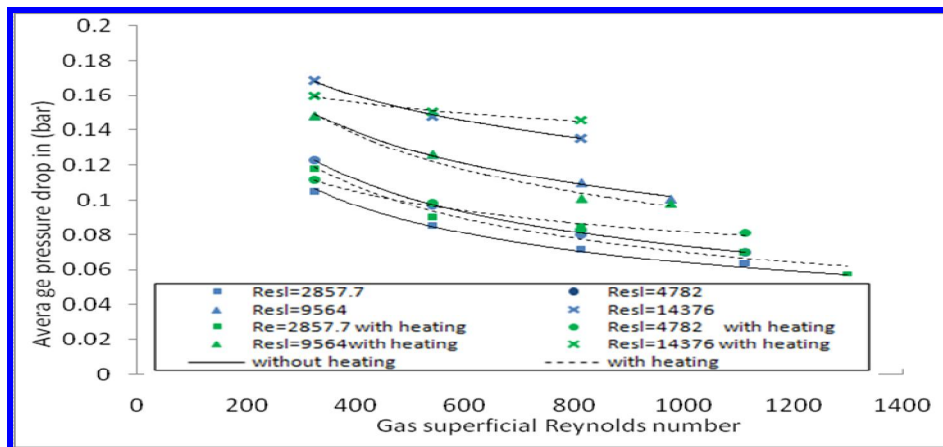


Fig.7: The Relation Between the Average Pressure Drop and Gas Superficial Reynolds in Slug Flow Regime.

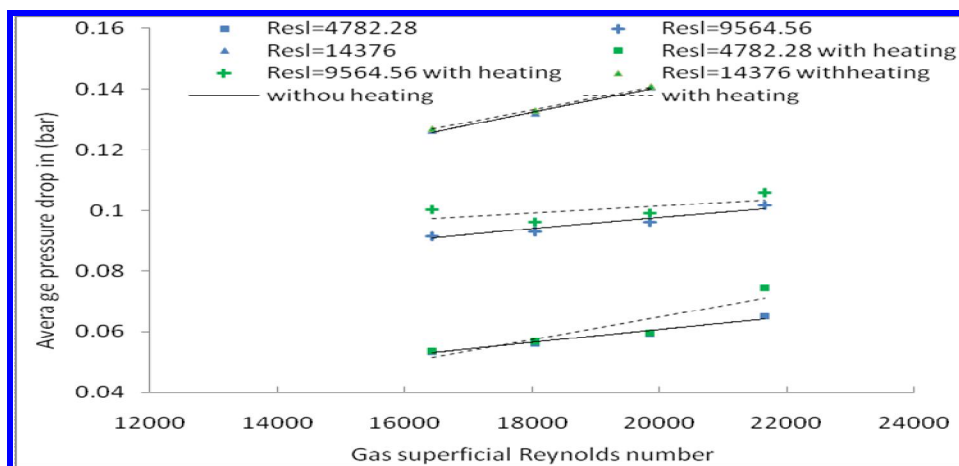


Fig. 8: The Relation between the Average Pressure Drop and Gas Superficial Reynolds in Annular Flow Regime.

Figures 6 to 8 illustrate the average pressure drop through the length of test pipe with gas superficial Reynolds number. These values were calculated in two cases of water before it was mixed with air. The first case, water was mixed with the air at room temperature, and when the water heated to 60 C°. **Figures 6 and 7** represent this relation at bubbly and slug flow regimes respectively. In these flow regimes an inverse power relationship was obtained. As the gas superficial Reynolds number increases the void fraction increases which will decrease the two phase density and the elevation pressure drop, when the liquid superficial Reynolds number increases the void fraction decreases and the total two phase density increases which will increase the elevation pressure drop, which is the main component in these two flow regimes. **Figure.8** shows the same relation for annular flow regime. The

relation is a direct upward power relation. As the superficial gas Reynolds number increases the pressure drop increases. This behavior is seen in this flow regime because the increase in gas superficial Reynolds number increases the frictional pressure drop, which is the major pressure drop components. Due to the large gas superficial Reynolds numbers used in annular flow regime, the void fraction is large and the two phase density is less than the values in bubbly and slug flow regimes, therefore; the elevation pressure drop effect will be minor. As gas and liquid superficial Reynolds numbers increase the frictional pressure drop increases.

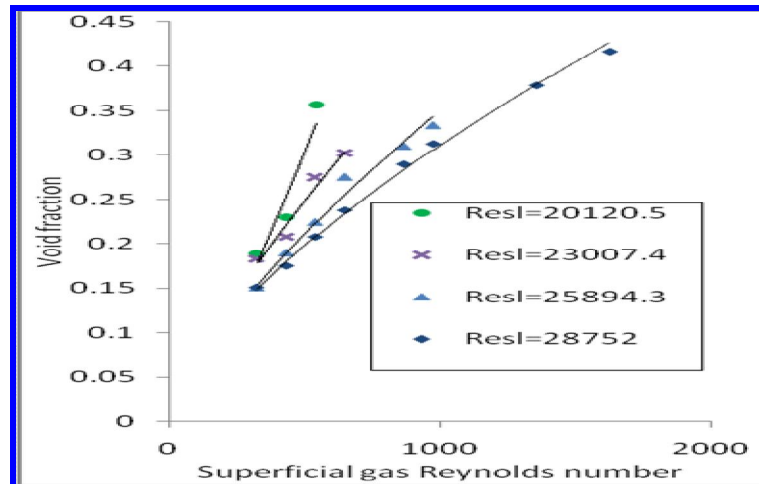


Fig.9 : Relationship Between the Void Fraction Values and Re_{SG} for Bubbly Regime.

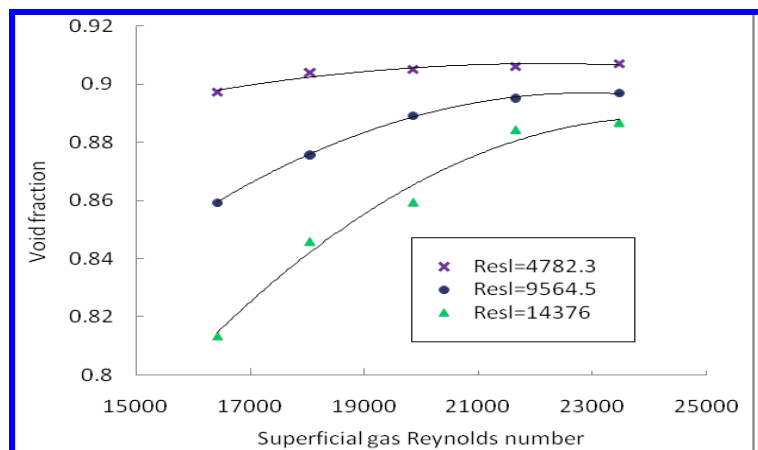


Fig.10 : Relationship Between the Void Fraction Values and Re_{SG} for Slug Regime.

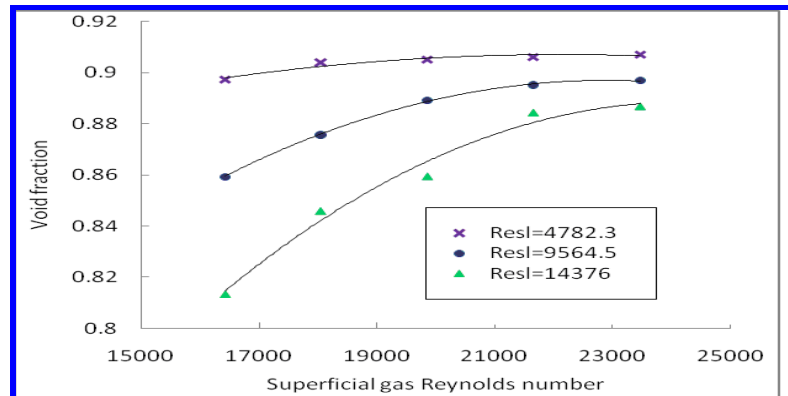


Fig.11: Relationship between the Void Fraction Values and Re_{SG} for Annular Regime.

Figures 9 to 11 demonstrate the relation between the void fraction and gas superficial Reynolds number. Heating of liquid does not have considerable effect on the void fraction values in all flow regimes discussed. So, the void fraction curves in case of liquid heating were not plotted. **Figure 9** shows this relation in bubbly flow regime. In this flow regime the measured void fraction values ranged from 0.15 to 0.416 because of the small quantity of gas flow rate required to obtain bubbly regime. **Figure 10** displays the void fraction values in slug flow regime. They are ranged from 0.29 to 0.7 because of the higher gas flow rate used in this flow regime if it compared with the previous regime. **Figure 11** represents the values of void fraction in annular flow regime. They are ranged from 0.79 to 0.907 because of the very high gas volume flow rate values used to obtain annular flow regime which were reached to 26 m³/hr. The relation is a direct relation in all these figures. As the gas superficial Reynolds number increases the void fraction values increase. Due to the increase in gas volume. It can be also observed, as the liquid superficial Reynolds number increases the void fraction values decrease for the same value of gas superficial Reynolds number.

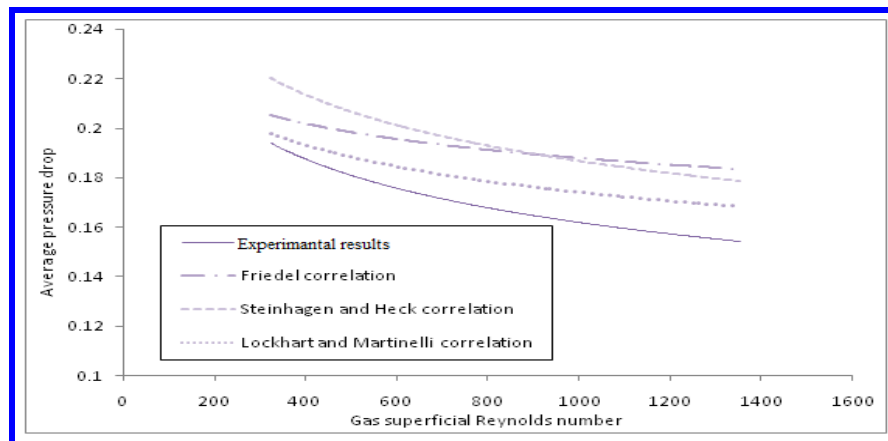


Fig.12 : Comparison Between the Experimental Results and Pressure Drop Empirical Correlations for Bubbly Flow Regime at $Re_{SL}=25894$.



Figs.12 to 14 represent the values of the experimental average pressure drop and the values obtained by [Friedel 1979], [Steinhagen and Heck1986], and [Lockhart and Martinelli 1949] pressure drop empirical correlations. These correlations were found by correlating the experimental results for frictional pressure drop component. In **Figure 12** the comparison is achieved in bubbly flow regime. The nearest correlation to the experimental values in this figure is the Lockhart and Martinelli correlation with average error of 3.93 %. **Figure 13** shows the comparison in slug flow regime. In this flow regime the average absolute error values were taken to diagnose the experimental results accuracy. Steinhagen and Heck correlation is the nearest with average absolute error (AAE) of 18.4 % .**Figure 14** represents the comparison in annular flow regime. In this flow regime the Steinhagen and Heck correlation also is the nearest with experimental results with an average absolute error (AAE) of 26%. The percentage errors were calculated with the whole experimental result not only the plotted values.

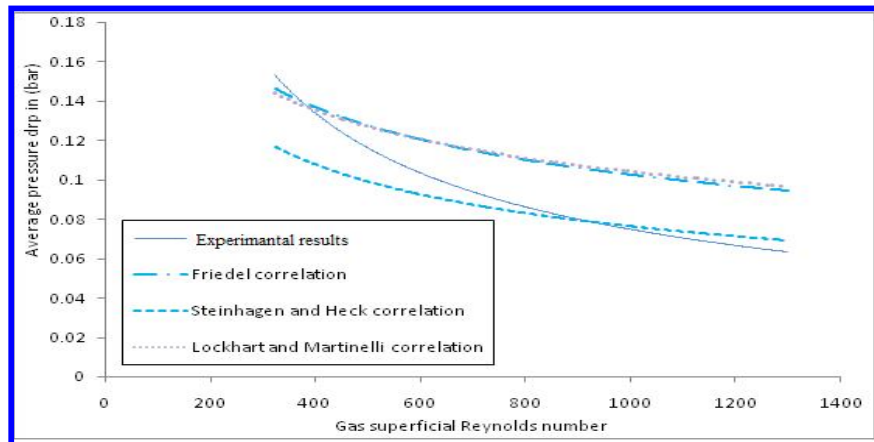


Fig.13: Comparison Between the Experimental Results and Pressure Drop Empirical Correlations for Slug Flow Regime at $Re_{SL}=2857$.

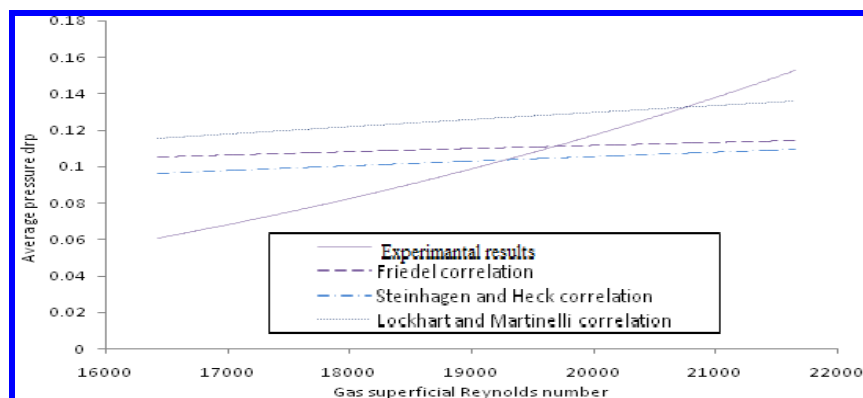


Fig.14; Comparison Between the Experimental Results and Pressure Drop Empirical Correlations for Annular Regime at $Re_{SL}= 9564$.



Figures 15 and 16 demonstrate the variation of the present experimental void fraction and that obtained by many void fraction correlations in bubbly flow regime with gas superficial velocity for liquid superficial velocity of 0.986 m/s. In Figure 15 the experimental results are compared with Hughmark [1962], [Bonnecaze et al. 1971], and [Chisholm 1973] empirical correlations. Hughmark correlation is the nearest with average error (AE) of 2.4% error. In Figure 16 the experimental results are compared with [Gregory and Scott 1969], [Smith 1969], and [Hoq and Loth 1982] correlations. Gregory and Scott correlation is the nearest to the experimental results with 2.7% average error value

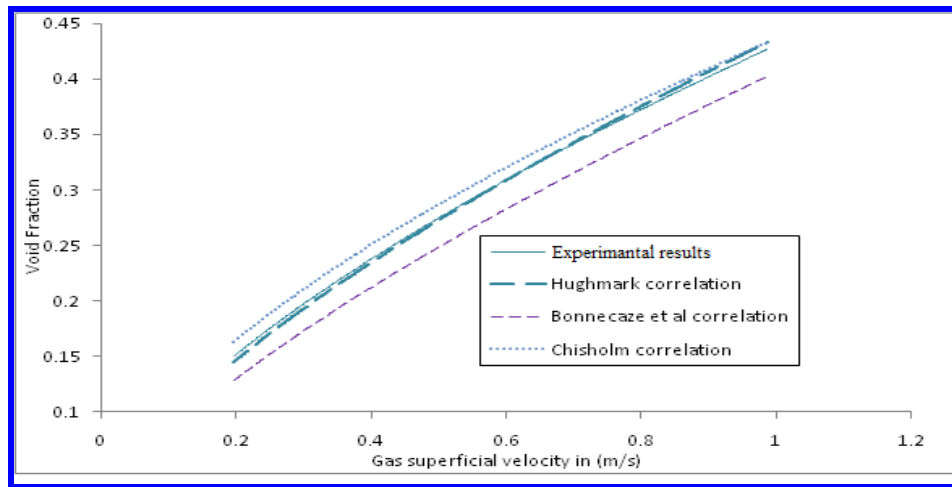


Fig.15: Comparison between the Experimental Void Fraction Results and That Obtained by Empirical Correlations in Bubbly Flow at $V_{SL}=0.986$ m/s.

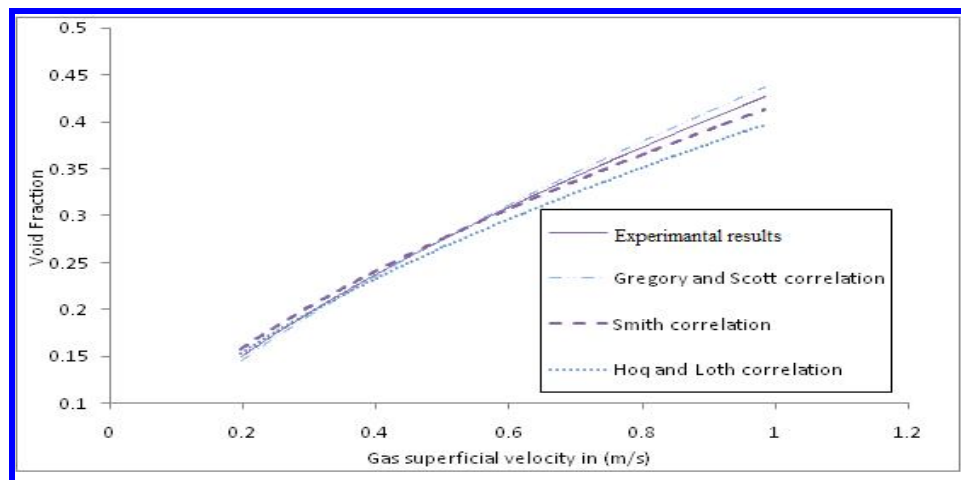


Fig.16: Comparison Between the Experimental Void Fraction Results and That Obtained by Empirical Correlations for Bubbly Flow at $V_{SL}=0.986$ m/s.



Figures 17 and 18 display the experimental void fraction values and that obtained by many void fraction correlations for slug flow regime at liquid superficial velocity of 0.098 m/s. In Figure 17 Hoq and Loth correlation is the nearest with average absolute error of 6.96 %. In Figure 18 the results obtained by Chisholm correlation are the nearest to the experimental results with average absolute error (AAE) of 6.91%. The results obtained by Bonnecaze et al. correlation are under the experimental results by an average absolute error of (AAE) of 19%. The Hughmark correlation makes an average absolute error of 6.94% above the experimental results.

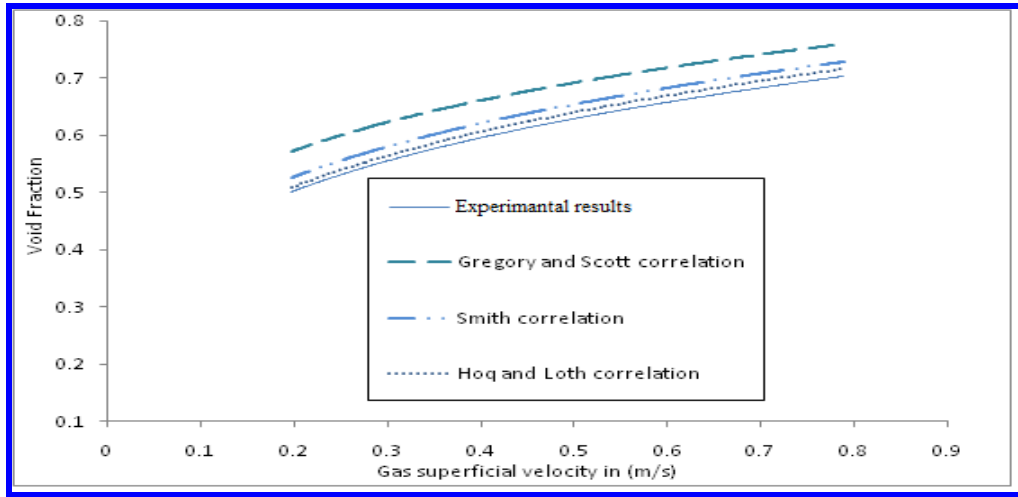


Fig.17 : Comparison Between the Experimental Void Fraction Results and That Obtained by Empirical Correlations for Slug Flow at $V_{SL}=0.098$.

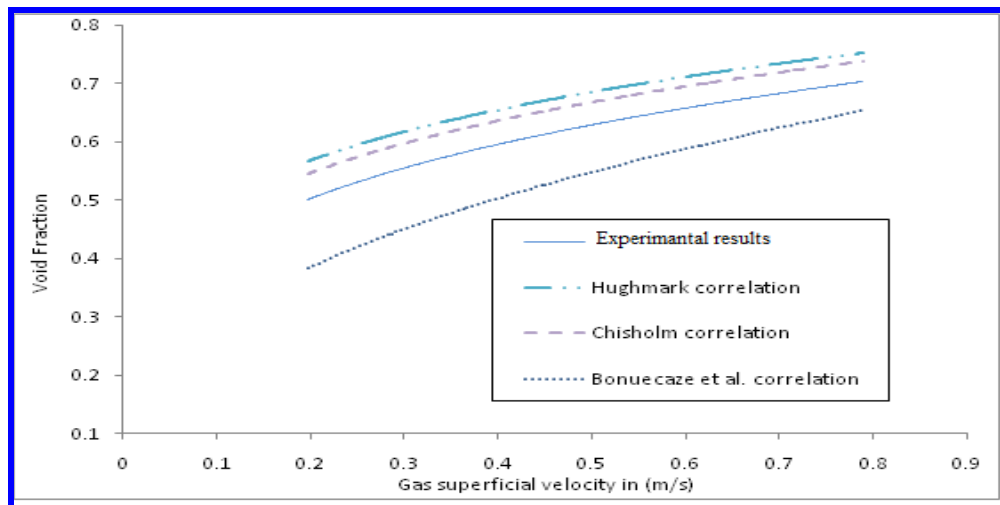


Fig.18 : Comparison Between the Experimental Void Fraction Results and That Obtained by Empirical Correlations for Slug Flow at $V_{SL}=0.098$.



Figures 19 and 20 illustrate the variation of the present experimental void fraction results with that obtained by many void fraction correlations for annular flow regime at liquid superficial velocity of 0.164 m/s. In Figure 19 the comparison achieved by Hughmark[6], Bonnecaze et al[7], and Chisholm[8] empirical correlations. Chisholm [8] correlation results are the nearest to that obtained experimentally with average absolute error of 1.9%. Figure 20 compares the experimental results with Gregory and Scott[10], Smith[11], and (Hoq and Loth) [12] correlations. Gregory and Scott results are under the experimental results with average absolute error of 7.09% while Smith[11], and (Hoq and Loth)[12] results are deviated by 1.7% and 2.6% respectively.

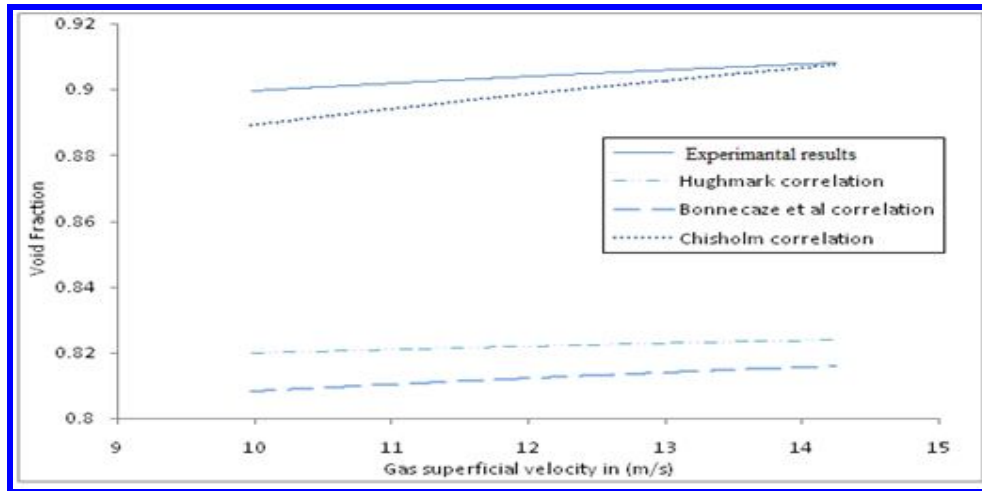


Fig.19: Comparison Between the Experimental Void Fraction Results and That Obtained by Empirical Correlations for Annular Flow at $V_{SL}=0.164$ m/s.

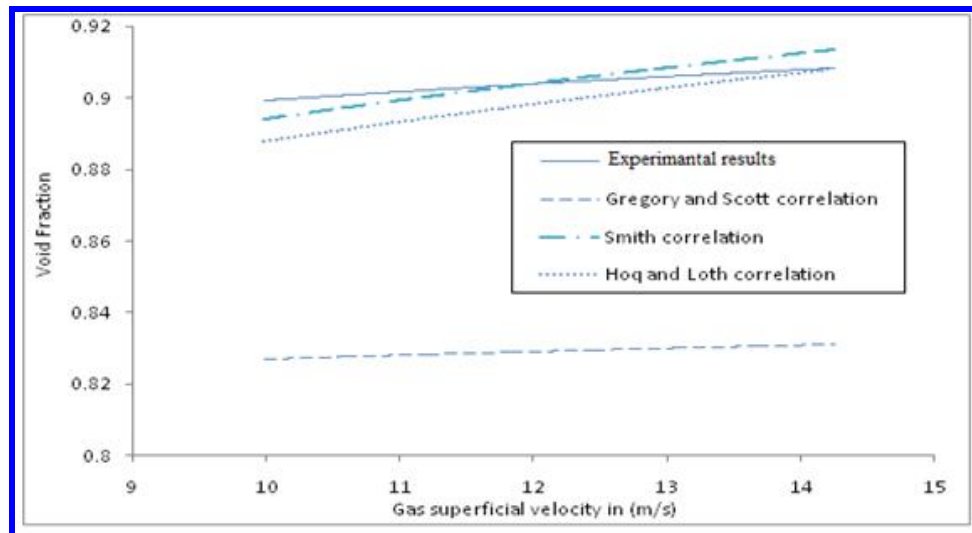


Fig.20: Comparison Between the Experimental Void Fraction Results and That Obtained by Empirical Correlations for Annular Flow Regime at $V_{SL}=0.164$ m/s.



1. Conclusions

It has been found that the measuring pressure drop through the distance of test pipe is proportional with the liquid superficial velocity in bubbly and slug flow regimes. While, it was inversely changed with gas superficial velocity. In annular flow regime, this relation was proportional with liquid and gas superficial velocities.

1. Heating of liquid phase was increased the measured values of pressure through the distance of rig pipe than the results without heating the liquid phase.
2. It has been found the void fraction proportional with gas superficial velocity but it is inversely changed with liquid superficial velocity in all flow regimes.
3. Through visual observations it can be observed that the bubble's quantity increases and the bubbles size decreases if the gas superficial velocity increases in bubbly flow regime.
4. The slug unit length in slug flow regime was proportional with the gas superficial velocity.
5. The liquid film thickness in annular flow regime was directly proportional to the liquid superficial velocity.
6. The experimental results are compared with many pressure drop and void fraction correlations. The pressure drop results in bubbly flow regime are nearest to the Lockhart and Martinelli correlation, while in slug and annular flow regimes the results are well predicted by Steinhagen and Heck correlation. Hughmark correlation is the nearest to the void fraction results in bubbly flow regime, while Chisholm and Smith correlations are the best in slug and annular flow regimes respectively.

Nomenclatures

AADP Average absolute difference percentage

DP Difference percentage.

D Pipe diameter

LLS Liquid slug region length

LTB Taylor bubble region length

ReSG Superficial Gas Reynolds number

ReSL Superficial liquid Reynolds number

VSG Superficial liquid velocity

VSL Superficial liquid velocity

$L_{1 \rightarrow 5}$ Pipe length between the first and the last sensor

Greek Letters

α Void fraction

v Volume

v_L Liquid volume

v_G Gas volume

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